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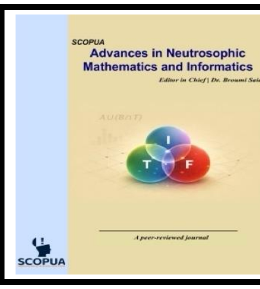

Cubic Spherical Neutrosophic Weighted Yager and Geometric Yager Aggregation Operators with Application in Groundwater Sustainability

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ABSTRACT

Decision-making problems involving uncertain and inconsistent information often require advanced aggregation techniques. In this paper, we propose new aggregation operators under the cubic spherical neutrosophic environment, namely the Cubic Spherical Neutrosophic Weighted Yager Aggregation Operator (CuSNWYAO) and the Cubic Spherical Neutrosophic Weighted Geometric Yager Aggregation Operator (CuSNWGYAO). These operators are developed by integrating Yager's norms with cubic spherical neutrosophic sets (CuSNS) to effectively combine uncertain, indeterminate and inconsistent information. The mathematical properties of the proposed operators including idempotency, boundedness and monotonicity are investigated and verified. To demonstrate the applicability of the proposed approach, a multi-criteria decision-making (MCDM) model is constructed for selecting the most suitable method to enhance groundwater levels. Six groundwater restoration alternatives are evaluated based on multiple criteria such as effectiveness, cost, land requirement, technical complexity, environmental impact, and social acceptability. Expert opinions are incorporated using linguistic variables and transformed into cubic spherical neutrosophic values. The proposed aggregation operators are then applied to obtain aggregated evaluations and final rankings of the alternatives. The results show that the proposed operators provide a flexible and effective framework for handling complex decision-making problems under uncertainty. Therefore, the developed method can serve as a valuable tool for solving practical decision-making problems in environmental management and other related fields.

Keywords:Cubic spherical neutrosophic set; Yager's norms; Cubic spherical neutrosophic weighted Yager aggregation operator; Cubic spherical neutrosophic weighted geometric Yager aggregation operator

1. Introduction

Decision-making in real-world situations often involves uncertainty, vagueness and incomplete information. Classical mathematical models are not always sufficient to represent such complex situations effectively. To overcome these limitations, various extensions of fuzzy set theory have been developed including intuitionistic fuzzy sets, neutrosophic sets, and spherical fuzzy sets. Among these frameworks, neutrosophic theory provides a powerful tool for modelling uncertainty by considering three independent components: truth, indeterminacy and falsity. Recently, CuSNSs have emerged as an advanced extension that combines the advantages of cubic structures and spherical neutrosophic information, offering greater flexibility in representing uncertain and inconsistent data. Due to these advantages, CuSNSs have attracted significant attention in the field of MCDM and practical problem-solving.

All living things need water to survive. The causes of water scarcity are climate change (droughts/ altered rainfall), population growth, excessive groundwater extraction, agricultural water overuse and poor water management system. Ground water is the purest water which is located under the surface of soil and rocks. More than 30 percent of fresh water is taken from the ground water. It is used for agricultural usage, industrial usage by construction works and it is extracted by borewells. The studies and research related to ground water is called hydrogeology. The main input source of ground water is infiltration from the surface water (river, streams). Restoring the ground water is divided into two ways called natural way and artificial way. Rain water harvesting is one of the primary natural way to restoring the ground water. Planting trees, restore wetlands & forest, urban green space maintain, natural drainage and maintain the soil health are natural ways. Trees and long rooted planets are reducing the soil erosion and help to infiltration of water to ground level. Wetlands are holding the water long lasting it seeps water through aquifers. Generate green environment like garden, park, fields are allowing rain water directly entering to the surface. Dry leaves, wooden dust, sand are help to filter the water and maintain the soil health with proper natural fertilization. The artificial restoring ground water is involving human ideas and contribution. It includes checking dams & percolation tanks, recharge wells & borewell recharge and managed aquifer recharge. Dams and percolation tanks are used to store a large amount water it allows water slowly into the surface area. In the recharge well method water is directly injecting the underground with the help of large pipes. Water is directly through the deeper aquifers is called managed aquifers recharge method.

We get the best alternative source to increasing the ground water level by using the proposed methodology. According to the above methods we selecting six major method is considering the alternatives of the application problem. The alternatives are rainwater harvesting, check dams & percolation tanks, recharge wells & borewell recharge, afforestation & soil conservation, managed aquifer recharge and urban green infrastructure. Selecting the best is based on their characteristics and benefit range for the society development. Effectiveness, cost land requirement, technical complexity, environmental impact and social acceptability are considering the major characteristics. These characteristics are taken as criteria in the application problem.

This paper is organized as follows: The paper is ordered as follows: the first section is introduction with sub sections. The second section is preliminaries which provides basic definitions and operators for the throughout paper. Third section introduces new aggregation operators and verified its properties based on the Yager's condition. Fourth section gives the simple procedure for calculate the final result. Fifth is the real life application (find the best source to increase the ground water level) of the proposed method with comparative and sensitive analysis. The sixth section provides the conclusion of this paper.

1.1. Research Gap and Problem Statement

Uncertainty and imprecision are common in many real-world decision-making problems. Classical mathematical models are often inadequate to represent such uncertain and indeterminate information. To overcome these limitations, several extensions of fuzzy set theory such as intuitionistic fuzzy sets, neutrosophic sets, and spherical fuzzy sets have been introduced. Among these, CuSNSs provide a more flexible framework for representing uncertainty because they combine the advantages of cubic structures and spherical neutrosophic information. In recent years, many researchers have developed various aggregation operators under different fuzzy and neutrosophic environments to solve multi-criteria decision-making problems. Although several aggregation operators exist for neutrosophic sets and spherical fuzzy sets, very limited work has been carried out for cubic spherical neutrosophic sets. In particular, the integration of Yager's norms and aggregation operators within the cubic spherical neutrosophic framework has not yet been fully explored in the literature.

Yager's operators are widely used in decision-making because of their flexibility and ability to handle complex preference information. However, no existing studies have developed Yager-based aggregation operators specifically for CuSNS. Therefore, there is a need to construct new aggregation operators that can effectively aggregate cubic spherical neutrosophic information and improve decision-making accuracy under uncertain environments. To fill this research gap, this study introduces new cubic spherical neutrosophic weighted aggregation operators under Yager norms and demonstrates their usefulness in solving a real-life decision-making problem related to groundwater resource management.

1.2. Main Contributions of the Study

The main contributions of this study are summarized as follows:

- Development of new aggregation operators.
- This study introduces two new operators namely the CuSNWYAO and CuSNWGYAO for aggregating cubic spherical neutrosophic information.
- Integration of Yager's norms with CuSNSs.
- The proposed operators integrate Yager's norms with CuSNSs, providing a flexible mathematical framework for handling uncertain and indeterminate information.
- Mathematical analysis of the proposed operators.
- Several important properties of the proposed operators, including idempotency, monotonicity, and boundedness, are established and verified through theoretical proofs.
- Development of an MCDM decision framework.

- A structured MCDM procedure based on the proposed operators is developed to evaluate alternatives under cubic spherical neutrosophic environments.
- Real - life application.
- The effectiveness of the proposed approach is demonstrated through a real-life application for selecting the most suitable method to increase groundwater levels considering multiple environmental and technical criteria.
- Practical decision support tool.
- The proposed model provides a useful decision-support tool for environmental management and sustainability problems, where uncertainty and incomplete information are common.

1.3. Related Work

Fuzzy set is introduced by LA Zadeh [23] at 1965 it had only one characteristics is called membership value it is mapping from zero to one. Then, KT Atanassov [3] introduced intuitionisitc fuzzy set which had two characteristics as membership value and non - membership value. The sum of membership and non – membership value is less than or equal to 1. To handling the uncertainty, vagueness, indeterminate values Florentin Smarandache introduced neutrosophic set [15] at 1998 which is the next stage of intuitionistc fuzzy set. It had three characteristic as membership value, non – membership value and indeterminacy value. It often called true, indeterminacy and false these value belongs to zero to one. Sum of these three is less than or equal to three. Later, neutrosophic set is expanded into various sets some of them are single – valued neutrosophic set [21], complex neutrosophic set [2], interval neutrosophic set [20].

Soft set is the primary mathematical tool which is help to dealing the uncertainty (in fuzzy, probability, interval, etc.) it is introduced at 1999 by Molodtsov [12]. Application of soft set is increasing when Maji [11] developed the operations and some prepositions with detailing examples. Later, this set is extended to hypersoft set and superhypersoft set. The hypersoft set is mapping from Cartesian product of the sets to power set of the universal set it is expanded by Smarandache [19]. The expansion of hypersoft set is superhypersoft set which is introduced by Smarandache [17] at 2023. These sets are collaborate with many sets some of them as fuzzy sets, neutrosophic sets, pilthoconic sets.

The CuSNS is introduced by Gomathi [7] at 2023. The operations and their properties also verified by the same author. This set is used in various application problems in recent years. The application of CuSNS is to select the suitable crop for different climate area in Tamil Nadu under various environmental, agricultural situations [18]. To choosing the best fertilizer, to enhancing the productivity, sustaining the tree health and maintaining the proper yield in coconut trees [5]. Reducing the environmental issues, world heating, improving the green environment we choose the best green suppliers to avoiding these type of problems [7]. In recent years, electric vehicles are affordable, without air pollution and easy chargeable. So, the demand of electric vehicles are high then to select the best electric vehicle is important one. In this paper [9], analyse this problem under the cubic spherical neutrosophic environment. Choosing the best fertilizer for banana and guava tree [14], if use the wrong one it leads to losing the planet and not receiving the proper yield or growth.

