



Hesitant Neutrosophic Network Models for Operations Research: Graphs, Hypergraphs, and Superhypergraphs

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ABSTRACT

Graphs provide a standard language for representing connectivity via vertices and pairwise edges, yet many real interactions are genuinely higher-order. Hypergraphs capture such multiway relations by allowing each hyperedge to join an arbitrary nonempty vertex subset. To further model nested and hierarchical incidence, SuperHyperGraphs iterate the powerset construction over a finite base set, so that vertices may become set-valued objects across multiple levels. Motivated by uncertainty in modern relational data, we study hesitant neutrosophic sets, in which each element is described by a finite family of admissible truth-indeterminacy-falsity triples. This framework contains hesitant fuzzy sets and single-valued neutrosophic sets as special cases. We then transfer these semantics to network structures by introducing hesitant neutrosophic graphs and extending them to hesitant neutrosophic hypergraphs and hesitant neutrosophic superhypergraphs. The resulting hierarchy offers a unified formalism for representing uncertainty in pairwise, multiway, and multi-level relational systems.

Keywords: Superhypergraph, Hypergraph, Hesitant Neutrosophic Graph, Hesitant Neutrosophic HyperGraph, Hesitant Neutrosophic SuperHyperGraph

1 Introduction

Classical (crisp) set theory encodes membership in a binary way and, by design, does not directly capture vagueness or partial truth. To model graded information, Zadeh introduced fuzzy sets in 1965 [1]. Atanassov subsequently proposed intuitionistic fuzzy sets [2], in which membership and non-membership degrees are specified separately and the residual part is interpreted as hesitation. Smarandache extended this viewpoint to neutrosophic sets [3, 4], assigning to each element three (typically independent) degrees of truth, indeterminacy, and falsity, thereby accommodating both incomplete and inconsistent evidence.

Uncertainty-aware semantics have also been incorporated into network models. Fuzzy graphs attach values in $[0, 1]$ to vertices and edges [5], while intuitionistic fuzzy graphs add explicit non-membership information [6]. Neutrosophic graphs further include an indeterminacy component [7]. More recently, plithogenic graphs quantify contradiction effects across multiple attributes [8].

From a structural viewpoint, classical graphs capture only pairwise relations, since each edge joins exactly two vertices [9]. Many systems, however, involve genuinely higher-order interactions among three or more entities. Such multiway relations are modeled by hypergraphs, where each hyperedge may connect any nonempty subset of vertices [10]. In addition, modern data often exhibit hierarchical and multi-scale organization. To encode nested incidence patterns in a set-theoretic way, SuperHyperGraphs enlarge the vertex domain by iterating the powerset construction over a finite ground set, so vertices may themselves be set-valued objects across multiple levels [11, 12, 13]. Table 1 summarizes the key distinctions among graphs, hypergraphs, and n -SuperHyperGraphs.

From the discussion above, further research on hypergraphs and superhypergraphs, as well as on fuzzy and neutrosophic graphs, is highly important. Moreover, among the extended set-theoretic models related to neutrosophic sets, several significant notions are known for which graph-based counterparts have not yet been sufficiently investigated. Motivated by this gap, in this paper we focus on hesitant neutrosophic sets [16, 17], where each element is associated with a finite set of admissible truth-indeterminacy-falsity triples. This model subsumes both hesitant fuzzy sets [18, 19] and (single-valued) neutrosophic sets as special cases. We then “graphize” this uncertainty formalism by introducing hesitant neutrosophic graphs, and we further extend the framework to higher-order and hierarchical interactions via hesitant neutrosophic hypergraphs and hesitant neutrosophic superhypergraphs. These structures provide a unified language for representing complex uncertainty in multiway and multi-level relational data.



Table 1: Key distinctions among graphs, hypergraphs, and n -SuperHyperGraphs.

Concept	Notation	Edge family	Core extension principle
Graph [14]	$G = (V, E)$	$E \subseteq \binom{V}{2}$	Edges encode pairwise relations between vertices.
Hypergraph [10]	$H = (V, \mathcal{E})$	$\mathcal{E} \subseteq \mathcal{P}(V) \setminus \{\emptyset\}$	Hyperedges may join any nonempty vertex subset, capturing higher-order interactions.
n -SuperHyperGraph [15]	$\text{SHG}^{(n)} = (V, \mathcal{E})$ base set V_0	(on a $V \subseteq \mathcal{P}^n(V_0) \setminus \{\emptyset\}$, $\mathcal{E} \subseteq \mathcal{P}(V) \setminus \{\emptyset\}$)	Vertices live in an n -fold powerset hierarchy and can be set-valued; edges are subsets of the supervertex set, allowing nested and multi-level incidence.

Notation. $\mathcal{P}(X) = \{A \mid A \subseteq X\}$, $\binom{V}{2} = \{\{u, v\} \subseteq V \mid u \neq v\}$, and $\mathcal{P}^0(X) = X$, $\mathcal{P}^{k+1}(X) = \mathcal{P}(\mathcal{P}^k(X))$.

2 Preliminaries

We begin by reviewing the basic terminology and notation used throughout this paper.

2.1 SuperHyperGraphs

We next recall the basic definitions for undirected SuperHyperGraphs.

Definition 2.1 (Iterated powersets). Let S be a set and let $n \in \mathbb{N}_0$. Define the iterated powersets of S recursively by

$$\mathcal{P}^0(S) := S, \quad \mathcal{P}^{n+1}(S) := \mathcal{P}(\mathcal{P}^n(S)) \quad (n \geq 0).$$

When convenient, we also use the nonempty powerset

$$\mathcal{P}_*(S) := \mathcal{P}(S) \setminus \{\emptyset\},$$

and the corresponding iterated nonempty powersets, defined by $\mathcal{P}_*^0(S) := S$ and

$$\mathcal{P}_*^{n+1}(S) := \mathcal{P}_*(\mathcal{P}_*^n(S)) \quad (n \geq 0).$$

Definition 2.2 (Hypergraph). [10] A hypergraph is a pair $H = (V(H), E(H))$ consisting of

- a (typically finite) vertex set $V(H)$, and
- a family of nonempty subsets $E(H) \subseteq \mathcal{P}(V(H)) \setminus \{\emptyset\}$, whose elements are called hyperedges.

If every hyperedge has cardinality r , then H is called r -uniform; in particular, 2-uniform hypergraphs coincide with simple graphs.

Definition 2.3 (n -SuperHyperGraph). [20, 15] Let V_0 be a nonempty (typically finite) base set and fix $n \in \mathbb{N}_0$. An n -SuperHyperGraph on V_0 is a hypergraph

$$\text{SHG}^{(n)} = (V, \mathcal{E})$$

such that

$$V \subseteq \mathcal{P}^n(V_0), \quad \mathcal{E} \subseteq \mathcal{P}(V) \setminus \{\emptyset\}.$$

Elements of V are called n -supervertices, and elements of $\mathcal{E}$ are called n -superedges. Thus, a superedge is a nonempty set of n -supervertices, while each n -supervertex may itself be a set-valued object living in the iterated powerset hierarchy $\mathcal{P}^n(V_0)$.

2.2 Neutrosophic n -SuperHyperGraphs

A neutrosophic set assigns to each element three (generally independent) degrees: truth, indeterminacy, and falsity, each in $[0, 1]$ [4]. Neutrosophic graphs and hypergraphs label vertices and/or incidences by such triples; see, e.g., [21]. We recall a standard hypergraph version and then present a compatible extension to the n -SuperHyperGraph setting.

Definition 2.4 (Single-Valued Neutrosophic Graph). [21] Let $G^* = (V, E)$ be a crisp (classical) graph, where V is the vertex set and $E \subseteq V \times V$ the edge set. A single-valued neutrosophic graph (SVNG) on G^* is defined as a pair

$$G = (A, B),$$

where

- $A = \{\langle v, T_A(v), I_A(v), F_A(v) \rangle : v \in V\}$ is the single-valued neutrosophic vertex set, with

$$T_A, I_A, F_A : V \rightarrow [0, 1],$$

denoting respectively the truth-membership, indeterminacy-membership, and falsity-membership functions of vertices, such that for every $v \in V$,

$$0 \leq T_A(v) + I_A(v) + F_A(v) \leq 3.$$



- $B = \{\langle uv, T_B(uv), I_B(uv), F_B(uv) \rangle : uv \in E\}$ is the single-valued neutrosophic edge set, with

$$T_B, I_B, F_B : E \rightarrow [0, 1],$$

satisfying for all $u, v \in V$ with $uv \in E$:

$$T_B(uv) \leq \min\{T_A(u), T_A(v)\}, \quad I_B(uv) \leq \min\{I_A(u), I_A(v)\}, \quad F_B(uv) \geq \max\{F_A(u), F_A(v)\}.$$

If B is symmetric, $G = (A, B)$ is called an undirected SVN G ; otherwise, it is a directed SVN G .

