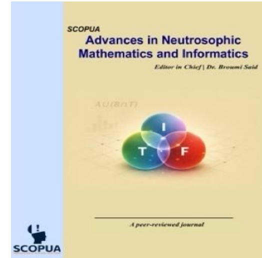
Advances in Neutrosophic Mathematics and Informatics (ANMI)

Vol.1, Issue 1, April 2026
DOI:https://doi.org/10.64060/ANMI1V1i3



Einstein Operators of Bipolar Neutrosophic Soft Matrix and its Application

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Received: 15 March 2026 / Revised: 04 April 2026 / Accepted: 08 April 2026 / Published online: 14 April 2026

ABSTRACT

Bipolar neutrosophic soft matrices (BNSMs) are currently a robust area in several industries. Moreover, bipolar neutrosophic soft matrices are a highly regarded topic in many fields of psychology, qualitative reasoning, multiple criteria decision-making, etc.; issues to be solved in a society can be both negative for membership and non-membership and positive for membership and non-membership. In this manuscript, we define Einstein operators on bipolar neutrosophic soft matrices (BNSMs). Certain properties are investigated, like commutative, associative, identities, etc. Finally, we construct commutative monoid designs on BNSM. Moreover, investigate one example that provides us a new way to propose MCDM issues using the Einstein sum operator over BNSM.

Keywords*:*Bipolar neutrosophic soft matrix (BNSM); Einstein sum; Einstein product; Value matrix; Score matrix; Total score

1. Introduction

Zadeh [1] first introduced fuzzy sets in 1965; fuzzy ideas have spread to almost every field. The idea of fuzzy matrices was initially suggested by Thomason [2] in 1977. Fuzzy matrix elements get values in the restricted range [0, 1]. Einstein operations on fuzzy matrices were investigated by Selvarajan et al. [3]. Additionally, they created certain properties. Selvarajan et al. [4,5] defined Einstein operators on IFMs and extended several algebraic properties.

Zhang [6] was the first to introduce the idea of bipolar fuzzy sets. Bipolar fuzzy matrices were first proposed by Pal and Sanjib Mondal [7] in 2019, and several results on BFMs are discussed. Muhammad Naveed Jafarr et al. [8] presented new technology in agriculture using neutrosophic soft matrices with the help of a score function. Einstein operators over picture fuzzy matrices are proposed by Ramakrishnan and Sriram [9]. Also, a few relations are elucidated.

Ramakrishnan and Sriram [10] discussed algebraic properties that are used to solve some equalities in Einstein operators of Pythagorean fuzzy matrices and their basic operations. Using Eistein operators, some significant properties on Pythagorean fuzzy matrices are discussed by Selvarajan and Ramya [11].

Anitha [12] defined the Einstein operators on BFMs and BIFMs. Several algebraic properties, like commutative, associative, identity, etc., are investigated. Muthuraji and Anitha [13] show that the application of various soils and a more score function have been applied to solve a feasible application of decision-making in agriculture.

Boobalan and Afrine N.S. Shiny [14] constructed two new operators, the Einstein sum and the Einstein product of fuzzy neutrosophic soft matrices. Muthuraji and Anitha [15] defined the weighted similarity measure (WSM) and expanded the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) procedure with totally uncertain criterion weights, which are presented by a new MCDM procedure using BIFMs. In this manuscript, we define Einstein operators on bipolar neutrosophic soft matrices. Several algebraic properties, like commutative, associative, identity, etc., are investigated. Here we are investigating one example that provides us a new way to propose MCDM issues using the Einstein sum operator over BNSMs.

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2. Preliminaries

Several fundamental definitions are presented in this section.

Definition 2.1 [6] A bipolar fuzzy set X is a pair of $(-x_n, x_p)$ where $-x_n : X \to [-1,0]$ and $x_p : X \to [0,1]$ are the respective negative and positive membership degrees of $x \in X$. It is denoted by $B_F(X)$.

Definition 2.2 [7] A Bipolar fuzzy matrix $R = [r_{ij}]$. where r_{ij} represents the expression $\langle -r_{ijn}, r_{ijp} \rangle$ where $r_{ijn}, r_{ijp} \in [0,1]$ $\forall$ i,j are the negative and positive membership values of the element $r_{ij,}$ respectively.

Definition 2.3 . [14] Let X be a neutrosophic set on the set A. It is defined to be as $X = < x_{ij}, x'_{ij}, x''_{ij} >$ such that $x_{ij}, x_{ij}', x_{ij}'' : A \to [0,1]$ where $0 \le x_{ij} + x'_{ij} + x''_{ij} \le 3$ denotes the degree of truth, indeterminacy, and falsity, respectively.