Yager [22] introduced the norms and co – norms, it is used to introduce various definition and operators. It is developed in different sets and it is also applicable in multi criteria decision making problems. To select the suitable lab for COVID – 19 test under the Fermatean fuzzy Yager's aggregation operators [4]. Selection of emerging technology enterprise under q-rung picture fuzzy Yager weighted arithmetic operator, q-rung picture fuzzy Yager ordered weighted arithmetic operator, q-rung picture fuzzy Yager hybrid weighted arithmetic operator, q-rung picture fuzzy Yager weighted geometric operator, q-rung picture fuzzy Yager ordered weighted geometric operator, q-rung picture fuzzy Yager hybrid weighted geometric operator [10]. Closed Circuit Television (CCTV) is help to video surveillance, choosing the optimal CCTV which s lower cost, long lasting, best technical support and easy to maintain. This selection is calculated under complex Pythagorean fuzzy Yager aggregation operator [1]. To choose the high ranking university in the education sector under Pythagorean fuzzy Yager weighted operator [13], the university gives the valuable studies, atmosphere, campus interview and healthy environment. Select the best solar power planet under single valued neutrosophic weighted Yager aggregation operator [8]. There is no study approach the Yager condition in CuSNS so we derive the CuSNWYAO and CuSNWGYAO.

2. Preliminaries

Definition 2.1 [7] Let Δ be universal set and its subset is defined as $\mathcal{A} = \left\{\left(y, \widehat{\mathrm{T}}(y), \underline{\mathcal{I}}(y), \dot{\mathrm{f}}(y); \ddot{\mathrm{r}}\right): y \in \Delta\right\}$ where $\widehat{\mathrm{T}}(y), \underline{\mathcal{I}}(y), \dot{\mathrm{f}}(y): \Delta \rightarrow [0,1]$. The sum of truth ($\widehat{\mathrm{T}}$), indeterminacy ($\underline{\mathcal{I}}$) and falsity ($\dot{\mathrm{f}}$) is less than or equal to 3 and the radius ($\ddot{\mathrm{r}}$) belongs to [0, 1]. The radius of the sphere with centre ($\widehat{\mathrm{T}}(y), \underline{\mathcal{I}}(y), \dot{\mathrm{f}}(y)$) inside the cube (or) cube inside the sphere is called cubic spherical neutrosophic set (CuSNS) $\mathcal{A}$.

The centre of the sphere is $\left(\widehat{\mathrm{T}}(y_r), \underline{\mathcal{I}}(y_r), \dot{\mathrm{f}}(y_r)\right) = \left(\frac{\sum_{s=1}^{p_r} \widehat{\mathrm{T}}_{r,s}}{p_r}, \frac{\sum_{s=1}^{p_r} \underline{\mathcal{I}}_{r,s}}{p_r}, \frac{\sum_{s=1}^{p_r} \dot{\mathrm{f}}_{r,s}}{p_r}\right)$.

The radius is

$$\ddot{\mathrm{r}}_r = \min\left\{\max_{1 \leq s \leq p_r} \sqrt{\left(\widehat{\mathrm{T}}(y_r) - \widehat{\mathrm{T}}_{r,s}\right)^{2} + \left(\underline{\mathcal{I}}(y_r) - \underline{\mathcal{I}}_{r,s}\right)^{2} + \left(\dot{\mathrm{f}}(y_r) - \dot{\mathrm{f}}_{r,s}\right)^{2}}, 1\right\}.$$

Operators:

1. $y_1 \oplus y_2 = \left(\hat{\mathrm{T}}_1 + \hat{\mathrm{T}}_2 - \hat{\mathrm{T}}_1\hat{\mathrm{T}}_2, \underline{\mathcal{I}}_1\underline{\mathcal{I}}_2, \dot{f}_1\dot{f}_2;\ \ddot{r}_1 + \ddot{r}_2 - \ddot{r}_1\ddot{r}_2\right)$.
2. $\oplus_{a=1}^{n} = \left(1 - \prod_{a=1}^{n}(1 - \hat{\mathrm{T}}_a),\ \prod_{a=1}^{n}\underline{\mathcal{I}}_a,\ \prod_{a=1}^{n}\dot{f}_a; 1 - \prod_{a=1}^{n}(1 - \ddot{r}_a)\right)$.
3. $y_1 \otimes y_2 = \left(\hat{\mathrm{T}}_1\hat{\mathrm{T}}_2,\ \underline{\mathcal{I}}_1 + \underline{\mathcal{I}}_2 - \underline{\mathcal{I}}_1\underline{\mathcal{I}}_2,\ \dot{f}_1 + \dot{f}_2 - \dot{f}_1\dot{f}_2;\ \ddot{r}_1\ddot{r}_2\right)$.
4. $\otimes_{a=1}^{n} = \left(\prod_{a=1}^{n}\hat{\mathrm{T}}_a, 1 - \prod_{a=1}^{n}(1 - \underline{\mathcal{I}}_a),\ 1 - \prod_{a=1}^{n}(1 - \dot{f}_a);\ \prod_{a=1}^{n}\ddot{r}_a\right)$.
5. $\sigma y = \left(1 - \left(1 - \hat{\mathrm{T}}\right)^{\sigma}, \underline{\mathcal{I}}^{\sigma}, \dot{f}^{\sigma}; 1 - (1 - \ddot{r})^{\sigma}\right)$.
6. $\sigma^{y} = \left(\hat{\mathrm{T}}^{\sigma}, 1 - \left(1 - \underline{\mathcal{I}}\right)^{\sigma}, 1 - (1 - \dot{f})^{\sigma};\ \ddot{r}^{\sigma}\right)$.

Definition 2.2 [16] The score value of the neutrosophic set ($\mathcal{A}$) is defined as $sv(\mathcal{A}) = \frac{2 + \hat{\mathrm{T}} - \underline{\mathcal{I}} - \dot{f}}{3}$ and the value lies between 0 and 1.

3. Proposed Cubic Spherical Neutrosophic Yager Aggregation Operators

In this section we introduce the new aggregation operators namely CuSNWYAO and CuSNWGYAO. These operators are proved with induction hypotheses method. The properties idempotency, monotonicity and boundedness are verified. In the decision making problems, aggregation operators are very important one to combine the different opinions. Currently, many researchers shown the interest in CuSNS sets and their applications.

Definition 3.1 [22] Yager's norms for any $f, g \in \mathbb{R}$ and $\tau \in (0, \infty)$

1) $\acute{T}(f,g) = 1 - min\left(1, ((1-f)^{\tau} + (1-g)^{\tau})^{\frac{1}{\tau}}\right)$

2) $\acute{S}(f,g) = min\left(1,\ (f^{\tau} - g^{\tau})^{\frac{1}{\tau}}\right)$.

Definition 3.2 Let $°\mathrm{F}_{t_1}, °\mathrm{F}_{t_2} \in CuSNS$ with $\sigma, \tau > 0$ and $°\mathrm{F}_t = \left(\hat{\mathrm{T}}(y), \underline{\mathcal{I}}(y), \dot{f}(y); \ddot{r}(y)\right)$. Then the Yager's operating laws are

1) $°\mathrm{F}_{t_1} \otimes °\mathrm{F}_{t_2} = \left\{1 - min\left(1, \left(\left(1 - \hat{\mathrm{T}}_1\right)^{\tau} + \left(1 - \hat{\mathrm{T}}_2\right)^{\tau}\right)^{\frac{1}{\tau}}\right), min\left(1, \left(\underline{\mathcal{I}}_1 + \underline{\mathcal{I}}_2\right)^{\frac{1}{\tau}}\right), min\left(1, (\dot{f}_1 + \dot{f}_2)^{\frac{1}{\tau}}\right); 1 - min\left(1, ((1 - \ddot{r}_1)^{\tau} + (1 - \ddot{r}_2)^{\tau})^{\frac{1}{\tau}}\right)\right\}$.

2) $°\mathrm{F}_{t_1} \oplus °\mathrm{F}_{t_2} = \left\{min\left(1, (\hat{\mathrm{T}}_1 + \hat{\mathrm{T}}_2)^{\frac{1}{\tau}}\right), 1 - min\left(1, \left(\left(1 - \underline{\mathcal{I}}_1\right)^{\tau} + \left(1 - \underline{\mathcal{I}}_2\right)^{\tau}\right)^{\frac{1}{\tau}}\right), 1 - min\left(1, ((1 - \dot{f}_1)^{\tau} + (1 - \dot{f}_2)^{\tau})^{\frac{1}{\tau}}\right);\ min\left(1, (\ddot{r}_1 + \ddot{r}_2)^{\frac{1}{\tau}}\right)\right\}$.

3) $°\mathrm{F}_{t_1}^{\sigma} = \left\{1 - min\left(1, \left(\sigma\left(1 - \hat{\mathrm{T}}_1\right)^{\tau}\right)^{\frac{1}{\tau}}\right), min\left(1, \left(\sigma\underline{\mathcal{I}}_1^{\tau}\right)^{\frac{1}{\tau}}\right), min\left(1, (\sigma\dot{f}_1^{\tau})^{\frac{1}{\tau}}\right);\ 1 - min\left(1, (\sigma(1 - \ddot{r}_1)^{\tau})^{\frac{1}{\tau}}\right)\right\}$.