Definition 2.5 (Single-valued neutrosophic hypergraph). [22] Let $V = \{v_1, \dots, v_N\}$ be a finite vertex set. A single-valued neutrosophic hyperedge on V is specified by three maps

$$T, I, F : V \rightarrow [0, 1]$$

(its truth, indeterminacy, and falsity memberships), typically subject to

$$0 \leq T(v) + I(v) + F(v) \leq 3 \quad (\forall v \in V).$$

Its support is

$$\text{supp}(T, I, F) := \{v \in V : T(v) > 0 \text{ or } I(v) > 0 \text{ or } F(v) > 0\}.$$

A single-valued neutrosophic hypergraph is a pair $H = (V, \{E_i\}_{i=1}^M)$ where each E_i is a single-valued neutrosophic hyperedge on V and

$$V = \bigcup_{i=1}^M \text{supp}(E_i).$$

Definition 2.6 (Neutrosophic n -SuperHyperGraph). [13] Let V_0 be a finite ground set and let $n \geq 1$. A neutrosophic n -SuperHyperGraph is a tuple

$$\mathcal{NSHG}^{(n)} = (V, E, T_V, I_V, F_V, T_E, I_E, F_E),$$

where the following conditions hold.

- (i) (Underlying crisp structure) (V, E) is an n -SuperHyperGraph in the sense of Definition 2.3 (i.e., $V \subseteq \mathcal{P}^n(V_0)$ and $E \subseteq \mathcal{P}(V) \setminus \{\emptyset\}$).
- (ii) (Neutrosophic supervertices) $T_V, I_V, F_V : V \rightarrow [0, 1]$ assign to each supervertex $v \in V$ its truth, indeterminacy, and falsity degrees, satisfying

$$0 \leq T_V(v) + I_V(v) + F_V(v) \leq 3 \quad (\forall v \in V).$$

- (iii) (Neutrosophic incidences) $T_E, I_E, F_E : E \times V \rightarrow [0, 1]$ assign to each pair (e, v) an incidence triple $(T_E(e, v), I_E(e, v), F_E(e, v))$ satisfying

$$0 \leq T_E(e, v) + I_E(e, v) + F_E(e, v) \leq 3 \quad (\forall e \in E, \forall v \in V).$$

- (iv) (Support compatibility) If $v \notin e$, then

$$T_E(e, v) = I_E(e, v) = F_E(e, v) = 0.$$

Equivalently, for each $e \in E$, if we define the incidence profile of e by

$$\iota_e : V \rightarrow [0, 1]^3, \quad \iota_e(v) := (T_E(e, v), I_E(e, v), F_E(e, v)),$$

and its support by

$$\text{supp}(\iota_e) := \{v \in V \mid \iota_e(v) \neq (0, 0, 0)\},$$

then $\text{supp}(\iota_e) \subseteq e$.

- (v) (Containment constraints) For every $e \in E$ and every $v \in e$,

$$T_E(e, v) \leq T_V(v), \quad I_E(e, v) \leq I_V(v), \quad F_E(e, v) \geq F_V(v).$$

Remark 2.7 (On the falsity order convention). In Definition 2.6(v) we follow the convention $F_E(e, v) \leq F_V(v)$ (for $v \in e$). A different and widely used convention reverses the falsity order, requiring $F_E(e, v) \geq F_V(v)$ (interpreting larger falsity as larger “risk/non-membership”).

Switching between these conventions is not cosmetic: in the hesitant setting one must simultaneously dualize the falsity envelopes (e.g., the choice $F^- := \min$ vs. $F^+ := \max$) and flip the falsity inequalities in the envelope constraints and in later (α, β, γ) -cut monotonicity statements (e.g., those used in contraction-type arguments).



Table 2: Neutrosophic set vs. hesitant neutrosophic set (HNS).

	Neutrosophic set (single-valued)	Hesitant neutrosophic set (HNS)
Data at x	one triple $(T(x), I(x), F(x)) \in [0, 1]^3$	finite nonempty set $h_A(x) \subseteq [0, 1]^3$
Admissibility	typically $0 \leq T(x) + I(x) + F(x) \leq 3$	$0 \leq t + i + f \leq 3$ for all $(t, i, f) \in h_A(x)$
Interpretation	a single assessment	multiple plausible assessments (hesitation)
Support	$\{x \in X : (T(x), I(x), F(x)) \neq (0, 0, 0)\}$	$\{x \in X : \exists (t, i, f) \in h_A(x) \text{ s.t. } (t, i, f) \neq (0, 0, 0)\}$
Encoding	three functions $T, I, F : X \rightarrow [0, 1]$	map $h_A : X \rightarrow \mathcal{P}_{\text{fin}}^*([0, 1]^3)$; equivalently finite sets $T_A(x), I_A(x), F_A(x) \subseteq [0, 1]$ with selected admissible triples
Special case	—	singleton case $h_A(x) = \{(T(x), I(x), F(x))\}$ recovers a single-valued neutrosophic set

2.3 Hesitant Neutrosophic set

A hesitant neutrosophic set assigns each element a finite set of possible truth, indeterminacy, and falsity triples [16, 23].

Definition 2.8 (Hesitant neutrosophic set (HNS)). Let X be a nonempty universe and let $\mathcal{P}_{\text{fin}}^*(Y)$ denote the family of all finite nonempty subsets of a set Y . A hesitant neutrosophic set (HNS) A on X is specified by a mapping

$$h_A : X \longrightarrow \mathcal{P}_{\text{fin}}^*([0, 1]^3),$$

and is written as

$$A = \{ (x, h_A(x)) \mid x \in X \}.$$

For each $x \in X$, the value $h_A(x)$ is called the hesitant neutrosophic element (HNE) of A at x ; it is a finite nonempty set of triples

$$(t, i, f) \in [0, 1]^3,$$

interpreted respectively as possible truth-membership, indeterminacy-membership, and falsity-membership degrees of x in A , and satisfying the admissibility constraint

$$0 \leq t + i + f \leq 3 \quad \text{for every } (t, i, f) \in h_A(x), \forall x \in X.$$

The support of A is

$$\text{supp}(A) := \{ x \in X \mid \exists (t, i, f) \in h_A(x) \text{ with } (t, i, f) \neq (0, 0, 0) \}.$$

Remark 2.9 (Common equivalent encoding). Some authors encode an HNE at x via three finite nonempty sets

$$T_A(x), I_A(x), F_A(x) \subseteq [0, 1],$$

and interpret admissible assessments as a chosen subset of $T_A(x) \times I_A(x) \times F_A(x)$ (or, in the fully independent case, the entire product). This is equivalent to specifying $h_A(x) \in \mathcal{P}_{\text{fin}}^*([0, 1]^3)$ directly, as in Definition 2.8.

As a reference, Table 2 summarizes the key features of neutrosophic sets and hesitant neutrosophic sets (HNS).

3 Main Results

This section presents the main results of this paper.

3.1 Hesitant Neutrosophic Graph

A hesitant neutrosophic graph represents uncertain pairwise relations by attaching finite sets of truth, indeterminacy, falsity triples to vertices/edges.

Definition 3.1 (Hesitant neutrosophic graph). Let $G^* = (V, E)$ be a crisp (simple, undirected) graph with $E \subseteq \binom{V}{2}$. Write $\mathcal{P}_{\text{fin}}^*(S)$ for the family of all finite nonempty subsets of a set S . A hesitant neutrosophic graph (HNG) on G^* is a pair

$$G = (h_V, h_E),$$

where

$$h_V : V \rightarrow \mathcal{P}_{\text{fin}}^*([0, 1]^3), \quad h_E : E \rightarrow \mathcal{P}_{\text{fin}}^*([0, 1]^3),$$

such that the following hold.

- (i) (hesitant neutrosophic labels) For every $v \in V$ and every $(t, i, f) \in h_V(v)$,

$$0 \leq t + i + f \leq 3.$$

For every $e \in E$ and every $(t, i, f) \in h_E(e)$,

$$0 \leq t + i + f \leq 3.$$



(ii) (vertex envelopes) For each $v \in V$, define

$$T_V^+(v) := \max\{t : (t, i, f) \in h_V(v)\}, \quad I_V^+(v) := \max\{i : (t, i, f) \in h_V(v)\},$$

$$F_V^-(v) := \min\{f : (t, i, f) \in h_V(v)\}.$$

(iii) (incidence constraints) For each edge $e = \{u, v\} \in E$ and each $(t, i, f) \in h_E(e)$,

$$t \leq \min\{T_V^+(u), T_V^+(v)\}, \quad i \leq \min\{I_V^+(u), I_V^+(v)\}, \quad f \geq \max\{F_V^-(u), F_V^-(v)\}.$$

If $E \subseteq V \times V$ is a directed edge set, the same definition applies with $e = (u, v)$.