Definition 2.4 [14] A Bipolar neutrosophic fuzzy matrix R of the form $0 \le (-r_{ijn}) + (-r'_{ijn}) + (rx''_{ijn}) + r_{ijp} + r'_{ijp} + r''_{ijp} \le 6$ satisfying the condition, $0 \le (-r_{ijn)} + (-r'_{ijn}) + (rx''_{ijn}) + r_{ijp} + r'_{ijp} + r''_{ijp} \le 6$ for all i,j.

Where $-r_{ijn}, -r_{ijn}', -r_{ijn}'' \in [-1,0]$ and $r_{ijp}, r_{ijp}', r_{ijp}'' \in [1,0]$ is called the Bipolar neutrosophic soft matrix (BNSM).

The set of all bipolar neutrosophic soft matrices denote by $B_{m\times n}$.

Definition 2.5 (Operators in BNSMs) [14]

Let R $=(< -r_{ijn}, -r'_{ijn}, -r''_{ijn} >, < r_{ijp}, r'_{ijp}, r''_{ijp} >)$ and T $=(< -t_{ijn}, -t'_{ijn}, -t''_{ijn} >, < t_{ijp}, t'_{ijp}, t''_{ijp} >) \in BNSM_{mn}$.

Then

$$(i)\ R \vee T = (< -min\,(r_{ijn}, t_{ijn}), -min(r'_{ijn}, t'_{ijn}), -max(r''_{ijn}, t''_{ijn}) >, < min\,(r_{ijp}, t_{ijp}), min(r'_{ijp}, t'_{ijp}), max(r''_{ijp}, t''_{ijp}) >)\ \forall\ i, j.$$

$$(ii)\ R \wedge T = (< -max\,(r_{ijn}, t_{ijn}), -max(r'_{ijn}, t'_{ijn}), -min(r''_{ijn}, t''_{ijn}) >, < max(r_{ijp}, t_{ijp}), max(r'_{ijp}, t'_{ijp}), min(r''_{ijp}, t''_{ijp}) >)\ \forall\ i, j.$$

(iii) R^C stands for the complement of R and is defined as

$$R^C = (< -r''_{ijn}, -r'_{ijn}, -r_{ijn} >, < r''_{ijp}, r'_{ijp}, r_{ijp} >)\ \forall\ i, j.$$

3. Einstein Operators on BNSMs

In this section,we define Einstein sum($\oplus_\in$) and Einstein product ($\otimes_\in$) on Bipolar neutrosophic soft matrices. Also, discuss certain properties over BNSMs.

Definition 3.1.

Let R $=(< -r_{ijn}, -r'_{ijn}, -r''_{ijn} >, < r_{ijp}, r'_{ijp}, r''_{ijp} >)$ and T $=(< -t_{ijn}, -t'_{ijn}, -t''_{ijn} >, < t_{ijp}, t'_{ijp}, t''_{ijp} >) \in BNSM_{mn}$.

Then

$$\text{(i)}\quad R \oplus_\in T = (< -(\frac{r_{ijn} + t_{ijn}}{1 + r_{ijn}\, t_{ijn}}),\ -(\frac{r'_{ijn} + t'_{ijn}}{1 + r'_{ijn}\, t'_{ijn}}), -(\frac{r''_{ijn}\, t''_{ijn}}{1 + (1 - r''_{ijn})(1 - t''_{ijn})}) >, < \frac{r_{ijp} + t_{ijp}}{1 + r_{ijp}\, t_{ijp}}, \frac{r'_{ijp} + t'_{ijp}}{1 + r'_{ijp}\, t'_{ijp}}, \frac{r''_{ijp} + t''_{ijp}}{1 + (1 - r''_{ijp})(1 - t''_{ijp})} >)\quad \forall\ i, j.$$

$$(ii)\quad R \otimes_\in T = (< -(\frac{r_{ijn}\ t_{ijn}}{1 + (1 - r_{ijn})(1 - t_{ijn})}),\ -(\frac{r'_{ijn}\, t'_{ijn}}{1 + (1 - r'_{ijn})(1 - t'_{ijn})}), -(\frac{r''_{ijn} + t''_{ijn}}{1 + r''_{ijn}\, t''_{ijn}}) >, < \frac{r_{ijp}\, t_{ijp}}{1 + (1 - r_{ijp})(1 - t_{ijp})}, \frac{r'_{ijp}\, t'_{ijp}}{1 + (1 - r'_{ijp})(1 - t'_{ijp})}, \frac{r''_{ijp} + t''_{ijp}}{1 + r''_{ijp}\, t''_{ijp}} >)\ \forall\ i, j.$$

The following example illustrates Einstein sum ($\oplus_\in$) and Einstein product($\otimes_\in$) on BNSMs