4) $\sigma.°\mathrm{F}_{t_1} = \left\{min\left(1, \left(\sigma\hat{\mathrm{T}}_1^{\tau}\right)^{\frac{1}{\tau}}\right), 1 - min\left(1, \left(\sigma\left(1 - \underline{\mathcal{I}}_1\right)^{\tau}\right)^{\frac{1}{\tau}}\right), 1 - min\left(1, (\sigma(1 - \dot{f}_1)^{\tau})^{\frac{1}{\tau}}\right); min\left(1, (\sigma\ddot{r}_1^{\tau})^{\frac{1}{\tau}}\right)\right\}$.

Now, we derive the new aggregation operators to handle the cubic spherical values under the Yager's operation laws.

Definition 3.3 Let $°\mathrm{F}_t = \left(\hat{\mathrm{T}}_t(y), \underline{\mathcal{I}}_t(y), \dot{f}_t(y); \ddot{r}_t(y)\right) \in CuSNS$. The CuSNWYAO as defined as:

$$CuSNWYAO(°\mathrm{F}_1, °\mathrm{F}_2, \dots, °\mathrm{F}_k) = \sigma_1°\mathrm{F}_1 \oplus \sigma_2°\mathrm{F}_2 \oplus \dots \sigma_k°\mathrm{F}_k = \sum_{t=1}^{k}\sigma_t°\mathrm{F}_t$$

where the weights (σ) is greater than 0 and sum of all weights is 1.

Theorem 3.1 Let $°\mathrm{F}_t = \left(\hat{\mathrm{T}}_t(y), \underline{\mathcal{I}}_t(y), \dot{f}_t(y); \ddot{r}_t(y)\right) \in CuSNS$. Then the CuSNWYAO

$$CuSNWYAO(°\mathrm{F}_1, °\mathrm{F}_2, \dots, °\mathrm{F}_k) = \begin{pmatrix} min\left(1, \left(\sum_{t=1}^{k}\sigma_t\hat{\mathrm{T}}_t^{\tau}\right)^{\frac{1}{\tau}}\right), \\ 1 - min\left(1, \left(\sum_{t=1}^{k}\sigma_t\left(1 - \underline{\mathcal{I}}_t\right)^{\tau}\right)^{\frac{1}{\tau}}\right), \\ 1 - min\left(1, \left(\sum_{t=1}^{k}\sigma_t(1 - \dot{f}_t)^{\tau}\right)^{\frac{1}{\tau}}\right); \\ min\left(1, \left(\sum_{t=1}^{k}\sigma_t\ddot{r}_t^{\tau}\right)^{\frac{1}{\tau}}\right) \end{pmatrix}.$$

Proof: To prove this theorem we use mathematical induction on k, for each t,

Since $°\mathrm{F}_t = \left(\hat{\mathrm{T}}_t(y), \underline{\mathcal{I}}_t(y), \dot{f}_t(y); \ddot{r}_t(y)\right) \in CuSNS$. First put k = 2

$CuSNWYAO(°\mathrm{F}_1, °\mathrm{F}_2) = \sigma_1°\mathrm{F}_1 \oplus \sigma_2°\mathrm{F}_2$

$$= \begin{pmatrix} min\left(1, \left(\sum_{t=1}^{2} \sigma_t \hat{\mathrm{T}}_t^{\tau}\right)^{\frac{1}{\tau}}\right), \\ 1 - min\left(1, \left(\sum_{t=1}^{2} \sigma_t \left(1 - \underline{\mathcal{I}}_t\right)^{\tau}\right)^{\frac{1}{\tau}}\right), \\ 1 - min\left(1, \left(\sum_{t=1}^{2} \sigma_t (1 - \dot{\mathrm{f}}_t)^{\tau}\right)^{\frac{1}{\tau}}\right); \\ min\left(1, \left(\sum_{t=1}^{2} \sigma_t \ddot{\mathrm{r}}_t^{\tau}\right)^{\frac{1}{\tau}}\right) \end{pmatrix}$$

Suppose that this equation holds k = n,

$$CuSNWYAO(°\mathrm{F}_1, °\mathrm{F}_2, \dots, °\mathrm{F}_n) = \begin{pmatrix} min\left(1, \left(\sum_{t=1}^{n} \sigma_t \hat{\mathrm{T}}_t^{\tau}\right)^{\frac{1}{\tau}}\right), \\ 1 - min\left(1, \left(\sum_{t=1}^{n} \sigma_t \left(1 - \underline{\mathcal{I}}_t\right)^{\tau}\right)^{\frac{1}{\tau}}\right), \\ 1 - min\left(1, \left(\sum_{t=1}^{n} \sigma_t (1 - \dot{\mathrm{f}}_t)^{\tau}\right)^{\frac{1}{\tau}}\right); \\ min\left(1, \left(\sum_{t=1}^{n} \sigma_t \ddot{\mathrm{r}}_t^{\tau}\right)^{\frac{1}{\tau}}\right) \end{pmatrix}$$

We have to prove that k = n+1,

$CuSNWYAO(°\mathrm{F}_1, °\mathrm{F}_2, \dots, °\mathrm{F}_{n+1}) = \sum_{t=1}^{n} \sigma_t °\mathrm{F}_t \oplus \sigma_{n+1} °\mathrm{F}_{n+1}$

$$= \begin{pmatrix} min\left(1, \left(\sum_{t=1}^{n} \sigma_t \hat{\mathrm{T}}_t^{\tau}\right)^{\frac{1}{\tau}}\right), \\ 1 - min\left(1, \left(\sum_{t=1}^{n} \sigma_t \left(1 - \underline{\mathcal{I}}_t\right)^{\tau}\right)^{\frac{1}{\tau}}\right), \\ 1 - min\left(1, \left(\sum_{t=1}^{n} \sigma_t (1 - \dot{\mathrm{f}}_t)^{\tau}\right)^{\frac{1}{\tau}}\right); \\ min\left(1, \left(\sum_{t=1}^{n} \sigma_t \ddot{\mathrm{r}}_t^{\tau}\right)^{\frac{1}{\tau}}\right) \end{pmatrix} \oplus \begin{pmatrix} min\left(1, \left(\sigma_{n+1} \hat{\mathrm{T}}_{n+1}^{\tau}\right)^{\frac{1}{\tau}}\right), \\ 1 - min\left(1, \left(\sigma_{n+1} \left(1 - \underline{\mathcal{I}}_{n+1}\right)^{\tau}\right)^{\frac{1}{\tau}}\right), \\ 1 - min\left(1, \left(\sigma_{n+1} (1 - \dot{\mathrm{f}}_{n+1})^{\tau}\right)^{\frac{1}{\tau}}\right); \\ min\left(1, \left(\sigma_{n+1} \ddot{\mathrm{r}}_{n+1}^{\tau}\right)^{\frac{1}{\tau}}\right) \end{pmatrix}$$

$$= \begin{pmatrix} min\left(1, \left(\sum_{t=1}^{n+1} \sigma_t \hat{\mathrm{T}}_t^{\tau}\right)^{\frac{1}{\tau}}\right), \\ 1 - min\left(1, \left(\sum_{t=1}^{n+1} \sigma_t \left(1 - \underline{\mathcal{I}}_t\right)^{\tau}\right)^{\frac{1}{\tau}}\right), \\ 1 - min\left(1, \left(\sum_{t=1}^{n+1} \sigma_t (1 - \dot{\mathrm{f}}_t)^{\tau}\right)^{\frac{1}{\tau}}\right); \\ min\left(1, \left(\sum_{t=1}^{n+1} \sigma_t \ddot{\mathrm{r}}_t^{\tau}\right)^{\frac{1}{\tau}}\right) \end{pmatrix}$$

$$CuSNWYAO(°\mathrm{F}_1, °\mathrm{F}_2, \dots, °\mathrm{F}_k) = \begin{pmatrix} min\left(1, \left(\sum_{t=1}^{k} \sigma_t \hat{\mathrm{T}}_t^{\tau}\right)^{\frac{1}{\tau}}\right), \\ 1 - min\left(1, \left(\sum_{t=1}^{k} \sigma_t \left(1 - \underline{\mathcal{I}}_t\right)^{\tau}\right)^{\frac{1}{\tau}}\right), \\ 1 - min\left(1, \left(\sum_{t=1}^{k} \sigma_t (1 - \dot{\mathrm{f}}_t)^{\tau}\right)^{\frac{1}{\tau}}\right); \\ min\left(1, \left(\sum_{t=1}^{k} \sigma_t \ddot{\mathrm{r}}_t^{\tau}\right)^{\frac{1}{\tau}}\right) \end{pmatrix}$$

That is, when k = n+1, equation holds.

Hence equation holds for any k. Hence proved the theorem.