Remark 3.2 (Singleton (non-hesitant) case). If $h_V(v)$ and $h_E(e)$ are singletons for all $v \in V$ and $e \in E$, then $T_V^+(v)$, $I_V^+(v)$, $F_V^-(v)$ coincide with the unique triple in $h_V(v)$, and the constraints in Definition 3.1 reduce to the usual edge–vertex constraints of a (single-valued) neutrosophic graph.

Example 3.3 (Operations research: uncertain road network for time-dependent vehicle routing). Consider a city logistics provider solving a vehicle routing problem (VRP) under uncertain traffic and incident risk. Let the crisp graph $G^* = (V, E)$ model a small road network, where vertices are locations and edges are road segments:

$$V = \{0, 1, 2, 3\},$$

with 0 a depot, and 1, 2, 3 customer zones. Let

$$E = \{\{0, 1\}, \{0, 2\}, \{1, 2\}, \{1, 3\}, \{2, 3\}\}.$$

A route-planning system must select paths, but each road segment has multiple plausible states (free-flow, moderate congestion, severe congestion/incident), and the dispatcher may hesitate among several assessments from different data sources (historical patterns, sensors, driver reports).

Hesitant neutrosophic vertex labels. Vertices may have uncertain service readiness (e.g., loading dock availability, queue length). For example, customer 1 may be assessed as

$$h_V(1) = \{(0.80, 0.10, 0.15), (0.60, 0.25, 0.20)\},$$

where (t, i, f) informally encode “service feasibility” (truth), ambiguity (indeterminacy), and risk of failure (falsity). Similarly, for the depot

$$h_V(0) = \{(0.95, 0.05, 0.05)\}.$$

Hesitant neutrosophic edge labels. Each edge $e \in E$ receives a finite set $h_E(e) \subseteq [0, 1]^3$ describing plausible travel reliability/uncertainty levels. For instance, the road segment $\{0, 1\}$ may have two competing assessments (normal vs. congestion expected):

$$h_E(\{0, 1\}) = \{(0.85, 0.10, 0.10), (0.55, 0.25, 0.30)\}.$$

A frequently congested segment $\{1, 3\}$ might be

$$h_E(\{1, 3\}) = \{(0.70, 0.15, 0.20), (0.40, 0.30, 0.40)\}.$$

Edges not present in E are simply absent from the model.

In a robust or multi-objective VRP, a planner can aggregate $h_E(e)$ along a candidate route to estimate (i) expected route feasibility (via truth degrees), (ii) ambiguity due to conflicting traffic information (via indeterminacy degrees), and (iii) disruption risk (via falsity degrees). The hesitant structure captures multiple plausible assessments rather than forcing a single probability or a single membership value, which is useful when data sources disagree or the state changes rapidly.

Consistency with Definition 3.1. All triples above satisfy $0 \leq t + i + f \leq 3$. Moreover, the edge assessments can be chosen so that each $(t, i, f) \in h_E(\{u, v\})$ respects the incidence constraints

$$t \leq \min\{T_V^+(u), T_V^+(v)\}, \quad i \leq \min\{I_V^+(u), I_V^+(v)\}, \quad f \geq \max\{F_V^-(u), F_V^-(v)\},$$

where T_V^+ , I_V^+ , F_V^- are computed from the hesitant vertex labels.

Theorem 3.4 (HNGs generalize single-valued neutrosophic graphs). Let $G^* = (V, E)$ be a crisp graph. Suppose $G = (A, B)$ is a single-valued neutrosophic graph on G^* , i.e.,

$$T_A, I_A, F_A : V \rightarrow [0, 1], \quad T_B, I_B, F_B : E \rightarrow [0, 1],$$

satisfy $0 \leq T_A(v) + I_A(v) + F_A(v) \leq 3$ for all $v \in V$ and, for each $e = \{u, v\} \in E$,

$$T_B(e) \leq \min\{T_A(u), T_A(v)\}, \quad I_B(e) \leq \min\{I_A(u), I_A(v)\}, \quad F_B(e) \geq \max\{F_A(u), F_A(v)\}.$$

Define hesitant labelings h_V, h_E by singleton sets:

$$h_V(v) := \{(T_A(v), I_A(v), F_A(v))\} \quad (v \in V), \quad h_E(e) := \{(T_B(e), I_B(e), F_B(e))\} \quad (e \in E).$$

Then (h_V, h_E) is a hesitant neutrosophic graph on G^* in the sense of Definition 3.1. Consequently, the class of single-valued neutrosophic graphs embeds as a special case of hesitant neutrosophic graphs.



Proof. Fix $v \in V$. Since $h_V(v)$ is a singleton, every element $(t, i, f) \in h_V(v)$ equals $(T_A(v), I_A(v), F_A(v))$, and the assumed bound $0 \leq T_A(v) + I_A(v) + F_A(v) \leq 3$ verifies Definition 3.1(i) for vertices. The same argument using $0 \leq T_B(e) + I_B(e) + F_B(e) \leq 3$ (which holds in standard single-valued neutrosophic graph models) verifies (i) for edges.

Moreover, by construction,

$$T_V^+(v) = T_A(v), \quad I_V^+(v) = I_A(v), \quad F_V^-(v) = F_A(v).$$

Now fix an edge $e = \{u, v\} \in E$ and the unique element $(t, i, f) = (T_B(e), I_B(e), F_B(e)) \in h_E(e)$. Then the single-valued edge constraints yield

$$\begin{aligned} t &= T_B(e) \leq \min\{T_A(u), T_A(v)\} = \min\{T_V^+(u), T_V^+(v)\}, \\ i &= I_B(e) \leq \min\{I_A(u), I_A(v)\} = \min\{I_V^+(u), I_V^+(v)\}, \\ f &= F_B(e) \geq \max\{F_A(u), F_A(v)\} = \max\{F_V^-(u), F_V^-(v)\}. \end{aligned}$$

Thus Definition 3.1(iii) holds, so (h_V, h_E) is an HNG. $\square$

3.2 Hesitant Neutrosophic HyperGraph

A hesitant neutrosophic hypergraph models multiway relations by assigning each vertex–hyperedge incidence finite possible truth, indeterminacy, falsity triples.

Definition 3.5 (Hesitant neutrosophic hyperedge profile). Let V be a nonempty set. A hesitant neutrosophic incidence profile on V is a map

$$h : V \longrightarrow \mathcal{P}_{\text{fin}}^*([0, 1]^3)$$

such that for every $v \in V$ and every $(t, i, f) \in h(v)$,

$$0 \leq t + i + f \leq 3.$$

Its support is

$$\text{supp}(h) := \left\{ v \in V : \exists (t, i, f) \in h(v) \text{ with } (t, i, f) \neq (0, 0, 0) \right\}.$$

Definition 3.6 (Hesitant neutrosophic hypergraph (HNHG)). Let $H^* = (V, \mathcal{E})$ be a crisp hypergraph, i.e., $\emptyset \neq V$ and $\mathcal{E} \subseteq \mathcal{P}(V) \setminus \{\emptyset\}$. A hesitant neutrosophic hypergraph on H^* is a tuple

$$\mathcal{H} = (V, \mathcal{E}, h_V, h_{\mathcal{E}}),$$

where

$$h_V : V \rightarrow \mathcal{P}_{\text{fin}}^*([0, 1]^3), \quad h_{\mathcal{E}} : \mathcal{E} \times V \rightarrow \mathcal{P}_{\text{fin}}^*([0, 1]^3),$$

satisfying the following conditions.

(i) (vertex admissibility) For every $v \in V$ and every $(t, i, f) \in h_V(v)$,

$$0 \leq t + i + f \leq 3.$$

(ii) (incidence admissibility) For every $e \in \mathcal{E}$, $v \in V$, and every $(t, i, f) \in h_{\mathcal{E}}(e, v)$,

$$0 \leq t + i + f \leq 3.$$

(iii) (support compatibility) If $v \notin e$, then

$$h_{\mathcal{E}}(e, v) = \{(0, 0, 0)\}.$$

(Equivalently, $\text{supp}(v \mapsto h_{\mathcal{E}}(e, v)) \subseteq e$.)

(iv) (endpoint envelope constraints) Define, for each $v \in V$,

$$T_V^+(v) := \max\{t : (t, i, f) \in h_V(v)\}, \quad I_V^+(v) := \max\{i : (t, i, f) \in h_V(v)\},$$

$$F_V^-(v) := \min\{f : (t, i, f) \in h_V(v)\}.$$

Then for every $e \in \mathcal{E}$, every $v \in e$, and every $(t, i, f) \in h_{\mathcal{E}}(e, v)$,

$$t \leq T_V^+(v), \quad i \leq I_V^+(v), \quad f \geq F_V^-(v).$$

If moreover

$$V = \bigcup_{e \in \mathcal{E}} \text{supp}(v \mapsto h_{\mathcal{E}}(e, v)),$$

then $\mathcal{H}$ is called vertex-spanning.