Example 3.2. For $R, T \in BNSM_{mn}$,

$$\text{Let R} = \begin{bmatrix} (-0.5, -0.4, -0.3, 0.6, 0.4, 0.2) & (-0.8, -0.5, -0.3, 0.1, 0.2, 0.3) \\ (-0.7, -0.1, -0.2, 0.6, 0.2, 0.5) & (-0.7, -0.4, -0.1, 0.5, 0.4, 0.3) \end{bmatrix} \text{ and}$$

$$\text{T} = \begin{bmatrix} (-0.3, -0.2, -0.1, 0.4, 0.5, 0.6) & (-0.5, -0.3, -0.1, 0.5, 0.7, 0.1) \\ (-0.2, -0.3, -0.5, 0.2, 0.3, 0.5) & (-0.6, -0.3, -0.2, 0.6, 0.4, 0.3) \end{bmatrix}$$

Then

$$R \oplus_\in \text{T} = \begin{bmatrix} (-0.69, -0.51, -0.01, 0.81, 0.75, 0.09) & (-0.93, -0.70, -0.41, 0.57, 0.79, 0.38) \\ (-0.79, -0.39, -0.07, 0.71, 0.47, 0.20) & (-0.91, -0.63, -0.01, 0.84, 0.69, 0.06) \end{bmatrix}$$

$$R \otimes_\in \text{T} = \begin{bmatrix} (-0.11, -0.05, -0.39, 0.19, 0.15, 0.71) & (-0.36, -0.11, -0.39, 0.03, 0.11, 0.39) \\ (-0.11, -0.02, -0.5, 0.09, 0.04, 0.8) & (-0.38, -0.08, -0.29, 0.24, 0.18, 0.40) \end{bmatrix}$$

These characteristics are evident. Both commutative and associative properties apply to the operations $\oplus_\in$ and $\otimes_\in$. The following theorems prove the existence of the identity elements with relation to $\oplus_\in$ and $\otimes_\in$.

Theorem 3.3. For R,T ∈ $BNSM_{mn}$,

(i) $R \oplus_{\in} T = T \oplus_{\in} R$.
(ii) $R \otimes_{\in} T = T \otimes_{\in} R$.

Theorem 3.4. For R ∈ $BNSM_{mn}$,

(i) $R \oplus_{\in} O = O \oplus_{\in} R = R$.
(ii) $R \otimes_{\in} J = J \otimes_{\in} R = R$.
(iii) $R \otimes_{\in} O = O$.
(iv) $R \oplus_{\in} J = J$.

Theorem 3.5. For R,S ∈ $BNSM_{mn}$,

(i) $(R \oplus_{\in} S)^t = R^t \oplus_{\in} S^t$
(ii) $(R \otimes_{\in} S)^t = R^t \otimes_{\in} S^t$, Where R^T is the transpose of R.

Theorem 3.6. For R,T ,S ∈ $BNSM_{mn}$,

(i) $(R \oplus_{\in} T) \oplus_{\in} S = R \oplus_{\in} (T \oplus_{\in} S)$
(ii) $(R \otimes_{\in} T) \otimes_{\in} S = R \otimes_{\in} (T \otimes_{\in} S)$

Thus ($BNSM_{mn} \oplus_{\in}$) and ($BNSM_{mn}, \otimes_{\in}$) form a commutative monoid.

Theorem 3.7. For R,T ∈ $BNSM_{mn}$,

(i) $R \wedge (R \oplus_{\in} T) = R$
(ii) $R \vee (R \otimes_{\in} T) = R$

Remark 3.8. For R,T ∈ $BNSM_{mn}$,

(i) $(R \oplus_{\in} T)^C \neq R^c \otimes_{\in} T^c$
(ii) $(R \otimes_{\in} T)^C \neq R^c \oplus_{\in} T^c$

Example 3.9. For R,T ∈ $BNSM_{mn}$, $(R \oplus_{\in} T)^C \neq R^c \otimes_{\in} T^c$

Proof:

$$\text{Let } R = \begin{bmatrix} (-0.5,-0.4,-0.3,0.6,0.4,0.2) & (-0.8,-0.5,-0.3,0.1,0.2,0.3) \\ (-0.7,-0.1,-0.2,0.6,0.2,0.5) & (-0.7,-0.4,-0.1,0.5,0.4,0.3) \end{bmatrix} \text{ and}$$

$$T = \begin{bmatrix} (-0.3,-0.2,-0.1,0.4,0.5,0.6) & (-0.5,-0.3,-0.1,0.5,0.7,0.1) \\ (-0.2,-0.3,-0.5,0.2,0.3,0.5) & (-0.6,-0.3,-0.2,0.6,0.4,0.3) \end{bmatrix}$$