Theorem 3.2 Let $°\mathrm{F}_t = \left(\hat{\mathrm{T}}_t(y), \underline{\mathcal{I}}_t(y), \dot{\mathrm{f}}_t(y); \ddot{\mathrm{r}}_t(y)\right) \in CuSNS$ such that $°\mathrm{F}_t = °\mathrm{F}$. Then

$$CuSNWYAO(°\mathrm{F}_1, °\mathrm{F}_2, \dots, °\mathrm{F}_k) = °\mathrm{F}.$$

Proof: Since $°\mathrm{F}_t = °\mathrm{F}$, then by the above theorem we have

$$CuSNWYAO(°\mathrm{F}_1, °\mathrm{F}_2, \dots, °\mathrm{F}_k) = \begin{pmatrix} min\left(1, \left(\sum_{t=1}^{k} \sigma_t \hat{\mathrm{T}}_t^{\tau}\right)^{\frac{1}{\tau}}\right), \\ 1 - min\left(1, \left(\sum_{t=1}^{k} \sigma_t \left(1 - \underline{\mathcal{J}}_t\right)^{\tau}\right)^{\frac{1}{\tau}}\right), \\ 1 - min\left(1, \left(\sum_{t=1}^{k} \sigma_t (1 - \dot{f}_t)^{\tau}\right)^{\frac{1}{\tau}}\right); \\ min\left(1, \left(\sum_{t=1}^{k} \sigma_t \ddot{\mathrm{r}}_t^{\tau}\right)^{\frac{1}{\tau}}\right) \end{pmatrix}$$

$$= \begin{pmatrix} min\left(1, \left(\sum_{t=1}^{k} \sigma_t \hat{\mathrm{T}}^{\tau}\right)^{\frac{1}{\tau}}\right), \\ 1 - min\left(1, \left(\sum_{t=1}^{k} \sigma_t \left(1 - \underline{\mathcal{J}}\right)^{\tau}\right)^{\frac{1}{\tau}}\right), \\ 1 - min\left(1, \left(\sum_{t=1}^{k} \sigma_t (1 - \dot{f})^{\tau}\right)^{\frac{1}{\tau}}\right); \\ min\left(1, \left(\sum_{t=1}^{k} \sigma_t \ddot{\mathrm{r}}^{\tau}\right)^{\frac{1}{\tau}}\right) \end{pmatrix}$$

$$= \begin{pmatrix} min\left(1, \left(\hat{\mathrm{T}}^{\tau}\right)^{\frac{1}{\tau}}\right), \\ 1 - min\left(1, \left(\left(1 - \underline{\mathcal{J}}\right)^{\tau}\right)^{\frac{1}{\tau}}\right), \\ 1 - min\left(1, ((1 - \dot{f})^{\tau})^{\frac{1}{\tau}}\right); \\ min\left(1, (\ddot{\mathrm{r}}^{\tau})^{\frac{1}{\tau}}\right) \end{pmatrix}$$

$$= \left(\hat{\mathrm{T}}(y), \underline{\mathcal{J}}(y), \dot{f}(y); \ddot{\mathrm{r}}(y)\right) = °\mathrm{F}.$$

Hence proved.

Theorem 3.3 Let $°\mathrm{F}_t = \left(\hat{\mathrm{T}}_t(y), \underline{\mathcal{J}}_t(y), \dot{f}_t(y); \ddot{\mathrm{r}}_t(y)\right)$,
$°\mathrm{F}_t^{+} = \left\{max\left(\hat{\mathrm{T}}_t(y)\right), min\left(\underline{\mathcal{J}}_t(y)\right), min\left(\dot{f}_t(y)\right); max\left(\ddot{\mathrm{r}}_t(y)\right)\right\}$ and
$°\mathrm{F}_t^{-} = \left\{min\left(\hat{\mathrm{T}}_t(y)\right), max\left(\underline{\mathcal{J}}_t(y)\right), max\left(\dot{f}_t(y)\right); min\left(\ddot{\mathrm{r}}_t(y)\right)\right\}$. Then
$°\mathrm{F}_t^{-} \leq CuSNWYAO(°\mathrm{F}_1, °\mathrm{F}_2, \dots, °\mathrm{F}_k) \leq °\mathrm{F}_t^{+}$.

Proof: Since $°\mathrm{F}_t \geq °\mathrm{F}_t^{-}$, then based on the above theorems,
$CuSNWYAO(°\mathrm{F}_1, °\mathrm{F}_2, \dots, °\mathrm{F}_k) \geq CuSNWYAO(°\mathrm{F}_1^{-}, °\mathrm{F}_2^{-}, \dots, °\mathrm{F}_k^{-}) = °\mathrm{F}_t^{-}$
$CuSNWYAO(°\mathrm{F}_1, °\mathrm{F}_2, \dots, °\mathrm{F}_k) \leq CuSNWYAO(°\mathrm{F}_1^{+}, °\mathrm{F}_2^{+}, \dots, °\mathrm{F}_k^{+}) = °\mathrm{F}_t^{+}$
Then, $°\mathrm{F}_t^{-} \leq CuSNWYAO(°\mathrm{F}_1, °\mathrm{F}_2, \dots, °\mathrm{F}_k) \leq °\mathrm{F}_t^{+}$.
Hence proved the boundedness.

Theorem 3.4 Let $°\mathrm{F}_t = \left(\hat{\mathrm{T}}_t(y), \underline{\mathcal{J}}_t(y), \dot{f}_t(y); \ddot{\mathrm{r}}_t(y)\right)$, $°\mathrm{F}_t^{\star} = \left(\hat{\mathrm{T}}_t^{\star}(y), \underline{\mathcal{J}}_t^{\star}(y), \dot{f}_t^{\star}(y); \ddot{\mathrm{r}}_t^{\star}(y)\right) \in CuSNS$. If $°\mathrm{F}_t \geq °\mathrm{F}_t^{\star}$ for all t, suppose $\hat{\mathrm{T}}_t \geq \hat{\mathrm{T}}_t^{\star}, \underline{\mathcal{J}}_t \leq \underline{\mathcal{J}}_t^{\star}, \dot{f}_t \leq \dot{f}_t^{\star}$ and $\ddot{\mathrm{r}}_t \geq \ddot{\mathrm{r}}_t^{\star}$ then $CuSNWYAO(°\mathrm{F}_1, °\mathrm{F}_2, \dots, °\mathrm{F}_k) = CuSNWYAO(°\mathrm{F}_1^{\star}, °\mathrm{F}_2^{\star}, \dots, °\mathrm{F}_k^{\star})$.

Proof: 1) Since $\hat{\mathrm{T}}_t \geq \hat{\mathrm{T}}_t^{\star}$ for all t,

$$\hat{\mathrm{T}}_t \geq \hat{\mathrm{T}}_t^{\star}$$
$$\hat{\mathrm{T}}_t^{\tau} \geq \hat{\mathrm{T}}_t^{\tau\star}$$
$$\sigma_t \hat{\mathrm{T}}_t^{\tau} \geq \sigma_t \hat{\mathrm{T}}_t^{\tau\star}$$
$$\sum_{t=1}^{k} \sigma_t \hat{\mathrm{T}}_t^{\tau} \geq \sum_{t=1}^{k} \sigma_t \hat{\mathrm{T}}_t^{\tau\star}$$
$$\left(\sum_{t=1}^{k} \sigma_t \hat{\mathrm{T}}_t^{\tau}\right)^{\frac{1}{\tau}} \geq \left(\sum_{t=1}^{n} \sigma_t \hat{\mathrm{T}}_t^{\tau\star}\right)^{\frac{1}{\tau}}$$
$$min\left(1, \left(\sum_{t=1}^{k} \sigma_t \hat{\mathrm{T}}_t^{\tau}\right)^{\frac{1}{\tau}}\right) \geq min\left(1, \left(\sum_{t=1}^{n} \sigma_t \hat{\mathrm{T}}_t^{\tau\star}\right)^{\frac{1}{\tau}}\right)$$

In this way, we prove the radius.

2) Since $\underline{\mathcal{J}}_t \leq \underline{\mathcal{J}}_t^{\star}$ for all t,

$$\underline{\mathcal{J}}_t \leq \underline{\mathcal{J}}_t^{\star}$$
$$1 - \underline{\mathcal{J}}_t \geq 1 - \underline{\mathcal{J}}_t^{\star}$$
$$\left(1 - \underline{\mathcal{J}}_t\right)^{\tau} \geq \left(1 - \underline{\mathcal{J}}_t^{\star}\right)^{\tau}$$

$$\sigma_t\left(1-\underline{\mathcal{J}}_t\right)^{\tau} \geq \sigma_t\left(1-\underline{\mathcal{J}}_t^{\star}\right)^{\tau}$$
$$\sum_{t=1}^{k}\sigma_t\left(1-\underline{\mathcal{J}}_t\right)^{\tau} \geq \sum_{t=1}^{k}\sigma_t\left(1-\underline{\mathcal{J}}_t^{\star}\right)^{\tau}$$
$$\left(\sum_{t=1}^{k}\sigma_t\left(1-\underline{\mathcal{J}}_t\right)^{\tau}\right)^{\frac{1}{\tau}} \geq \left(\sum_{t=1}^{k}\sigma_t\left(1-\underline{\mathcal{J}}_t^{\star}\right)^{\tau}\right)^{\frac{1}{\tau}}$$
$$min\left(1,\left(\sum_{t=1}^{k}\sigma_t\left(1-\underline{\mathcal{J}}_t\right)^{\tau}\right)^{\frac{1}{\tau}}\right) \geq min\left(1,\left(\sum_{t=1}^{n}\sigma_t\left(1-\underline{\mathcal{J}}_t^{\star}\right)^{\tau}\right)^{\frac{1}{\tau}}\right)$$
$$1-min\left(1,\left(\sum_{t=1}^{k}\sigma_t\left(1-\underline{\mathcal{J}}_t\right)^{\tau}\right)^{\frac{1}{\tau}}\right) \leq 1-min\left(1,\left(\sum_{t=1}^{n}\sigma_t\left(1-\underline{\mathcal{J}}_t^{\star}\right)^{\tau}\right)^{\frac{1}{\tau}}\right)$$

Similarly, we prove the false.