Remark 3.7 (Edge-constant specialization). If for each $e \in \mathcal{E}$ the incidence set $h_{\mathcal{E}}(e, v)$ is independent of $v \in e$ (i.e., $h_{\mathcal{E}}(e, u) = h_{\mathcal{E}}(e, v)$ for all $u, v \in e$), then each hyperedge carries a single hesitant triple-set label. This specialization is convenient for embedding hesitant neutrosophic graphs into the hypergraph framework.



Example 3.8 (Operations research: emergency medical dispatch with uncertain team feasibility). Consider an emergency medical service (EMS) dispatcher who must decide which teams of resources should be sent together to incident types under uncertain and partially conflicting information (e.g., traffic conditions, staff availability, equipment readiness, and fluctuating demand).

Crisp hypergraph layer. Let the vertex set collect dispatchable resources

$$V = \{a_1, a_2, a_3\} \cup \{v_1, v_2\} \cup \{d_1\},$$

where a_i are ambulances, v_j are volunteer responders, and d_1 is a drone carrying an AED. Let the hyperedge family encode multi-resource response bundles that can jointly cover a given incident type:

$$\mathcal{E} = \{e_1, e_2, e_3\},$$

with

$$\begin{aligned} e_1 &= \{a_1, v_1\} \quad (\text{cardiac arrest: ambulance} + \text{volunteer}), & e_2 &= \{a_2, d_1\} \quad (\text{cardiac arrest: ambulance} + \text{AED drone}), \\ e_3 &= \{a_3, v_2, d_1\} \quad (\text{mass event: ambulance} + \text{volunteer} + \text{drone}). \end{aligned}$$

Thus, each hyperedge represents a coalition of resources whose simultaneous dispatch is operationally meaningful.

Hesitant neutrosophic semantics. Decision makers may hesitate among several plausible assessments of reliability or feasibility. We therefore attach to each resource $x \in V$ a finite set $h_V(x) \subseteq [0, 1]^3$ of possible (truth, indeterminacy, falsity) triples, and to each incidence (e, x) a finite set $h_{\mathcal{E}}(e, x) \subseteq [0, 1]^3$ describing how confidently resource x can contribute to bundle e .

For instance, for ambulance a_1 we may have two assessments reflecting good vs. congested traffic:

$$h_V(a_1) = \{(0.85, 0.10, 0.10), (0.60, 0.25, 0.25)\}.$$

For volunteer v_1 (availability is uncertain) and drone d_1 (battery and weather uncertainty) we may set

$$h_V(v_1) = \{(0.70, 0.20, 0.20), (0.50, 0.35, 0.25)\}, \quad h_V(d_1) = \{(0.75, 0.15, 0.20), (0.55, 0.30, 0.25)\}.$$

For the coalition $e_1 = \{a_1, v_1\}$, the incidence profiles may reflect different estimates of coordination:

$$h_{\mathcal{E}}(e_1, a_1) = \{(0.80, 0.10, 0.15)\}, \quad h_{\mathcal{E}}(e_1, v_1) = \{(0.65, 0.20, 0.20), (0.55, 0.30, 0.25)\}.$$

For $x \notin e$ we set $h_{\mathcal{E}}(e, x) = \{(0, 0, 0)\}$.

A dispatcher (or an optimization model) can treat each hyperedge $e \in \mathcal{E}$ as a candidate response plan and aggregate hesitant neutrosophic incidence information to score e by expected feasibility (truth), ambiguity (indeterminacy), and risk of failure (falsity). This supports robust coalition selection (choose e), staffing (activate $x \in e$), and contingency planning under simultaneous multi-resource interactions that cannot be reduced to pairwise edges.

Theorem 3.9 (HNHG generalizes hesitant neutrosophic graphs). Let $G^* = (V, E)$ be a crisp (simple, undirected) graph, viewed as a 2-uniform hypergraph $H^* = (V, \mathcal{E})$ with $\mathcal{E} = E \subseteq \binom{V}{2}$. Assume $G = (h_V, h_E)$ is a hesitant neutrosophic graph on G^* in the sense that

$$h_V : V \rightarrow \mathcal{P}_{\text{fin}}^*([0, 1]^3), \quad h_E : E \rightarrow \mathcal{P}_{\text{fin}}^*([0, 1]^3),$$

satisfy admissibility, and for each edge $e = \{u, v\} \in E$ and each $(t, i, f) \in h_E(e)$,

$$t \leq \min\{T_V^+(u), T_V^+(v)\}, \quad i \leq \min\{I_V^+(u), I_V^+(v)\}, \quad f \geq \max\{F_V^-(u), F_V^-(v)\},$$

where T_V^+, I_V^+, F_V^- are computed from h_V as in Definition 3.6(iv). Define $h_{\mathcal{E}} : \mathcal{E} \times V \rightarrow \mathcal{P}_{\text{fin}}^*([0, 1]^3)$ by

$$h_{\mathcal{E}}(e, x) := \begin{cases} h_E(e), & x \in e, \\ \{(0, 0, 0)\}, & x \notin e. \end{cases}$$

Then $\mathcal{H} = (V, \mathcal{E}, h_V, h_{\mathcal{E}})$ is a hesitant neutrosophic hypergraph (Definition 3.6). Hence, every hesitant neutrosophic graph canonically embeds into the class of hesitant neutrosophic hypergraphs.

Proof. Conditions (i) and (ii) in Definition 3.6 hold because $h_V(v)$ and $h_E(e)$ satisfy the hesitant neutrosophic admissibility constraints, and $\{(0, 0, 0)\}$ is also admissible. Condition (iii) holds by the definition of $h_{\mathcal{E}}$.

For (iv), fix $e = \{u, v\} \in \mathcal{E}$ and $x \in e$. Any $(t, i, f) \in h_{\mathcal{E}}(e, x) = h_E(e)$ satisfies

$$t \leq \min\{T_V^+(u), T_V^+(v)\} \leq T_V^+(x), \quad i \leq \min\{I_V^+(u), I_V^+(v)\} \leq I_V^+(x),$$

and

$$f \geq \max\{F_V^-(u), F_V^-(v)\} \geq F_V^-(x),$$

so the endpoint envelope constraints in Definition 3.6(iv) are satisfied. Therefore $\mathcal{H}$ is an HNHG. $\square$



Theorem 3.10 (HNHG generalize single-valued neutrosophic hypergraphs). Let V be a finite set. Consider a single-valued neutrosophic hypergraph given by

$$H = (V, \{E_j\}_{j=1}^m),$$

where each hyperedge E_j is specified by three maps

$$T_j, I_j, F_j : V \rightarrow [0, 1]$$

(with $0 \leq T_j(v) + I_j(v) + F_j(v) \leq 3$ for all $v \in V$), and such that $V = \bigcup_{j=1}^m \text{supp}(T_j, I_j, F_j)$, where $\text{supp}(T_j, I_j, F_j) = \{v : T_j(v) > 0 \text{ or } I_j(v) > 0 \text{ or } F_j(v) > 0\}$. Define a crisp hypergraph $H^* = (V, \mathcal{E})$ by

$$\mathcal{E} := \{\text{supp}(T_j, I_j, F_j) : j = 1, \dots, m\}.$$

Define $h_V : V \rightarrow \mathcal{P}_{\text{fin}}^*([0, 1]^3)$ by the constant singleton labeling

$$h_V(v) := \{(1, 1, 0)\} \quad (v \in V),$$

and define $h_{\mathcal{E}} : \mathcal{E} \times V \rightarrow \mathcal{P}_{\text{fin}}^*([0, 1]^3)$ as follows: for $e = \text{supp}(T_j, I_j, F_j) \in \mathcal{E}$ set

$$h_{\mathcal{E}}(e, v) := \begin{cases} \{(T_j(v), I_j(v), F_j(v))\}, & v \in e, \\ \{(0, 0, 0)\}, & v \notin e. \end{cases}$$

Then $\mathcal{H} = (V, \mathcal{E}, h_V, h_{\mathcal{E}})$ is a vertex-spanning hesitant neutrosophic hypergraph (Definition 3.6). In particular, the class of single-valued neutrosophic hypergraphs is contained (via singleton hesitation) in the class of hesitant neutrosophic hypergraphs.

Proof. Vertex admissibility (i) holds since $(1, 1, 0) \in [0, 1]^3$ and $1 + 1 + 0 = 2 \leq 3$. Incidence admissibility (ii) holds because each $(T_j(v), I_j(v), F_j(v))$ lies in $[0, 1]^3$ and satisfies the assumed bound $0 \leq T_j(v) + I_j(v) + F_j(v) \leq 3$; also $(0, 0, 0)$ is admissible. Support compatibility (iii) is built into the definition of $h_{\mathcal{E}}$.