Then

$$R^C = \begin{bmatrix} (-0.3,-0.4,-0.5,\ 0.2,0.4,0.6) & (-0.3,-0.5,-0.8,0.3,0.2,0.1) \\ (-0.2,-0.1,-0.7,0.5,0.2,0.6) & (-0.1,-0.4,-0.7,0.3,0.4,0.5) \end{bmatrix}$$

$$T^C = \begin{bmatrix} (-0.1,-0.2,-0.4,\ 0.6,0.5,0.4) & (-0.1,-0.3,-0.5,0.1,0.7,0.5) \\ (-0.5,-0.3,-0.2,0.5,0.3,0.2) & (-0.2,-0.3,-0.6,0.3,0.4,0.6) \end{bmatrix}$$

Also

$$R \oplus_{\in} T = \begin{bmatrix} (-0.69,-0.51,-0.01,0.81,0.75,0.09) & (-0.93,-0.70,-0.41,0.57,0.79,0.38) \\ (-0.79,-0.39,-0.07,0.71,0.47,0.20) & (-0.91,-0.63,-0.01,0.84,0.69,0.06) \end{bmatrix}$$

$$(R \oplus_{\in} T)^C = \begin{bmatrix} (-0.01,-0.51,-0.69,\ 0.09,0.75,0.81) & (-0.41,-0.70,-0.93,0.38,0.79,0.57) \\ (-0.07,-0.39,-0.79,0.20,0.47,0.71) & (-0.01,-0.63,-0.91,0.06,0.69,0.84) \end{bmatrix}$$

$$R^c \otimes_{\in} T^c = \begin{bmatrix} (-0.02,-0.05,-0.70,\ 0.09,0.15,0.81) & (-0.03,-0.11,-0.93,0.02,0.11,0.41) \\ (-0.07,-0.02,-0.79,0.20,0.03,0.61) & (-0.01,-0.08,-0.92,0.06,0.12,0.8) \end{bmatrix}$$

Therefore $(R \oplus_{\in} T)^C \neq R^c \otimes_{\in} T^c$

Theorem 3.10. For R,T ∈ $BNSM_{mn}$,

(i) $(R \wedge T) \oplus_{\in} (R \vee T) = R \oplus_{\in} T$
(ii) $(R \wedge T) \otimes_{\in} (R \vee T) = R \otimes_{\in} T$

Proof:

$$\text{(i)}(R \wedge T) \oplus_{\in} (R \vee T) = (< -\min(r_{ijn}, t_{ijn}), -\min(r'_{ijn}, t'_{ijn}), -\max(r''_{ijn}, t''_{ijn}) >,$$
$$< \min(r_{ijp}, t_{ijp}), \min(r'_{ijp}, t'_{ijp}), \max(r''_{ijp}, t''_{ijp}) >) \oplus_{\in}$$
$$(< -\max(r_{ijn}, t_{ijn}), -\max(r'_{ijn}, t'_{ijn}), -\min(r''_{ijn}, t''_{ijn}) >,$$
$$< \max(r_{ijp}, t_{ijp}), \max(r'_{ijp}, t'_{ijp}), \min(r''_{ijp}, t''_{ijp}) >) .$$

$$= (< -(\frac{\min(r_{ijn}, t_{ijn}) + \max(r_{ijn}, t_{ijn})}{1+\min(r_{ijn}, t_{ijn})\max(r_{ijn}, t_{ijn})}), -(\frac{\min(r'_{ijn}, t'_{ijn}) + \max(r'_{ijn}, t'_{ijn})}{1+\min(r'_{ijn}, t'_{ijn})\max(r'_{ijn}, t'_{ijn})}), -(\frac{\max(r''_{ijn}, t''_{ijn})\min(r''_{ijn}, t''_{ijn})}{1+(1-\max(r''_{ijn}, t''_{ijn})(1-\min(r''_{ijn}, t''_{ijn}))}) >,$$
$$< -(\frac{\min(r_{ijp}, t_{ijp}) + \max(r_{ijp}, t_{ijp})}{1+\min(r_{ijp}, t_{ijp})\max(r_{ijp}, t_{ijp})}), -(\frac{\min(r'_{ijp}, t'_{ijp}) + \max(r'_{ijp}, t'_{ijp})}{1+\min(r'_{ijp}, t'_{ijp})\max(r'_{ijp}, t'_{ijp})}), -(\frac{\max(r''_{ijp}, t''_{ijp})\min(r''_{ijp}, t''_{ijp})}{1+(1-\max(r''_{ijp}, t''_{ijp})(1-\min(r''_{ijp}, t''_{ijp}))}) >)$$

$$= (< -(\frac{r_{ijn} + t_{ijn}}{1+r_{ijn}\, t_{ijn}}),\ -(\frac{r'_{ijn} + t'_{ijn}}{1+r'_{ijn}\, t'_{ijn}}), -(\frac{r''_{ijn}\, t''_{ijn}}{1+(1-r''_{ijn})(1-t''_{ijn})}) >,$$

$$< \frac{r_{ijp} + t_{ijp}}{1 + r_{ijp}\, t_{ijp}}, \quad \frac{r'_{ijp} + t'_{ijp}}{1 + r'_{ijp}\, t'_{ijp}}, \quad \frac{r''_{ijp} + t''_{ijp}}{1 + (1 - r''_{ijp})(1 - t''_{ijp})} >)$$

$$= R \oplus_{\in} T$$

(ii) It can be proved similarly.