Definition 3.4 Let $°\mathrm{F}_t = \left(\widehat{\mathrm{T}}_t(y), \underline{\mathcal{J}}_t(y), \dot{\mathfrak{f}}_t(y); \ddot{\mathrm{r}}_t(y)\right) \in CuSNS$. Then the CuSNWGYAO as defined as:

$$CuSNWGYAO(°\mathrm{F}_1, °\mathrm{F}_2, \dots, °\mathrm{F}_k) = \sigma_1°\mathrm{F}_1 \oplus \sigma_2°\mathrm{F}_2 \oplus \dots \sigma_k°\mathrm{F}_k$$
$$= \sum_{t=1}^{k}(°\mathrm{F}_t)^{\sigma_t}$$

where the weights (σ) is greater than 0 and sum of all weights is 1.

Theorem 3.5 Let $°\mathrm{F}_t = \left(\widehat{\mathrm{T}}_t(y), \underline{\mathcal{J}}_t(y), \dot{\mathfrak{f}}_t(y); \ddot{\mathrm{r}}_t(y)\right) \in CuSNS$. Then the CuSNWGYAO

$$CuSNWGYAO(°\mathrm{F}_1, °\mathrm{F}_2, \dots, °\mathrm{F}_k) = \begin{pmatrix} 1-min\left(1,\left(\sum_{t=1}^{k}\sigma_t\left(1-\widehat{\mathrm{T}}_t\right)^{\tau}\right)^{\frac{1}{\tau}}\right), \\ min\left(1,\left(\sum_{t=1}^{k}\sigma_t\underline{\mathcal{J}}_t^{\tau}\right)^{\frac{1}{\tau}}\right), \\ min\left(1,\left(\sum_{t=1}^{k}\sigma_t\dot{\mathfrak{f}}_t^{\tau}\right)^{\frac{1}{\tau}}\right); \\ 1-min\left(1,\left(\sum_{t=1}^{k}\sigma_t(1-\ddot{\mathrm{r}}_t)^{\tau}\right)^{\frac{1}{\tau}}\right) \end{pmatrix}$$

Proof: This proof is similar to theorem 3.1. The properties idempotency, monotoncity and boundedness also verified.

4. Proposed Decision-Making Procedure

- Initiate the problem.
- Select alternatives and respective criteria.
- Expert opinion through linguistic table.
- Convert neutrosophic value into cubic spherical values.
- Using the CuSNWYAO to get the aggregated value.
- Calculate score value and ranking the results.

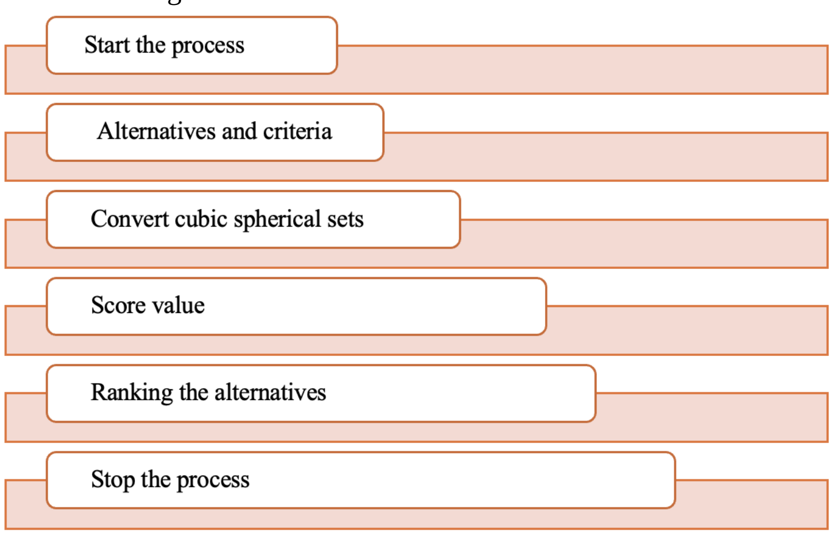

Figure 1: Algorithm

The figure 1 representing the simple procedure for start the problem to ranking the alternatives. It gives the throughout process in simple steps which lead us to easy understanding.

5. Case Study: Groundwater Sustainability Decision Problem

In this section we given the real life example under the usage of introduce operators. We take the problem is to finding the best source for increasing the ground water level. There is lot of water scarcity and lack of rain falling around the world. Solving these type of problems increasing the ground water level is one of the solution. Here we take the six alternatives as rainwater harvesting, check dams & percolation tanks, recharge wells & borewell recharge, afforestation & soil conservation, managed aquifer recharge and urban green infrastructure. The criteria are effectiveness, cost, land requirement, technical complexity, environmental impact and social acceptability. A case study is complex decision making problem in the real world, so we take the case to increase the ground water level and it analysis under the cubic spherical neutrosophic environment. There is no longer using the yager's aggregation operator in cubic spherical neutrosophic situation so we try to fill that research gap. The proposed method is very flexible and easy way to get the final results with more accuracy. Assured drinking water for communities and enough water for agriculture, reduce the drought impact and enhance the ecosystem by increasing the ground water level. To considering this benefits, we find the best source to increase the ground water level.

Alternatives	Criteria	sub – criteria
• rainwater harvesting (az1) • check dams & percolation tanks (zy2) • recharge wells & borewell recharge (yx3) • afforestation & soil conservation (xw4) • managed aquifer recharge (wv5) • urban green infrastructure (vu6)	effectiveness (#1)	soil infiltration rate (#011)
		rainfall intensity (#012)
		water availability (#013)
	cost (#2)	installation cost (#021)
		operation & maintenance cost (#022)
		lifespan of infrastructure (#023)
	land requirement (#3)	urban verses rural space availability (#031)
		land ownership & usage rights (#032)
	technical complexity (#4)	need for experts (#041)
		local labour availability (#042)
		regulatory approvals (#043)
	environmental impact (#5)	soil health (#051)
		vegetation support (#052)
		wildlife support (#053)
	social acceptability (#6)	aesthetic impact (#061)
		public cooperation (#062)
		awareness & education needs (#063)

Table 1: Alternatives and criteria

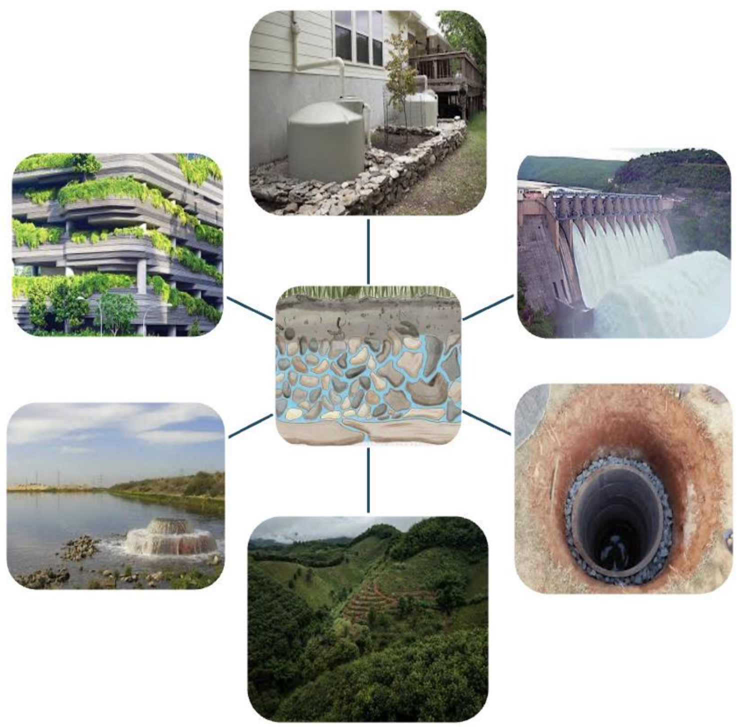

Figure 2: Image representation of alternatives

The table 1 gives the alternative, criteria and their sub – criteria. The figure 2 is visual representation of six respective alternatives. It is easily known the how the alternatives are workout. We assist the four hydrogeologists to analyse the problems and deliver their opinions. They are experts in the groundwater study, including its occurrence, movement and artificial recharge. So they are helpful to find the best alternative. The hydrogeologists weights are based on their experience of field working. According to that, the experience of hydrogeologists are 10 years, 7 years, 11 years and 5 years respectively. So, the weights are 0.32, 0.16, 0.41, 0.11. The total weight of the hydrogeologists are 1.

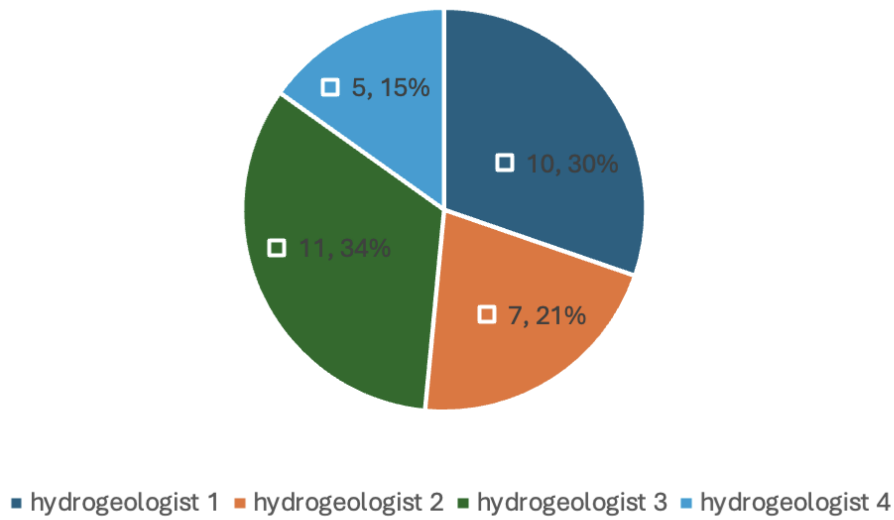

Figure 3: Weights of hydrogeologists

The figure 3 gives the pie chart view of four hydrogeologists experiences. The first one experience is 10 years which is 30 percentage out of hundred. The second one is 7 years with 21 percentage. Third is 11 years with 34 percentage and fourth is 5 years with 15 percentage. The table 2 gives linguistic term and their values with proper notation.