For (iv), we have for every $v \in V$:

$$T_V^+(v) = 1, \quad I_V^+(v) = 1, \quad F_V^-(v) = 0.$$

Hence for any $e \in \mathcal{E}$, any $v \in e$, and the unique triple $(t, i, f) = (T_j(v), I_j(v), F_j(v)) \in h_{\mathcal{E}}(e, v)$,

$$t \leq 1 = T_V^+(v), \quad i \leq 1 = I_V^+(v), \quad f \geq 0 = F_V^-(v),$$

so the endpoint envelope constraints hold.

Finally, because $V = \bigcup_{j=1}^m \text{supp}(T_j, I_j, F_j)$ by assumption and $\text{supp}(v \mapsto h_{\mathcal{E}}(e, v)) = e$ for each $e \in \mathcal{E}$, we obtain

$$V = \bigcup_{e \in \mathcal{E}} \text{supp}(v \mapsto h_{\mathcal{E}}(e, v)),$$

so $\mathcal{H}$ is vertex-spanning. □

3.3 Hesitant Neutrosophic SuperHyperGraph

A Hesitant Neutrosophic SuperHyperGraph is a superhypergraph whose nested set-valued vertices and incidences carry finite sets of truth, indeterminacy, falsity degrees, enabling hesitation under uncertainty.

Definition 3.11 (Hesitant neutrosophic incidence profile). Let X be a nonempty set. A hesitant neutrosophic profile on X is a map

$$h : X \longrightarrow \mathcal{P}_{\text{fin}}^*([0, 1]^3)$$

such that for every $x \in X$ and every $(t, i, f) \in h(x)$,

$$0 \leq t + i + f \leq 3.$$

Its support is

$$\text{supp}(h) := \{x \in X : \exists (t, i, f) \in h(x) \text{ with } (t, i, f) \neq (0, 0, 0)\}.$$

Definition 3.12 (Hesitant neutrosophic n -SuperHyperGraph). Let V_0 be a finite nonempty ground set and let $n \in \mathbb{N}_0$. Let $\text{SHG}^{(n)} = (V, E)$ be an n -SuperHyperGraph on V_0 in the sense that

$$V \subseteq \mathcal{P}^n(V_0), \quad E \subseteq \mathcal{P}(V) \setminus \{\emptyset\}.$$

A hesitant neutrosophic n -SuperHyperGraph (HN- n SHG) on $\text{SHG}^{(n)}$ is a tuple

$$\mathcal{H} = (V, E, h_V, h_E),$$

where

$$h_V : V \rightarrow \mathcal{P}_{\text{fin}}^*([0, 1]^3), \quad h_E : E \times V \rightarrow \mathcal{P}_{\text{fin}}^*([0, 1]^3),$$

satisfying the following axioms.

(i) (vertex admissibility) For every $v \in V$ and every $(t, i, f) \in h_V(v)$,

$$0 \leq t + i + f \leq 3.$$



(ii) (incidence admissibility) For every $e \in E$, $v \in V$, and every $(t, i, f) \in h_E(e, v)$,

$$0 \leq t + i + f \leq 3.$$

(iii) (support compatibility) If $v \notin e$, then

$$h_E(e, v) = \{(0, 0, 0)\}.$$

Equivalently, for each $e \in E$, the support of the profile $v \mapsto h_E(e, v)$ is contained in e .

(iv) (containment via envelopes) Define for each $v \in V$ the envelopes

$$T_V^+(v) := \max\{t : (t, i, f) \in h_V(v)\}, \quad I_V^+(v) := \max\{i : (t, i, f) \in h_V(v)\},$$

$$F_V^-(v) := \min\{f : (t, i, f) \in h_V(v)\}.$$

Then for every $e \in E$, every $v \in e$, and every $(t, i, f) \in h_E(e, v)$,

$$t \leq T_V^+(v), \quad i \leq I_V^+(v), \quad f \geq F_V^-(v).$$

If moreover

$$V = \bigcup_{e \in E} \text{supp}(v \mapsto h_E(e, v)),$$

then $\mathcal{H}$ is called vertex-spanning.

Remark 3.13 (Level-agnostic nature). Definition 3.12 is combinatorially level-agnostic: the hesitant neutrosophic degrees are attached to supervertices $v \in V \subseteq \mathcal{P}^n(V_0)$ and to incidences (e, v) with $v \in e$, regardless of the internal nesting depth of v . This matches the standard practice for neutrosophic labeling on set-valued vertex domains.

Example 3.14 (Operations research: hierarchical procurement and subcontracting under uncertain feasibility). Consider a project-based manufacturer planning procurement for a complex product (e.g., a vehicle or a plant), where decisions are naturally hierarchical: parts aggregate into modules, modules aggregate into subsystems, and subsystems aggregate into a final deliverable. In operations research, such settings appear in supplier selection, subcontracting, and multi-tier bill-of-materials (BOM) planning under uncertainty.

Ground set (level 0). Let

$$V_0 = \{s_1, s_2, s_3, \ell_1, \ell_2, w_1\},$$

where s_i are candidate suppliers, ℓ_j are logistics providers, and w_1 is a warehouse.

Supervertices (level $n = 2$). Let $n = 2$ and select supervertices in $V \subseteq \mathcal{P}^2(V_0)$ that represent bundled options and bundles of bundles. For example,

$$v_A = \{\{s_1, \ell_1\}, \{s_2, \ell_1\}\} \in \mathcal{P}^2(V_0) \quad (\text{two alternative supplier-logistics packages for Module A}),$$

$$v_B = \{\{s_2, \ell_2\}, \{s_3, \ell_2\}\} \in \mathcal{P}^2(V_0) \quad (\text{two alternative packages for Module B}),$$

$$v_S = \{\{v_A, w_1\}, \{v_B, w_1\}\} \in \mathcal{P}^2(V_0) \quad (\text{a subsystem choice: module choice combined with warehousing}).$$

Let

$$V = \{v_A, v_B, v_S\} \subseteq \mathcal{P}^2(V_0).$$

Superedges (multiway coordination constraints). Let $E \subseteq \mathcal{P}(V) \setminus \{\emptyset\}$ encode coordination requirements among these hierarchical options. For instance,

$$e_1 = \{v_A, v_B\} \quad (\text{Modules A and B must be jointly feasible within the same schedule window}),$$

$$e_2 = \{v_A, v_B, v_S\} \quad (\text{full subsystem plan must be feasible end-to-end}).$$

Thus $\text{SHG}^{(2)} = (V, E)$ is a crisp 2-SuperHyperGraph.

Hesitant neutrosophic assessments. Because feasibility is uncertain (capacity, lead times, disruptions, contract risk), managers may hesitate among several plausible evaluations from different data sources or experts. Attach to each supervertex $v \in V$ a finite set $h_V(v) \subseteq [0, 1]^3$ of (truth, indeterminacy, falsity) triples, e.g.

$$h_V(v_A) = \{(0.80, 0.15, 0.10), (0.60, 0.30, 0.25)\}, \quad h_V(v_B) = \{(0.75, 0.15, 0.15), (0.55, 0.25, 0.30)\},$$

$$h_V(v_S) = \{(0.70, 0.20, 0.20), (0.50, 0.35, 0.30)\}.$$

Similarly, for each incidence (e, v) with $v \in e$, assign a finite set $h_E(e, v)$ reflecting how confidently v can participate in the multiway constraint represented by e , for example

$$h_E(e_2, v_A) = \{(0.75, 0.15, 0.15)\}, \quad h_E(e_2, v_B) = \{(0.70, 0.20, 0.20)\}, \quad h_E(e_2, v_S) = \{(0.65, 0.25, 0.20)\}.$$

For $v \notin e$ set $h_E(e, v) = \{(0, 0, 0)\}$.

A procurement optimizer can treat each 2-supervertex as a hierarchical decision option (alternative packages; packages-of-packages), and each superedge as a joint feasibility constraint. Aggregating hesitant neutrosophic values supports robust selection of module/subsystem plans by balancing: (i) feasibility (truth), (ii) ambiguity due to conflicting forecasts (indeterminacy), and (iii) disruption or contract failure risk (falsity), in a way that explicitly preserves expert hesitation.



Theorem 3.15 (HN-0SHGs are exactly hesitant neutrosophic hypergraphs). Let V_0 be a finite nonempty set and let $n = 0$. Then 0-SuperHyperGraphs on V_0 are precisely crisp hypergraphs (V_0, E) , and Definition 3.12 specializes to Definition 3.6 (i.e., hesitant neutrosophic hypergraphs). Consequently, hesitant neutrosophic hypergraphs embed canonically into hesitant neutrosophic n -SuperHyperGraphs (by taking $n = 0$).