Decision-making on BNSMs

In this section, define value matrix, score matrix and total score matrix. Using the definitions to extend MCDM problems.

Definition 3.11. Value Matrix

Suppose R = $(< -r_{ijn}, -r'_{ijn}, -r''_{ijn} >, < r_{ijp}, r'_{ijp}, r''_{ijp} >) \in BNSM_{mn}$, then R is called the value of BNSM if $V(X) = (< -(r_{ijn} + r'_{ijn}, -r''_{ijn}) >, < r_{ijp} + r'_{ijp} - r''_{ijp} >)$ for all i and j respectively, $1 \le i \le m$, $1 \le j \le n$.

It is denoted by V (R).

Definition 3.12. Score Matrix

Suppose R= $(< -r_{ijn}, -r'_{ijn}, -r''_{ijn} >, < r_{ijp}, r'_{ijp}, r''_{ijp} >)$ and

T = $(< -t_{ijn}, -t'_{ijn}, -t''_{ijn} >, < t_{ijp}, t'_{ijp}, t''_{ijp} >) \in BNSM_{mn}$, then score matrix of R and T is denoted by $S_{(R,T)}$ and it is defined as $S_{(R,T)}$ = V (R) − V (T).

Definition 3.13. Total Score

Suppose R= $(< -r_{ijn}, -r'_{ijn}, -r''_{ijn} >, < r_{ijp}, r'_{ijp}, r''_{ijp} >)$ and

T = $(< -t_{ijn}, -t'_{ijn}, -t''_{ijn} >, < t_{ijp}, t'_{ijp}, t''_{ijp} >) \in BNSM_{mn}$

then their corresponding value matrix be V(R), V(T) and their score matrix be $S_{(R,T)}$. Then the total score,

$$S_i = \sum_{j=1}^{p} (V(R) - V(T))$$

4. Methodology

Let A = $\{(c_1,c_2,c_3)\}$ be a set of cars. And D is a set of parameters for fuel efficiency (e_1), speed (e_2), engine capacity (e_3). All three cars are going from chennai to bangalore. I will find out which automobile arrives first. Consider how the matrices R and T relate to the bipolar neutrosophic soft sets (S_R,D) and (S_T ,D). Furthermore, compute the complement matrices R^C and T^C using the complements of bipolar neutrosophic soft sets $(S_R,D)^c$ and $(S_T ,D)^c$ respectively. After that, determine the matrices $R \oplus_{\in} T$, $R^C \oplus_{\in} T^C$, the value matrices $V(R \oplus_{\in} T)$ and $V(R^C \oplus_{\in} T^C)$ as well as the score matrix $S_{((R\oplus_{\in}T)(R^C\oplus_{\in}T^C))}$ and the overall score S_i for each cars in A. Finally, S_k = Max(S_i) computes the cars S_k to determine which car will arrive first.

Algorithm

Step 1: Determine the bipolar neutrosophic soft set (S_R, D),(S_T, D), and then identify the BNSMs R and T corresponding to the (S_R, D) and (S_T, D) respectively.

Step 2: Determine the bipolar neutrosophic soft sets $(S_R, D)^c$,$(S_T, D)^c$, and identify the BNSMs R^c and T^c corresponding to the $(S_R, D)^c$ and $(S_T, D)^c$ respectively.

Step 3: Calculate $R \oplus_{\in} T$, $R^C \oplus_{\in} T^C$, V $(R \oplus_{\in} T)$, V $(R^C \oplus_{\in} T^C)$ and $S_{((R\oplus_{\in}T)(R^C\oplus_{\in}T^C))}$

Step 4: Determine the total score S_i for each s_i in A.

Step 5: S_k = Max(S_i) and then select the first arriving car S_k has the maximum value.

5. Results on Einstein Operators

In this section, construct the value matrix, score matrix and total score matrix. along with Einstein operator and compute which car will arrive first.