Term	value	Notation
Higher	(1.00, 0.00, 0.00)	Ӥa
maximum	(0.92, 0.02, 0.06)	Ӥb
above medium	(0.81, 0.06, 0.13)	Ӥc
Medium	(0.73, 0.08, 0.19)	Ӥd
below medium	(0.54, 0.09, 0.37)	Ӥe
minimum	(0.39, 0.08, 0.53)	Ӥf
Lower	(0.00, 0.00, 1.00)	Ӥg

Table 2: Linguistic values and notations

Taking the one set of combination and finding the best alternative for that combination by using the described steps in section 3.

The combination is [#013, #022, #032, #041, #053, #062].

Now, we collect the opinions from four hydrogeologists through the linguistic value table.

Hydrogeologists	alternative	#013	#022	#032	#041	#053	#062
ϴ1	az1	Ӥa	Ӥb	Ӥa	Ӥd	Ӥe	Ӥa
	zy2	Ӥb	Ӥd	Ӥf	Ӥa	Ӥb	Ӥd
	yx3	Ӥc	Ӥf	Ӥc	Ӥe	Ӥg	Ӥb
	xw4	Ӥf	Ӥa	Ӥe	Ӥd	ӥf	Ӥb
	wv5	Ӥe	Ӥd	Ӥb	Ӥa	Ӥc	ӥf
	vu6	Ӥd	Ӥc	Ӥf	Ӥe	Ӥd	Ӥg
ϴ2	az1	Ӥf	Ӥc	Ӥe	Ӥc	Ӥd	Ӥb
	zy2	Ӥd	Ӥf	Ӥe	Ӥf	Ӥc	Ӥg
	yx3	Ӥe	Ӥb	Ӥa	Ӥb	Ӥd	Ӥa
	xw4	Ӥd	Ӥb	Ӥb	Ӥc	Ӥe	Ӥf
	wv5	Ӥc	Ӥc	Ӥa	Ӥf	Ӥc	Ӥc
	vu6	Ӥc	Ӥf	Ӥb	Ӥf	Ӥg	Ӥd
ϴ3	az1	Ӥf	Ӥe	Ӥc	Ӥc	Ӥf	Ӥd
	zy2	Ӥb	Ӥc	Ӥf	Ӥd	Ӥc	Ӥc
	yx3	Ӥa	Ӥg	Ӥe	Ӥd	Ӥg	Ӥd
	xw4	Ӥb	Ӥf	Ӥd	Ӥc	Ӥf	Ӥa
	wv5	Ӥc	Ӥc	Ӥf	Ӥd	Ӥc	Ӥb
	vu6	Ӥf	Ӥd	Ӥd	Ӥb	Ӥd	Ӥc
ϴ4	az1	Ӥe	Ӥd	Ӥe	Ӥf	Ӥd	Ӥf
	zy2	Ӥd	Ӥc	Ӥd	Ӥg	Ӥc	Ӥe
	yx3	Ӥf	Ӥd	Ӥc	Ӥb	Ӥd	Ӥd
	xw4	Ӥd	Ӥc	Ӥc	ӥg	Ӥc	Ӥf
	wv5	Ӥe	Ӥf	Ӥf	Ӥa	Ӥf	Ӥd
	vu6	Ӥa	Ӥe	Ӥb	Ӥa	Ӥb	Ӥe

Table 3: Hydrogeologists opinion

The table 3 is the four hydrogeologists opinion for all criteria. Criteria weights based on our own opinion about the character of the criteria. The total criteria weight is one. The weights of the #1, #2, #3, #4, #5, #6 are 0.231, 0.161, 0.134, 0.127, 0.185, 0.162 respectively.

hydrogeologists	**alternative**	**#013**	**#022**	**#032**	**#041**	**#053**	**#062**
ϴ1	az1	(1.00, 0.00, 0.00)	(0.92, 0.02, 0.06)	(1.00, 0.00, 0.00)	(0.73, 0.08, 0.19)	(0.54, 0.09, 0.37)	(1.00, 0.00, 0.00)
	zy2	(0.92, 0.02, 0.06)	(0.73, 0.08, 0.19)	(0.39, 0.08, 0.53)	(1.00, 0.00, 0.00)	(0.92, 0.02, 0.06)	(0.73, 0.08, 0.19)
	yx3	(0.81, 0.06, 0.13)	(0.39, 0.08, 0.53)	(0.81, 0.06, 0.13)	(0.54, 0.09, 0.37)	(0.00, 0.00, 1.00)	(0.92, 0.02, 0.06)
	xw4	(0.39, 0.08, 0.53)	(1.00, 0.00, 0.00)	(0.54, 0.09, 0.37)	(0.73, 0.08, 0.19)	(0.39, 0.08, 0.53)	(0.92, 0.02, 0.06)
	wv5	(0.54, 0.09, 0.37)	(0.73, 0.08, 0.19)	(0.92, 0.02, 0.06)	(1.00, 0.00, 0.00)	(0.81, 0.06, 0.13)	(0.39, 0.08, 0.53)
	vu6	(0.73, 0.08, 0.19)	(0.81, 0.06, 0.13)	(0.39, 0.08, 0.53)	(0.54, 0.09, 0.37)	(0.73, 0.08, 0.19)	(0.00, 0.00, 1.00)
ϴ2	az1	(0.39, 0.08, 0.53)	(0.81, 0.06, 0.13)	(0.54, 0.09, 0.37)	(0.81, 0.06, 0.13)	(0.73, 0.08, 0.19)	(0.92, 0.02, 0.06)
	zy2	(0.73, 0.08, 0.19)	(0.39, 0.08, 0.53)	(0.54, 0.09, 0.37)	(0.39, 0.08, 0.53)	(0.81, 0.06, 0.13)	(0.00, 0.00, 1.00)
	yx3	(0.54, 0.09, 0.37)	(0.92, 0.02, 0.06)	(1.00, 0.00, 0.00)	(0.92, 0.02, 0.06)	(0.73, 0.08, 0.19)	(1.00, 0.00, 0.00)
	xw4	(0.73, 0.08, 0.19)	(0.92, 0.02, 0.06)	(0.92, 0.02, 0.06)	(0.81, 0.06, 0.13)	(0.54, 0.09, 0.37)	(0.39, 0.08, 0.53)
	wv5	(0.81, 0.06, 0.13)	(0.81, 0.06, 0.13)	(1.00, 0.00, 0.00)	(0.39, 0.08, 0.53)	(0.81, 0.06, 0.13)	(0.81, 0.06, 0.13)
	vu6	(0.81, 0.06, 0.13)	(0.39, 0.08, 0.53)	(0.92, 0.02, 0.06)	(0.39, 0.08, 0.53)	(0.00, 0.00, 1.00)	(0.73, 0.08, 0.19)
ϴ3	az1	(0.39, 0.08, 0.53)	(0.54, 0.09, 0.37)	(0.81, 0.06, 0.13)	(0.81, 0.06, 0.13)	(0.39, 0.08, 0.53)	(0.73, 0.08, 0.19)
	zy2	(0.92, 0.02, 0.06)	(0.81, 0.06, 0.13)	(0.39, 0.08, 0.53)	(0.73, 0.08, 0.19)	(0.81, 0.06, 0.13)	(0.81, 0.06, 0.13)
	yx3	(1.00, 0.00, 0.00)	(0.00, 0.00, 1.00)	(0.54, 0.09, 0.37)	(0.73, 0.08, 0.19)	(0.00, 0.00, 1.00)	(0.73, 0.08, 0.19)
	xw4	(0.92, 0.02, 0.06)	(0.39, 0.08, 0.53)	(0.73, 0.08, 0.19)	(0.81, 0.06, 0.13)	(0.39, 0.08, 0.53)	(1.00, 0.00, 0.00)
	wv5	(0.81, 0.06, 0.13)	(0.81, 0.06, 0.13)	(0.39, 0.08, 0.53)	(0.73, 0.08, 0.19)	(0.81, 0.06, 0.13)	(0.92, 0.02, 0.06)
	vu6	(0.39, 0.08, 0.53)	(0.73, 0.08, 0.19)	(0.73, 0.08, 0.19)	(0.92, 0.02, 0.06)	(0.73, 0.08, 0.19)	(0.81, 0.06, 0.13)
ϴ4	az1	(0.54, 0.09, 0.37)	(0.73, 0.08, 0.19)	(0.54, 0.09, 0.37)	(0.39, 0.08, 0.53)	(0.73, 0.08, 0.19)	(0.39, 0.08, 0.53)
	zy2	(0.73, 0.08, 0.19)	(0.81, 0.06, 0.13)	(0.73, 0.08, 0.19)	(0.00, 0.00, 1.00)	(0.81, 0.06, 0.13)	(0.54, 0.09, 0.37)
	yx3	(0.39, 0.08, 0.53)	(0.73, 0.08, 0.19)	(0.81, 0.06, 0.13)	(0.92, 0.02, 0.06)	(0.73, 0.08, 0.19)	(0.73, 0.08, 0.19)
	xw4	(0.73, 0.08, 0.19)	(0.81, 0.06, 0.13)	(0.81, 0.06, 0.13)	(0.00, 0.00, 1.00)	(0.81, 0.06, 0.13)	(0.39, 0.08, 0.53)
	wv5	(0.54, 0.09, 0.37)	(0.39, 0.08, 0.53)	(0.39, 0.08, 0.53)	(1.00, 0.00, 0.00)	(0.39, 0.08, 0.53)	(0.73, 0.08, 0.19)
	vu6	(1.00, 0.00, 0.00)	(0.54, 0.09, 0.37)	(0.92, 0.02, 0.06)	(1.00, 0.00, 0.00)	(0.92, 0.02, 0.06)	(0.54, 0.09, 0.37)

Table 4: Neutrosophic value

The above table 4 is neutrosophic for all hydrogeologists opinion. Using the definition to transform neutrosophic value to cubic spherical neutrosophic value. To calculate the centre of the neutrosophic set then calculate radius from the centre value.