Proof. When $n = 0$, we have $\mathcal{P}^0(V_0) = V_0$, so $V \subseteq V_0$. In the usual hypergraph convention we take $V = V_0$ (or replace V_0 by V). Also $E \subseteq \mathcal{P}(V) \setminus \{\emptyset\}$ is exactly a crisp hyperedge family. The hesitant label maps in Definition 3.12 are

$$h_V : V \rightarrow \mathcal{P}_{\text{fin}}^*([0, 1]^3), \quad h_E : E \times V \rightarrow \mathcal{P}_{\text{fin}}^*([0, 1]^3),$$

with the same admissibility, support-compatibility, and envelope containment constraints as in Definition 3.6. Hence the notions coincide. $\square$

Definition 3.16 (Vertex-induced sub-HN- n SHG). Let $U \subseteq V$. Define the induced crisp edge family

$$E[U] := \{e \in E \mid e \subseteq U\}.$$

The vertex-induced sub-HN- n SHG is

$$H[U] := (U, E[U], h_V|_U, h_E|_{E[U] \times U}).$$

Proposition 3.17 (Closure under vertex-induced restriction). For every $U \subseteq V$, the structure $H[U]$ is an HN- n SHG (with underlying crisp n -SuperHyperGraph $(U, E[U])$).

Proof. Vertex admissibility and incidence admissibility are inherited by restriction. Support compatibility holds because if $e \in E[U] \subseteq E$ and $v \in U$ with $v \notin e$, then already in H one has $h_E(e, v) = \{(0, 0, 0)\}$, hence the same is true after restriction. For envelope containment: the envelopes $T_V^+(v), I_V^+(v), F_V^-(v)$ depend only on $h_V(v)$, which is unchanged for $v \in U$, and every $(t, i, f) \in h_E(e, v)$ remains the same; therefore the inequalities required in the HN- n SHG axioms continue to hold. $\square$

Definition 3.18 (HN- n SHG isomorphism). Let $H = (V, E, h_V, h_E)$ and $H' = (V', E', h'_V, h'_E)$ be HN- n SHGs. An isomorphism is a bijection $\varphi : V \rightarrow V'$ such that:

- (i) $e \in E \iff \varphi(e) := \{\varphi(v) : v \in e\} \in E'$,
- (ii) $h'_V(\varphi(v)) = h_V(v)$ for all $v \in V$,
- (iii) $h'_E(\varphi(e), \varphi(v)) = h_E(e, v)$ for all $e \in E$ and $v \in V$.

Proposition 3.19 (Isomorphism invariants: connectivity, paths, cycles, cuts). If $\varphi : H \rightarrow H'$ is an isomorphism, then the underlying crisp n -SuperHyperGraphs $\underline{H}$ and $\underline{H}'$ are isomorphic. Consequently, any property of $\underline{H}$ defined purely in terms of (V, E) —such as existence of (Berge) paths, connectivity, (Berge) cycles, and cut-edges with respect to a vertex bipartition—is preserved under φ .

Proof. Condition (i) is exactly the statement that $e \mapsto \varphi(e)$ gives a bijection $E \rightarrow E'$ compatible with vertex incidence. Thus φ is an isomorphism of the underlying crisp hypergraphs/superhypergraphs. Standard invariance of combinatorial properties under graph/hypergraph isomorphism yields the claim. $\square$

Definition 3.20 (Crisp and envelope-extended degrees). The crisp degree of $v \in V$ is

$$\deg_{\underline{H}}(v) := |\{e \in E : v \in e\}|.$$

The truth-, indeterminacy-, and falsity-extended degrees are

$$\deg_T(v) := \sum_{e \in E : v \in e} T_E^+(e, v), \quad \deg_I(v) := \sum_{e \in E : v \in e} I_E^+(e, v), \quad \deg_F(v) := \sum_{e \in E : v \in e} F_E^-(e, v).$$

Proposition 3.21 (Incidence-envelope bounds and degree bounds). For every $e \in E$ and $v \in e$,

$$T_E^+(e, v) \leq T_V^+(v), \quad I_E^+(e, v) \leq I_V^+(v), \quad F_E^-(e, v) \geq F_V^-(v).$$

Consequently, for every $v \in V$,

$$\deg_T(v) \leq \deg_{\underline{H}}(v) T_V^+(v), \quad \deg_I(v) \leq \deg_{\underline{H}}(v) I_V^+(v), \quad \deg_F(v) \geq \deg_{\underline{H}}(v) F_V^-(v).$$

Proof. Fix $e \in E$ and $v \in e$. By the envelope containment axiom, every $(t, i, f) \in h_E(e, v)$ satisfies $t \leq T_V^+(v)$, $i \leq I_V^+(v)$, and $f \geq F_V^-(v)$. Taking the maximum of the t -coordinates over $h_E(e, v)$ gives $T_E^+(e, v) \leq T_V^+(v)$; similarly for I . Taking the minimum of the f -coordinates gives $F_E^-(e, v) \geq F_V^-(v)$. Summing these inequalities over all $e \ni v$ yields the stated degree bounds. $\square$

Proposition 3.22 (Isomorphism invariance of degree sequences). If $\varphi : H \rightarrow H'$ is an isomorphism, then for all $v \in V$,

$$\deg_{\underline{H}}(v) = \deg_{\underline{H}'}(\varphi(v)), \quad \deg_T(v) = \deg'_T(\varphi(v)), \quad \deg_I(v) = \deg'_I(\varphi(v)), \quad \deg_F(v) = \deg'_F(\varphi(v)).$$

Proof. The crisp degree is preserved because φ is an isomorphism of $\underline{H}$. For the extended degrees, use that φ bijects incidences (e, v) with $(\varphi(e), \varphi(v))$ and preserves the incidence triple-sets: $h'_E(\varphi(e), \varphi(v)) = h_E(e, v)$, hence the envelopes T_E^+, I_E^+, F_E^- match term-by-term in the defining sums. $\square$



Definition 3.23 (Set-valued incidence matrix and envelope matrices). Assume $V = \{v_1, \dots, v_N\}$ and $E = \{e_1, \dots, e_M\}$ are finite. The hesitant neutrosophic incidence matrix of H is the $N \times M$ matrix

$$\mathbf{M}(H) := (m_{ij}), \quad m_{ij} := h_E(e_j, v_i) \in \mathcal{P}_{\text{fin}}^*([0, 1]^3).$$

Define three real $N \times M$ envelope incidence matrices by

$$\mathbf{T}(H) := (T_{ij}), \quad T_{ij} := T_E^+(e_j, v_i), \quad \mathbf{I}(H) := (I_{ij}), \quad I_{ij} := I_E^+(e_j, v_i), \quad \mathbf{F}(H) := (F_{ij}), \quad F_{ij} := F_E^-(e_j, v_i).$$

Proposition 3.24 (Vertex-envelope domination in matrix form). For every i, j with $v_i \in e_j$, one has

$$T_{ij} \leq T_V^+(v_i), \quad I_{ij} \leq I_V^+(v_i), \quad F_{ij} \geq F_V^-(v_i).$$

If $v_i \notin e_j$, then $m_{ij} = \{(0, 0, 0)\}$ and $T_{ij} = I_{ij} = F_{ij} = 0$.

Proof. Immediate from the HN- n SHG envelope containment axiom and support compatibility. $\square$

Definition 3.25 (A generic scalarization (score/accuracy template)). Let $\sigma : [0, 1]^3 \rightarrow \mathbb{R}$ be any function. For a finite nonempty set $S \subseteq [0, 1]^3$, define the max-scalarization

$$\sigma^{\max}(S) := \max\{\sigma(t, i, f) : (t, i, f) \in S\}.$$

Applying $\sigma^{\max}$ entrywise to $\mathbf{M}(H)$ yields a real matrix $\mathbf{M}_\sigma(H) := (\sigma^{\max}(m_{ij}))$. Typical choices include

$$\sigma_{\text{score}}(t, i, f) := t - f, \quad \sigma_{\text{acc}}(t, i, f) := t + (1 - f) - i.$$

Proposition 3.26 (Monotonicity of max-scalarization). If $S_1 \subseteq S_2$ are finite nonempty subsets of $[0, 1]^3$, then $\sigma^{\max}(S_1) \leq \sigma^{\max}(S_2)$ for every σ . Consequently, for two HN- n SHGs on the same (V, E) with $h_E^{(1)}(e, v) \subseteq h_E^{(2)}(e, v)$ for all (e, v) , one has $\mathbf{M}_\sigma(H_1) \leq \mathbf{M}_\sigma(H_2)$ entrywise.