Application in decision-making

Due to the limits of standard mathematical methods, researchers use a variety of approaches to solve problems involving decision-making. One of the methods utilized in this article is BNSM, a device that many researchers employ to solve MCDM difficulties. Suppose (S_R, D) and (S_T,D) are two bipolar neutrosophic soft sets showing the set of three soils that are selected, the set A = $\{c_1,c_2,c_3\}$. Suppose D = $\{e_1,e_2,e_3\}$ is the set of assignments representing the various aspects of fuel efficiency (e_1), speed (e_2), and engine capacity (e_3). All three cars are going from Chennai to Bangalore. In this section, I will determine which car will arrive first.

Step 1:

Construction of bipolar neutrosophic soft sets

$(S_R,D) = \{S_R(e_1),S_R(e_2),S_R(e_3)\}$ where,

$S_R(e_1) = \{(s_1,-0.8,-0.5,-0.4,0.5,0.6,0.1),$

$(s_2,-0.2,-0.7,-0.6,0.7,0.8,0.2),(s_3,-0.3,-0.4,-0.7,0.2,0.4,0.5)\}$

$S_R(e_2) = \{(s_1,-0.5,-0.4,-0.3,0.6,0.4,0.2),$
$(s_2,-0.7,-0.1,-0.2,0.6,0.2,0.5),(s_3,-0.2,-0.4,-0.6,0.7,0.1,0.2)\}$

$S_R(e_3) = \{(s_1,-0.8,-0.5,-0.3,0.1,0.2,0.3),$
$(s_2,-0.7,-0.4,-0.1,0.5,0.4,0.3),(s_3,-0.3,-0.2,-0.1,0.7,0.4,0.1)\}$

$(S_T,D) = \{S_T(e_1),S_T(e_2),S_T(e_3)\}$ where,

$S_T(e_1) = \{(t_1,-0.2,-0.5,-0.7,0.4,0.3,0.9),$
$(t_2,-0.3,-0.4,-0.1,0.5,0.6,0.7), (t_3,-0.7,-0.4,-0.3,0.6,0.5,0.3)\}$

$S_T(e_2) = \{(t_1,-0.3,-0.2,-0.1,0.4,0.5,0.6),$
$(t_2,-0.2,-0.3,-0.5,0.2,0.3,0.5), (t_3,-0.6,-0.5,-0.4,0.2,0.3,0.5)\}$

$S_T(e_3) = \{(t_1,-0.5,-0.3,-0.1,0.5,0.7,0.1),$
$(s_2,-0.6,-0.3,-0.2,0.6,0.4,0.3), (s_3,-0.5,-0.7,-0.6,0.6,0.3,0.2)\}$

These are bipolar neutrosophic soft matrices of above soft sets:

$$R = \begin{bmatrix} (-0.8,-0.5,-0.4,0.5,0.6,0.1) & (-0.5,-0.4,-0.3,0.6,0.4,0.2) & (-0.8,-0.5,-0.3,0.1,0.2,0.3) \\ (-0.2,-0.7,-0.6,0.7,0.8,0.2) & (-0.7,-0.1,-0.2,0.6,0.2,0.5) & (-0.7,-0.4,-0.1,0.5,0.4,0.3) \\ (-0.3,-0.4,-0.7,0.2,0.4,0.5) & (-0.2,-0.4,-0.6,0.7,0.1,0.2) & (-0.3,-0.2,-0.1,0.7,0.4,0.1) \end{bmatrix}$$

$$T = \begin{bmatrix} (-0.2,-0.5,-0.7,0.4,0.3,0.9) & (-0.3,-0.2,-0.1,0.4,0.5,0.6) & (-0.5,-0.3,-0.1,0.5,0.7,0.1) \\ (-0.3,-0.4,-0.1,0.5,0.6,0.7) & (-0.2,-0.3,-0.5,0.2,0.3,0.5) & (-0.6,-0.3,-0.2,0.6,0.4,0.3) \\ (-0.7,-0.4,-0.3,0.6,0.5,0.3) & (-0.6,-0.5,-0.4,0.2,0.3,0.5) & (-0.5,-0.7,-0.6,0.6,0.3,0.2) \end{bmatrix}$$

Step 2: The bipolar neutrosophic soft complement matrices are

$$R^C = \begin{bmatrix} (-0.4,-0.5,-0.8,0.1,0.6,0.5) & (-0.3,-0.4,-0.5,0.2,0.4,0.6) & (-0.3,-0.5,-0.8,0.3,0.2,0.1) \\ (-0.6,-0.7,-0.2,0.2,0.8,0.7) & (-0.2,-0.1,-0.7,0.5,0.2,0.6) & (-0.1,-0.4,-0.7,0.3,0.4,0.5) \\ (-0.7,-0.4,-0.3,0.5,0.4,0.2) & (-0.6,-0.4,-0.2,0.2,0.1,0.7) & (-0.1,-0.2,-0.3,0.1,0.4,0.7) \end{bmatrix}$$