Alternative	**#013**	**#022**	**#032**	**#041**	**#053**	**#062**
az1	(0.580, 0.063, 0.358; 0.555)	(0.75, 0.063, 0.188; 0.280)	(0.723, 0.060, 0.218; 0.358)	(0.685, 0.070, 0.245; 0.410)	(0.598, 0.083, 0.320; 0.295)	(0.760, 0.045, 0.195; 0.500)
zy2	(0.825, 0.050, 0.125; 0.119)	(0.685, 0.070, 0.245; 0.410)	(0.513, 0.083, 0.405; 0.306)	(0.530, 0.040, 0.430; 0.779)	(0.838, 0.050, 0.133; 0.102)	(0.520, 0.058, 0.423; 0.779)
yx3	(0.685, 0.058, 0.258; 0.411)	(0.510, 0.045, 0.445; 0.755)	(0.790, 0.053, 0.158; 0.330)	(0.778, 0.530, 0.170; 0.313)	(0.365, 0.040, 0.595; 0.547)	(0.845, 0.045, 0.110; 0.195)
xw4	(0.693, 0.065, 0.243; 0.418)	(0.780, 0.040, 0.180; 0.526)	(0.750, 0.063, 0.188; 0.280)	(0.588, 0.050, 0.363; 0.868)	(0.533, 0.078, 0.390; 0.381)	(0.675, 0.045, 0.280; 0.431)
wv5	(0.675, 0.075, 0.250; 0.181)	(0.685, 0.070, 0.245; 0.410)	(0.675, 0.045, 0.280; 0.431)	(0.780, 0.040, 0.180; 0.526)	(0.705, 0.065, 0.230; 0.435)	(0.713, 0.060, 0.228; 0.443)
vu6	(0.733, 0.055, 0.213; 0.468)	(0.618, 0.078, 0.305; 0.320)	(0.740, 0.050, 0.210; 0.475)	(0.713, 0.048, 0.240; 0.435)	(0.595, 0.045, 0.360; 0.875)	(0.520, 0.058, 0.423; 0.779)

Table 5: Cubic spherical neutrosophic value

Table 5 is the cubic spherical value for table 4. Using the cubic spherical neutrosophic definition [7], first taking the alternative one with criteria one across the four hydroleogists opinion, then finding the centre and radius values. The transformation of neutrosophic value to cubic spherical neutrosophic value is simple. The definition of cubic spherical neutrosophic set is given in preliminaries section. According to that, first find the centre value of the neutrosophic sets. Then using that centre then get the radius value. Combination centre value and radius value is called cubic spherical neutrosophic value.

$$\text{centre} = \left(\frac{1+0.39+0.39+0.54}{4}, \frac{0+0.08+0.08+0.09}{4}, \frac{0+0.53+0.53+0.37}{4}\right) = (0.580, 0.063, 0.358).$$

$$\text{radius} = min\left(max\left(\begin{array}{c} \sqrt{(0.580-1)^2+(0.063-0)^2+(0.358-0)^2}, \\ \sqrt{(0.580-0.39)^2+(0.063-0.08)^2+(0.358-0.08)^2}, \\ \sqrt{(0.580-0.39)^2+(0.063-0.08)^2+(0.358-0.08)^2}, \\ \sqrt{(0.580-0.54)^2+(0.063-0.09)^2+(0.358-0.09)^2}, \end{array} \right), 1 \right)$$

$$= \min(0.555,1) = 0.555.$$

Cubic spherical neutrosophic value = (0.580, 0.063, 0.358; 0.555).

In the same way, we find all other table values.

alternative	aggregate value	score value	ranking
az1	(0.672, 0.064, 0.264; 0.409)	0.938	4
zy2	(0.676, 0.063, 0.267; 0.379)	0.931	6
yx3	(0.649, 0.049, 0.302; 0.433)	0.933	5
xw4	(0.669, 0.058, 0.274; 0.469)	0.952	2
wv5	(0.702, 0.061, 0.237; 0.385)	0.947	3
vu6	(0.653, 0.056, 0.292; 0.567)	0.968	1

Table 6: Score value and ranking

Table 6 is the aggregated value, score value and ranking for each alternative. Highest score value is consider as the best alternative. Using the derived theorem 3.1, first take the true value for all criteria

$$\text{True} = min\left(1,\ \begin{pmatrix}(0.231*(0.580)^1)+(0.161*(0.750)^1)+(0.134*(0.723)^1)\\(0.127*(0.685)^1)+(0.185*(0.598)^1)+(0.162*(0.760)^1)\end{pmatrix}^{\frac{1}{1}}\right)$$

$$= \min(1, 0.672) = 0.672.$$

$$\text{Indeterminacy} = 1 - min\left(1, \begin{pmatrix}(0.231*(1-0.063)^1)+(0.161*(1-0.063)^1)+\\(0.134*(1-0.060)^1)+(0.127*(1-0.070)^1)+\\(0.185*(1-0.083)^1)+(0.162*(1-0.045)^1)\end{pmatrix}^{\frac{1}{1}}\right)$$

$$= 1 - \min(1, 0.936) = 0.064.$$

$$\text{False} = 1 - min\left(1, \begin{pmatrix}(0.231*(1-0.358)^1)+(0.161*(1-0.188)^1)+\\(0.134*(1-0.218)^1)+(0.127*(1-0.245)^1)+\\(0.185*(1-0.320)^1)+(0.162*(1-0.195)^1)\end{pmatrix}^{\frac{1}{1}}\right)$$

$$= 1 - \min(1, 0.736) = 0.264.$$

$$\text{Radius} = min\left(1,\ \begin{pmatrix}(0.231*(0.555)^1)+(0.161*(0.280)^1)+(0.134*(0.358)^1)\\(0.127*(0.410)^1)+(0.185*(0.295)^1)+(0.162*(0.500)^1)\end{pmatrix}^{\frac{1}{1}}\right)$$

$$= \min(1, 0.409) = 0.409.$$

Then the first alternative aggregated value is (0.672, 0.064, 0.264; 0.409).

Here we using the score value, to combine the cubic spherical neutrosophic value then returning the single value. The limit of the score value is zero to one. Now, we calculate the score value by using definition,

Score value $= \frac{3+0.672-0.064-0.264+0.409}{4} = 0.938.$

Similarly, we calculate all other tabular values. Ranking the alternatives based on the score value. Here highest score value is considering the best alternative.

Ranking order = $vu6 > xw4 > wv5 > az1 > yx3 > zy2$. From using these steps, the result shows alternative 6 urban green infrastructure is the source to increasing the ground water level according to four hydrogeologists opinions.

Now, we take one more combination [#011, #023, #032, #043, #051, #061]

The final ranking is $wv5 > zy2 > xw4 > az1 > vu6 > yx3$.

5.1. Comparative and Sensitive Analysis

To show the strongest of this proposed method we comparing the existing cubic spherical aggregation operators. Here we take the six different aggregation operator to comparing our proposed operators. The below table 7 shows the ranking order under different cubic spherical operators.