Proof. The maximum of σ over a set cannot decrease when the set is enlarged. Apply this entrywise. $\square$

Definition 3.27 ((α, β, γ)-vertex core and induced cut superhypergraph). Fix thresholds $\alpha, \beta, \gamma \in [0, 1]$. Define the (α, β, γ)-vertex core by

$$V_{\alpha, \beta, \gamma} := \{v \in V : T_V^+(v) \geq \alpha, I_V^+(v) \geq \beta, F_V^-(v) \leq \gamma\}.$$

Define the (α, β, γ)-incidence-admissible edge family

$$E_{\alpha, \beta, \gamma} := \left\{ e \in E \mid e \subseteq V_{\alpha, \beta, \gamma} \text{ and } \forall v \in e : T_E^+(e, v) \geq \alpha, I_E^+(e, v) \geq \beta, F_E^-(e, v) \leq \gamma \right\}.$$

The (α, β, γ)-cut (crisp) n -SuperHyperGraph extracted from H is

$$\underline{H}_{\alpha, \beta, \gamma} := (V_{\alpha, \beta, \gamma}, E_{\alpha, \beta, \gamma}).$$

Theorem 3.28 ((α, β, γ)-cuts are well-defined and monotone). For every $\alpha, \beta, \gamma \in [0, 1]$, the pair $\underline{H}_{\alpha, \beta, \gamma}$ is a (crisp) n -SuperHyperGraph (in the sense that $V_{\alpha, \beta, \gamma} \subseteq V$ and $E_{\alpha, \beta, \gamma} \subseteq \mathcal{P}(V_{\alpha, \beta, \gamma}) \setminus \{\emptyset\}$). Moreover, if

$$\alpha_1 \geq \alpha_2, \quad \beta_1 \geq \beta_2, \quad \gamma_1 \leq \gamma_2,$$

then

$$V_{\alpha_1, \beta_1, \gamma_1} \subseteq V_{\alpha_2, \beta_2, \gamma_2} \quad \text{and} \quad E_{\alpha_1, \beta_1, \gamma_1} \subseteq E_{\alpha_2, \beta_2, \gamma_2},$$

hence $\underline{H}_{\alpha_1, \beta_1, \gamma_1}$ is a (vertex- and edge-)subsuperhypergraph of $\underline{H}_{\alpha_2, \beta_2, \gamma_2}$.

Proof. By definition, $V_{\alpha, \beta, \gamma} \subseteq V$. Also each $e \in E_{\alpha, \beta, \gamma}$ satisfies $e \subseteq V_{\alpha, \beta, \gamma}$ and $e \neq \emptyset$ because $e \in E$ is nonempty in the underlying crisp superhypergraph. Thus $E_{\alpha, \beta, \gamma} \subseteq \mathcal{P}(V_{\alpha, \beta, \gamma}) \setminus \{\emptyset\}$.

For monotonicity: if $\alpha_1 \geq \alpha_2$, $\beta_1 \geq \beta_2$, and $\gamma_1 \leq \gamma_2$, then any vertex meeting the stricter inequalities also meets the weaker ones, so $V_{\alpha_1, \beta_1, \gamma_1} \subseteq V_{\alpha_2, \beta_2, \gamma_2}$. Similarly, if e satisfies the stricter incidence-envelope thresholds, it satisfies the weaker ones, giving $E_{\alpha_1, \beta_1, \gamma_1} \subseteq E_{\alpha_2, \beta_2, \gamma_2}$. $\square$

Corollary 3.29 (Connectivity/cycle persistence under threshold relaxation). With $\alpha_1 \geq \alpha_2$, $\beta_1 \geq \beta_2$, $\gamma_1 \leq \gamma_2$ as above:

- (i) Any (Berge) path or cycle in $\underline{H}_{\alpha_1, \beta_1, \gamma_1}$ is also a (Berge) path or cycle in $\underline{H}_{\alpha_2, \beta_2, \gamma_2}$.
- (ii) If $\underline{H}_{\alpha_1, \beta_1, \gamma_1}$ is connected, then $\underline{H}_{\alpha_2, \beta_2, \gamma_2}$ is connected.

Proof. This is immediate from the edge/vertex inclusions in the preceding theorem and the fact that paths/cycles/connectivity are monotone under adding vertices/edges. $\square$

Definition 3.30 (Union and (conditional) intersection). Let $H_1 = (V, E, h_V^{(1)}, h_E^{(1)})$ and $H_2 = (V, E, h_V^{(2)}, h_E^{(2)})$ be HN- n SHGs on the same underlying (V, E) . Define the union-labeled structure

$$H_1 \cup H_2 := (V, E, h_V^\cup, h_E^\cup), \quad h_V^\cup(v) := h_V^{(1)}(v) \cup h_V^{(2)}(v), \quad h_E^\cup(e, v) := h_E^{(1)}(e, v) \cup h_E^{(2)}(e, v).$$

If additionally $h_V^{(1)}(v) \cap h_V^{(2)}(v) \neq \emptyset$ for all $v \in V$ and $h_E^{(1)}(e, v) \cap h_E^{(2)}(e, v) \neq \emptyset$ for all $(e, v) \in E \times V$, define the intersection-labeled structure

$$H_1 \cap H_2 := (V, E, h_V^\cap, h_E^\cap), \quad h_V^\cap(v) := h_V^{(1)}(v) \cap h_V^{(2)}(v), \quad h_E^\cap(e, v) := h_E^{(1)}(e, v) \cap h_E^{(2)}(e, v).$$



Proposition 3.31 (Closure under union; closure under intersection with nonemptiness hypotheses). $H_1 \cup H_2$ is an HN- n SHG. Under the stated nonemptiness hypotheses, $H_1 \cap H_2$ is also an HN- n SHG.

Proof. Union: vertex/incident admissibility is preserved because we only collect triples already admissible in each factor. Support compatibility is preserved because if $v \notin e$ then each $h_E^{(k)}(e, v) = \{(0, 0, 0)\}$, hence so is the union. For envelope containment: if $(t, i, f) \in h_E^\cup(e, v)$ then it lies in some $h_E^{(k)}(e, v)$ and satisfies the required inequalities with respect to the corresponding envelopes of $h_V^{(k)}(v)$; but $h_V^\cup(v)$ enlarges the vertex triple-set, so its envelopes dominate appropriately, ensuring containment. Intersection (under hypotheses): the same verifications apply, noting that nonemptiness ensures the codomain $\mathcal{P}_{\text{fin}}^*([0, 1]^3)$ is respected. $\square$

Definition 3.32 (A swap-complement). Define $c : [0, 1]^3 \rightarrow [0, 1]^3$ by $c(t, i, f) := (f, i, t)$. For a finite nonempty $S \subseteq [0, 1]^3$, set $c(S) := \{c(x) : x \in S\}$. Define the swap-complement of H as

$$H^c := (V, E, h_V^c, h_E^c), \quad h_V^c(v) := c(h_V(v)), \quad h_E^c(e, v) := c(h_E(e, v)).$$

Proposition 3.33 (Closure under swap-complement). H^c is an HN- n SHG. Moreover, its vertex envelopes satisfy

$$T_{V^c}^+(v) = \max\{f : (t, i, f) \in h_V(v)\}, \quad F_{V^c}^-(v) = \min\{t : (t, i, f) \in h_V(v)\},$$

and analogous identities hold for incidence envelopes.

Proof. Admissibility is preserved because $t + i + f$ is invariant under swapping t and f . Support compatibility is preserved because $c(0, 0, 0) = (0, 0, 0)$. For envelope containment: if $(t', i', f') \in h_E^c(e, v)$, then $(t', i', f') = (f, i, t)$ for some $(t, i, f) \in h_E(e, v)$. By the envelope containment axiom in H , we have $f \leq \max\{f : (\cdot) \in h_V(v)\} = T_V^+(v)$ and $t \geq \min\{t : (\cdot) \in h_V(v)\} = F_V^-(v)$, while $i' = i \leq I_V^+(v)$ holds similarly. Thus H^c satisfies the HN- n SHG axioms. $\square$

Definition 3.34 (Direct product (tagged base construction)). Let $H_1 = (V_1, E_1, h_V^{(1)}, h_E^{(1)})$ be an HN- n SHG on a base set $V_0^{(1)}$, and $H_2 = (V_2, E_2, h_V^{(2)}, h_E^{(2)})$ an HN- n SHG on a base set $V_0^{(2)}$. Let $U_0 := (V_0^{(1)} \times \{1\}) \dot{\cup} (V_0^{(2)} \times \{2\})$ be the tagged disjoint union base. Define embeddings $\tau_n^{(k)} : \mathcal{P}^n(V_0^{(k)}) \rightarrow \mathcal{P}^n(U_0)$ recursively by

$$\tau_0^{(k)}(x) := (x, k), \quad \tau_{m+1}^{(k)}(A) := \{\tau_m^{(k)}(a) : a \in A\}.$$

For $v_1 \in V_1$ and $v_2 \in V_2$, define the product supervertex

$$v_1 \boxtimes v_2 := \tau_n^{(1)}(v_1) \cup \tau_n^{(2)}(v_2) \in \mathcal{P}^n(U_0).$$

Set

$$V := \{v_1 \boxtimes v_2 : v_1 \in V_1, v_2 \in V_2\}, \quad E := \{e_1 \boxtimes e_2 : e_1 \in E_1, e_2 \in E_2\},$$

where $e_1 \boxtimes e_2 := \{v_1 \boxtimes v_2 : v_1 \in e_1, v_2 \in e_2\}$. For triples $x = (t_1, i_1, f_1)$ and $y = (t_2, i_2, f_2)$ define

$$x \otimes y := (\min\{t_1, t_2\}, \min\{i_1, i_2\}, \max\{f_1, f_2\}).$$

Finally define labelings on V and incidences by

$$h_V(v_1 \boxtimes v_2) := \{x \otimes y : x \in h_V^{(1)}(v_1), y \in h_V^{(2)}(v_2)\},$$

$$h_E(e_1 \boxtimes e_2, v_1 \boxtimes v_2) := \{x \otimes y : x \in h_E^{(1)}(e_1, v_1), y \in h_E^{(2)}(e_2, v_2)\}.$$

The resulting structure is denoted $H_1 \boxtimes H_2$.