$$T^C = \begin{bmatrix} (-0.7,-0.5,-0.2,0.9,0.3,0.4) & (-0.1,-0.2,-0.3,0.6,0.5,0.4) & (-0.1,-0.3,-0.5,0.1,0.7,0.5) \\ (-0.1,-0.4,-0.3,0.7,0.6,0.5) & (-0.5,-0.3,-0.2,0.5,0.3,0.2) & (-0.2,-0.3,-0.6,0.3,0.4,0.6) \\ (-0.3,-0.4,-0.7,0.3,0.5,0.6) & (-0.4,-0.5,-0.6,0.5,0.3,0.2) & (-0.6,-0.7,-0.5,0.2,0.3,0.6) \end{bmatrix}$$

Step 3: Construction of value matrix

$$R \oplus_\in T = \begin{bmatrix} (-0.86,-0.80,-0.24,0.75,0.70,0.92) & (-0.69,-0.51,-0.01,0.81,0.75,0.09) & (-0.93,-0.70,-0.41,0.57,0.79,0.38) \\ (-0.47,-0.86,-0.51,0.89,0.95,0.11) & (-0.79,-0.39,-0.07,0.71,0.47,0.20) & (-0.91,-0.63,-0.01,0.84,0.69,0.06) \\ (-0.83,-0.65,-0.17,0.61,0.69,0.11) & (-0.71,-0.75,-0.19,0.79,0.39,0.07) & (-0.70,-0.79,-0.04,0.91,0.63,0.01) \end{bmatrix}$$

$$R^C \oplus_\in T^C = \begin{bmatrix} (-0.86,-0.80,-0.14,0.92,0.76,0.15) & (-0.39,-0.56,-0.70,0.71,0.75,0.81) & (-0.39,-0.70,-0.93,0.39,0.79,0.66) \\ (-0.66,-0.86,-0.32,0.79,0.95,0.30) & (-0.64,-0.39,-0.79,0.80,0.47,0.71) & (-0.29,-0.63,-0.92,0.55,0.69,0.85) \\ (-0.83,-0.65,-0.17,0.59,0.69,0.09) & (-0.81,-0.75,-0.71,0.64,0.39,0.79) & (-0.66,-0.79,-0.70,0.29,0.63,0.92) \end{bmatrix}$$

$$V(R \oplus_\in T) = \begin{bmatrix} (-1.42,0.53) & (-1.19,1.47) & (-1.22,0.98) \\ (-0.82,1.73) & (-1.11,0.98) & (-1.53,1.47) \\ (-1.31,1.19) & (-0.65,1.11) & (-1.45,1.53) \end{bmatrix}$$

$$V(R^C \oplus_\in T^C) = \begin{bmatrix} (-1.52,1.53) & (-0.25,0.65) & (-0.16,0.52) \\ (-0.16,0.52) & (-0.24,0.56) & (0,0.39) \\ (-1.31,1.19) & (-0.85,0.24) & (-0.75,0) \end{bmatrix}$$

Step 4: Construction of score matrix

$$S_{((R \oplus_\in T)(R^C \oplus_\in T^C))} = \begin{bmatrix} (-2.00,1.28) \\ (-2.40,1.50) \\ (-0.51,2.40) \end{bmatrix}.$$

Step 5: Using the foregoing results, Figure 1 shows a ranking of fast-moving cars. The best option is to use the highest score, $S_k = 2.4$. As a result, the car (c_3) scored greater than the engine capacity (e_3). The car(c_3) was chosen as the first to arrive.