Operators	Ranking
CSNWAO[5]	$vu6 > zy2 > xw4 > yx3 > wv5 > az1$
CSNWGO[5]	$vu6 > xw4 > wv5 > az1 > yx3 > zy2$
$CSNWA_{\delta}^{A}$[9]	$vu6 > xw4 > wv5 > az1 > zy2 > yx3$
$CSNWA_{\rho}^{A}$[9]	$vu6 > xw4 > wv5 > az1 > zy2 > yx3$
$CSNWG_{\delta}^{A}$[9]	$vu6 > xw4 > wv5 > az1 > zy2 > yx3$
$CSNWG_{\rho}^{A}$[9]	$vu6 > xw4 > wv5 > az1 > zy2 > yx3$
CSNWAAO [7]	$vu6 > xw4 > zy2 > wv5 > yx3 > az1$
CSNWGAO [7]	$vu6 > xw4 > wv5 > az1 > yx3 > zy2$
CSNWGHM [13]	$vu6 > xw4 > yx3 > wv5 > zy2 > az1$
CSNWA [6]	$vu6 > az1 > xw4 > wv5 > yx3 > zy2$
CSNWG [6]	$vu6 > az1 > xw4 > wv5 > yx3 > zy2$
$CuSNWYAO$[proposed]	$vu6 > xw4 > wv5 > az1 > yx3 > zy2$

$CuSNWGYAO$[proposed]	$vu6 > xw4 > wv5 > az1 > yx3 > zy2$

Table 7: Comparison

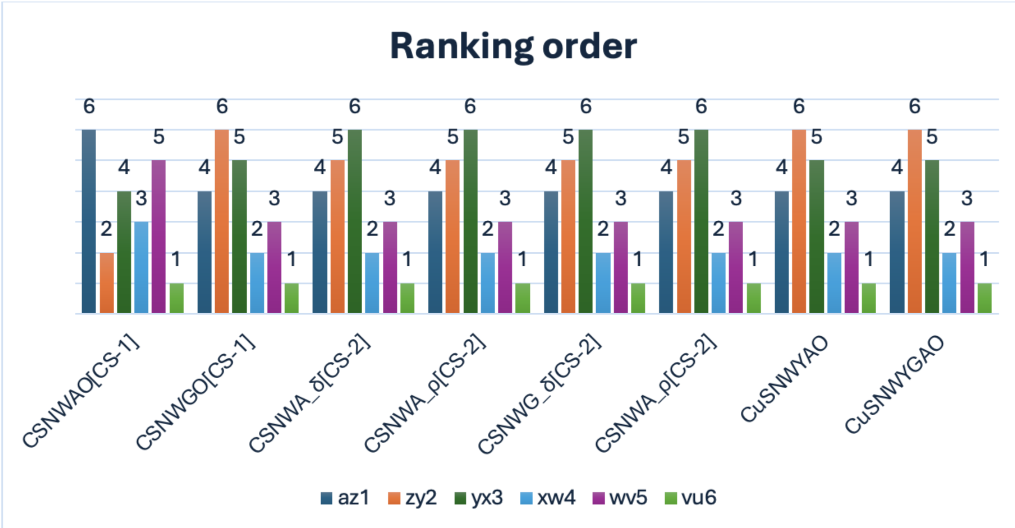

Figure 4: Bar chart representation

The above bar chart (figure 4) shows the ranking order for all alternative. In all the operator, the sixth alternative (vu6) is the best one. The bar chart gives the easy way to understanding the final results. Table 7 (comparison) is shows the different ranking order by using the different cubic spherical neutrosophic aggregation operators. The results shows, the alternative 6 is best source when using various operators. It shows the strongest of the proposed operator.

Here we present the different parameter values to show the ranking order. According to the parameter value there is no change in the best alternative. Alternative 6 (urban green infrastructure) is the best way to increasing the ground water level while changing the different parameter value.

Parameter value	Ranking
p = 1.5	$vu6 > xw4 > wv5 > zy2 > yx3 > az1$
p = 1.75	$vu6 > xw4 > zy2 > wv5 > yx3 > az1$
p= 2	$vu6 > xw4 > zy2 > yx3 > wv5 > az1$
p = 2.25	$vu6 > zy2 > xw4 > yx3 > wv5 > az1$
p = 2.5	$vu6 > zy2 > xw4 > yx3 > wv5 > az1$
p = 2.75	$vu6 > zy2 > xw4 > yx3 > wv5 > az1$
p = 3	$vu6 > zy2 > xw4 > yx3 > wv5 > az1$
p = 3.25	$vu6 > zy2 > xw4 > yx3 > wv5 > az1$
p = 3.5	$vu6 > zy2 > yx3 > xw4 > wv5 > az1$
p = 3.75	$vu6 > zy2 > yx3 > xw4 > wv5 > az1$
p = 4	$vu6 > zy2 > yx3 > xw4 > wv5 > az1$
p = 5	$vu6 > zy2 > yx3 > xw4 > wv5 > az1$
p = 5.5	$vu6 > zy2 > yx3 > xw4 > wv5 > az1$

Table 8: Different parameter value

The above table 8 is ranking order table when using the different parameter value. In all the situations, best alternative is sixth (urban green infrastructure) and the worst alternative is first (rainwater harvesting).

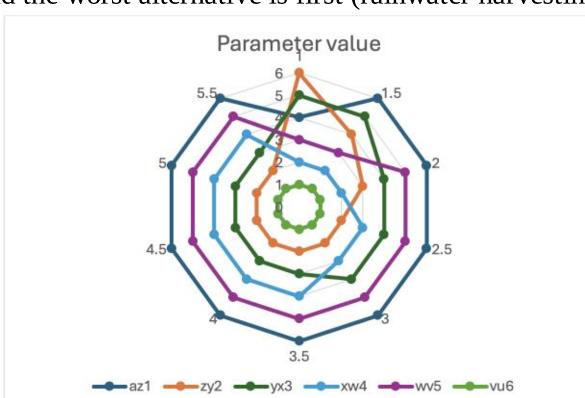

Figure 5: Radar representation

The above radar representation (figure 5) shows that, if changing the parameter value then get smaller difference in the ranking order. Here number one to six is the alternative ranking order. The green line is alternative six which is in the first rank all the parameters. The best alternative is urban green infrastructure when changing the parameter value one to six. This shows the strong and efficiency of the proposed method.

To show the strength of this paper, we choose different weights of the criteria then calculate the final ranking which is shown in table 9.

Critria weights	Ranking
(0.254, 0.145, 0.121, 0.140, 0.195, 0.145)	$vu6 > xw4 > wv5 > az1 > zy2 > yx3$
(0.168, 0.123, 0.234, 0.112, 0.164, 0.199)	$vu6 > wv5 > xw4 > az1 > yx3 > zy2$
(0.134, 0.208, 0.198, 0.161, 0.124, 0.175)	$xw4 > vu6 > wv5 > az1 > yx3 > zy2$
(0.111, 0.198, 0.147, 0.222, 0.201, 0.121)	$vu6 > wv5 > xw4 > az1 > yx3 > zy2$
(0.210, 0.187, 0.141, 0.155, 0.129, 0.178)	$vu6 > xw4 > wv5 > az1 > yx3 > zy2$
(0.165, 0.231, 0.176, 0.191, 0.201, 0.036)	$vu6 > xw4 > wv5 > az1 > yx3 > zy2$
(0.175, 0.119, 0.243, 0.167, 0.121, 0.175)	$vu6 > xw4 > wv5 > az1 > yx3 > zy2$
(0.188, 0.111, 0.176, 0.158, 0.101, 0.266)	$vu6 > xw4 > wv5 > az1 > yx3 > zy2$
(0.199, 0.188, 0.177, 0.166, 0.155, 0.115)	$vu6 > xw4 > wv5 > az1 > yx3 > zy2$
(0.168, 0.192, 0.239, 0.127, 0.142, 0.132)	$vu6 > xw4 > wv5 > az1 > yx3 > zy2$
(0.194, 0.177, 0.128, 0.289, 0.126, 0.086)	$xw4 > vu6 > wv5 > yx3 > zy2 > az1$

Table 9: Different weights

According to the table results, the most the entry gives alternative 6 is the best alternative. This shows the strength and strong of the proposed method. Obtaining the same top ranked alternative shows the strength and effectiveness of the proposed method. If we get different top ranked alternative from different operators how to conclude which alternative is best or which operator is best one.

Now we compare the score value for each alternative.From the table 10 results we get the same best alternative.

Operator	$az1$	$zy2$	$yx3$	$xw4$	$wu5$	$uv6$	best alternative
CSNWAO[7]	0.946	0.971	0.964	0.969	0.951	0.989	$uv6$
CSNWGO[7]	0.933	0.899	0.911	0.943	0.941	0.957	$uv6$
$CSNWA_{\delta}^{A}$[8]	0.931	0.886	0.875	0.957	0.942	0.963	$uv6$
$CSNWA_{\rho}^{A}$[8]	0.951	0.879	0.855	0.961	0.958	0.977	$uv6$
$CSNWG_{\delta}^{A}$[8]	0.958	0.923	0.876	0.977	0.964	0.988	$uv6$
$CSNWG_{\rho}^{A}$[8]	0.941	0.899	0.871	0.959	0.944	0.961	$uv6$
CSNWAAO [7]	0.911	0.881	0.895	0.910	0.898	0.911	$uv6$
CSNWGAO [7]	0.904	0.855	0.861	0.902	0.896	0.904	$uv6$
CSNWGHM [13]	0.806	0.808	0.818	0.824	0.816	0.832	$uv6$
CSNWA [6]	0.942	0.957	0.945	0.965	0.950	0.985	$uv6$
CSNWG [6]	0.936	0.912	0.929	0.948	0.942	0.961	$uv6$
$CuSNWYAO$[proposed]	0.938	0.931	0.933	0.952	0.947	0.968	$uv6$
$CuSNWGYAO$[proposed]	0.938	0.931	0.933	0.952	0.947	0.968	$uv6$

Table 10: Comparison of score value

6. Conclusion

Increasing the effectiveness and accuracy of cubic spherical values we derive`e the CuSNWYAO and CSNWGYAO by using the Yager's condition. The theorem is proved by induction hypothesis method and the fundamental properties as idempotency, monotonicity and boundedness are verified.To apply the proposed operator in real life we taken six alternatives and six criteria with corresponding sub – criteria, to finding the best source for increasing the ground water level. Applying the described steps we get urban green infrastructure is the best source to increasing the ground water level among other five alternatives. The comparative analysis and sensitive analysis is strengthen and ability of the proposed method because there is no change in the best alternative. It help us to use the proposed aggregation operators in various cubic spherical neutrosophic situations.

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Declarations

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