Theorem 3.35 (Closure under the direct product $\boxtimes$). $H_1 \boxtimes H_2$ is an HN- n SHG on the tagged base set U_0 . Moreover, for every $v_1 \in V_1$ and $v_2 \in V_2$,

$$T_V^+(v_1 \boxtimes v_2) = \min\{T_{V_1}^+(v_1), T_{V_2}^+(v_2)\}, \quad I_V^+(v_1 \boxtimes v_2) = \min\{I_{V_1}^+(v_1), I_{V_2}^+(v_2)\},$$

$$F_V^-(v_1 \boxtimes v_2) = \max\{F_{V_1}^-(v_1), F_{V_2}^-(v_2)\},$$

and analogous formulas hold for incidence envelopes.

Proof. By construction, $V \subseteq \mathcal{P}^n(U_0)$ and $E \subseteq \mathcal{P}(V) \setminus \{\emptyset\}$, so (V, E) is a crisp n -SuperHyperGraph on U_0 .

Admissibility: if $x = (t_1, i_1, f_1)$ and $y = (t_2, i_2, f_2)$ are admissible, then $x \otimes y$ has coordinates in $[0, 1]$ and satisfies $\min(t_1, t_2) + \min(i_1, i_2) + \max(f_1, f_2) \leq (t_1 + i_1 + f_1) + (t_2 + i_2 + f_2) \leq 6$, hence in particular it is an admissible triple under the same $[0, 1]^3$ constraint (the HN- n SHG condition is $0 \leq t + i + f \leq 3$ for each triple; if one prefers to enforce this bound strictly, one may replace $\otimes$ by any standard t-norm/t-conorm combination that preserves $t + i + f \leq 3$). Support compatibility holds because if $v_1 \boxtimes v_2 \notin e_1 \boxtimes e_2$, then either $v_1 \notin e_1$ or $v_2 \notin e_2$, so at least one factor incidence set equals $\{(0, 0, 0)\}$ and thus the product incidence set reduces to $\{(0, 0, 0)\}$.

Envelope containment: for any $(t, i, f) = x \otimes y$ in $h_E(e_1 \boxtimes e_2, v_1 \boxtimes v_2)$, we have $t = \min(t_1, t_2) \leq \min(T_{V_1}^+(v_1), T_{V_2}^+(v_2)) = T_V^+(v_1 \boxtimes v_2)$, similarly for i , and $f = \max(f_1, f_2) \geq \max(F_{V_1}^-(v_1), F_{V_2}^-(v_2)) = F_V^-(v_1 \boxtimes v_2)$. The envelope identities follow by taking maxima/minima over the product sets. $\square$



Definition 3.36 (Superedge contraction (for $n \geq 1$)). Assume $n \geq 1$. Fix a superedge $e_0 \in E$ and define the contracted supervertex

$$v_* := \bigcup_{v \in e_0} v.$$

(For $n \geq 1$, each $v \in V \subseteq \mathcal{P}^n(V_0)$ is a subset of $\mathcal{P}^{n-1}(V_0)$, hence v_* is again a subset of $\mathcal{P}^{n-1}(V_0)$ and so $v_* \in \mathcal{P}^n(V_0)$.) Define the contracted vertex set

$$V' := (V \setminus e_0) \cup \{v_*\}.$$

For each $e \in E$, define its contraction

$$c_{e_0}(e) := (e \setminus e_0) \cup \begin{cases} \{v_*\}, & e \cap e_0 \neq \emptyset, \\ \emptyset, & e \cap e_0 = \emptyset, \end{cases}$$

and set

$$E' := \{c_{e_0}(e) \mid e \in E, c_{e_0}(e) \neq \emptyset\}.$$

Define labelings by

$$h'_V(v) := \begin{cases} h_V(v), & v \in V \setminus e_0, \\ \bigcup_{u \in e_0} h_V(u), & v = v_*, \end{cases} \quad h'_E(c_{e_0}(e), v) := \begin{cases} h_E(e, v), & v \in V \setminus e_0, \\ \bigcup_{u \in e \cap e_0} h_E(e, u), & v = v_*, \end{cases}$$

with the convention that if $e \cap e_0 = \emptyset$ then the second case does not occur. The resulting structure is denoted $H/e_0 := (V', E', h'_V, h'_E)$.

Theorem 3.37 (Closure under superedge contraction and cut-monotonicity). Let $n \geq 1$. Then H/e_0 is an HN- n SHG. Moreover, for any thresholds (α, β, γ) , the contraction map induces an inclusion of crisp cuts: the image of $\underline{H}_{\alpha, \beta, \gamma}$ under c_{e_0} is a subsuperhypergraph of $(H/e_0)_{\alpha, \beta, \gamma}$.

Proof. Crisp structure: by the remark in the definition, $v_* \in \mathcal{P}^n(V_0)$, so $V' \subseteq \mathcal{P}^n(V_0)$. Every $c_{e_0}(e)$ is a nonempty subset of V' , hence $E' \subseteq \mathcal{P}(V') \setminus \{\emptyset\}$.

Admissibility: $h'_V(v)$ and $h'_E(e', v)$ are unions of admissible triple-sets, hence admissible. Support compatibility: if $v \notin e'$ in the contracted structure, then either $v \neq v_*$ and the claim follows from support compatibility in H , or $v = v_*$ and e' has empty intersection with e_0 , so $v_* \notin e'$ by construction.

Envelope containment: for unchanged vertices/incidences it is inherited. For v_* , the envelopes of $h'_V(v_*) = \bigcup_{u \in e_0} h_V(u)$ satisfy

$$T_{V'}^+(v_*) = \max_{u \in e_0} T_V^+(u), \quad I_{V'}^+(v_*) = \max_{u \in e_0} I_V^+(u), \quad F_{V'}^-(v_*) = \min_{u \in e_0} F_V^-(u).$$

If $(t, i, f) \in h'_E(c_{e_0}(e), v_*) = \bigcup_{u \in e \cap e_0} h_E(e, u)$, then for some $u \in e \cap e_0$, $(t, i, f) \in h_E(e, u)$, hence $t \leq T_V^+(u) \leq T_{V'}^+(v_*)$, similarly $i \leq I_{V'}^+(v_*)$, and $f \geq F_V^-(u) \geq F_{V'}^-(v_*)$, proving containment.

For cut-monotonicity: since $T_{V'}^+(v_*)$ is a maximum over the contracted block and $F_{V'}^-(v_*)$ is a minimum, the contracted vertex is (weakly) easier to satisfy the vertex thresholds than any individual $u \in e_0$. Likewise, the contracted incidence envelopes are unions and thus (weakly) increase T_E^+/I_E^+ and (weakly) decrease F_E^- . Therefore, any vertex/edge that survives the cut before contraction maps to a vertex/edge that survives after contraction, yielding the stated inclusion. $\square$

4 Conclusion

We transferred hesitant neutrosophic semantics to network structures by introducing hesitant neutrosophic graphs and extending them to hesitant neutrosophic hypergraphs and hesitant neutrosophic superhypergraphs. In future work, we expect further developments including extensions based on plithogenic sets[24, 8], quantitative evaluation supported by computational experiments, and studies in additional settings such as directed graphs.

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Data Availability

This research is purely theoretical, involving no data collection or analysis. We encourage future researchers to pursue empirical investigations to further develop and validate the concepts introduced here.

Ethical Approval

As this research is entirely theoretical in nature and does not involve human participants or animal subjects, no ethical approval is required. I have not engaged in any unethical conduct.



On the Use of AI

LLM-based software was used for language polishing (e.g., grammar and clarity checks), formatting verification, and assistance in diagnosing potential L^AT_EX compilation errors. It was not used to produce results or in any manner that would compromise research integrity.

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