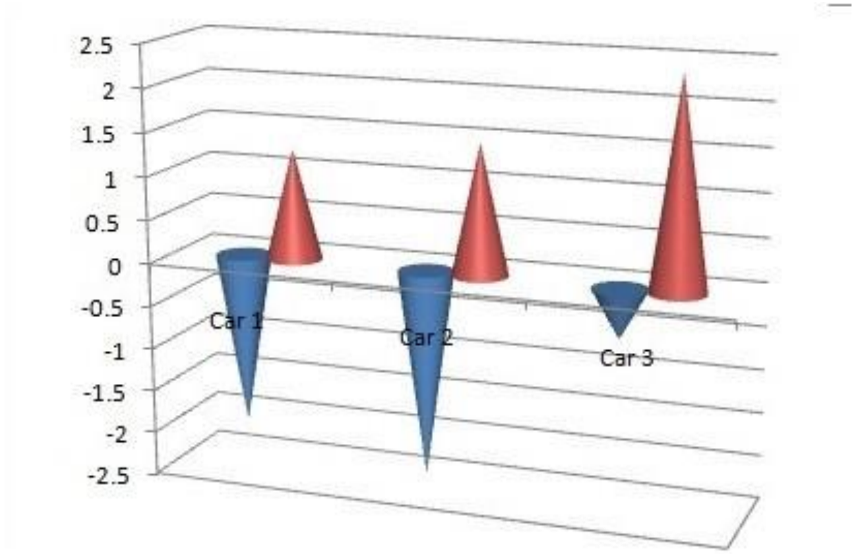

Figure 1: Ranking of moving fast cars

6. Conclusion

This manuscript defines Einstein operators for bipolar neutrosophic soft matrices. Bipolar neutrosophic soft matrices are being investigated for special properties. Finally, the car (c_3) received additional points for engine capacity (e_3). This ensures that the car (c_3) arrives first. In many real-world circumstances, when the goal is to make decisions quickly, this procedure looks to be quite feasible. This method not only simplifies decision-making but also improves the accuracy of results. Researchers can use bipolar neutrosophic soft matrices to effectively analyze complex data and gain relevant insights for a variety of applications.

References

[1] L.A.Zadeh,Fuzzysets,Informationand Control,8(3) (1965),338-353,https://doi.org/10.1016/S0019-9958(65)90241-X.

[2] M.G.Thomason, Convergence of powers of fuzzy matrix, J.Mathematical Analysis and Applications 57(2) (1977), 476-480. https://doi.org/10.1016/0022-247X(77)90274-8

[3] T.M.Selvarajan, S.Sriram and R.S.Ramya, Einstein operations of fuzzy matrices, International Journal of Engineering and Technology, 7(4), (2018), 813-816.

[4] T.M.Selvarajan, S.Sriram and R.S.Ramya, Some equalities on einstein of intuitionistic fuzzy matrices, International Journal of Pure and Applied Mathematics, 119(18), (2018), 261271.

[5] T.M.Selvarajan, S.Sriram and R.S.Ramya, Einstein operations of intuitionistic fuzzy matrices, International Journal of Recent Technology and Engineering, 8, (2019), 1104-1106.

[6] W-R Zhang, Bipolar fuzzy sets and relations, First International Joint Conference of The North American Fuzzy Information Processing Society Biannual Conference, (1994) 305309, DOI: 10.1109/IJCF.1994.375115 • Source: IEEE Xplore.

[7] M.Pal and Sanjib Mondal, Bipolar fuzzy matrices, Soft Computing, (2019), 9885 -9897, https://doi.org/10.1007/s00500-019-039129.

[8] Muhammad Naveed Jafar, Muhammad Saqlain, Ahmad Raza Shafiq, Muhammad Khalid, Hamza Akbar and Aamir Naveed, New Technology in Agriculture Using Neutrosophic Soft Matrices with the Help of Score Function, International Journal of Neutrosophic Science, 3(2), 2020, 78-88.

[9] M.Ramakrishnan and S.Sriram, Einstein operations of picture fuzzy matrices, International Journal of Early Childhood Special Education, 14, (2022), 3336-3345.

[10] M.Ramakrishnan and S.Sriram, Some equalities on einstein operations of pythagorean fuzzy matrices, Stochastic Modeling and Applications, 26(3), (2022), 564-578.

[11] T.M.Selvarajan and R.S.Ramya, Einstein operations on pythgorean fuzzy matrices, Advances in Mathematics: Scientific Journal, 8(2), (2019), 634-640.

[12] A.Anitha, A study on algebraic designs and applications of certain operators in generalized bipolar fuzzy matrices, Ph.D Thesis, Department of Mathematics, Annamalai University, (2025).

[13] T.Muthuraji and A.Anitha, Recent Technology in Agriculture using the Bipolar Fuzzy Matrices with the Aid of Score Function, Fifth International Conference on Electrical, Computer and Communication Technologies, IEEE Proceedings, (2023), 1-4, DOI: 10.1109/ICECCT56650.2023.10179734.

[14] J.Boobalan and Afrine N.S Shiny, Einstein operators of fuzzy neutrosophic soft matrix of type A, Advances in Nonlinear Variational Inequalities, 28(2), (2025), 599-608.

[15] T.Muthuraji and A.Anitha, Multiple Criteria Decision Making using Bipolar Intuitionistic Fuzzy Matrices, the Weighted Similarity Measure and the Expanded TOPSIS Method, First International Conference on Advances in Electrical, Electronics and Computational Intelligence, IEEE Proceedings, (2024), 1-7, DOI:10.1109/ICAEECI5827.2023.10370950.

Declarations

Competing Interests: The author declares that she has no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.
Declaration of AI Use: Not Applicable.
Funding Statement: This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.
Data Availability Statement: Data will be made available on request from the corresponding author.
Acknowledgments: Not Applicable.

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