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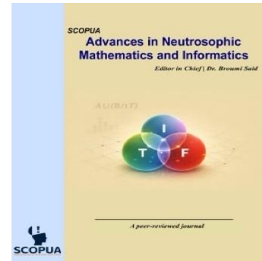

Bellman Ford Algorithm for the Shortest Path Problem in a Neutrosophic Pythagorean Fuzzy Network

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ABSTRACT

The study describes a method for discovering the shortest path in networks that employs Neutrosophic Pythagorean Fuzzy Graph and Triangular Fuzzy Numbers (TFN). This method uses neutrosophic logic and Pythagorean fuzzy sets for handling ambiguous and imprecise data. The Neutrosophic Pythagorean Fuzzy Shortest Path (NPFSP) algorithm calculates the shortest path while accounting for membership functions for truth, indeterminacy, and falsity. The technique provides improved flexibility and accuracy when dealing with uncertainty.

Keywords*:*Neutrosophic Pythagorean Fuzzy Graph (NPFG); Neutrosophic Pythagorean Fuzzy Triangular Fuzzy Numbers (NPTFNs); Neutrosophic Pythagorean Triangular Fuzzy Shortest Path Problem (NPTFSPP); Network Optimization

1. Introduction

The shortest path problem is a critical component of network optimisation in Industries like transportation, logistics, telecommunications, and robotics. Traditional algorithms assume deterministic edge weights, but real-world networks are inherently uncertain due to factors such as traffic fluctuations, environmental changes, and insufficient data. NPFG and TFN are proposed as novel approaches to coping with these uncertainties. NPFG combines the flexibility of Pythagorean fuzzy Graph with the detailed uncertainty modelling of Neutrosophic Fuzzy Graph (NFG), whereas TFNs represent the unknown weights of network edges. This method offers a powerful and adaptable framework for calculating the shortest path in unexpected networks.

Zadeh [9] introduced fuzzy set theory to address uncertainty and imprecise data in shortest path problems. Following this development, many researchers have explored different techniques to solve SP problems in fuzzy environments. Okada [14] proposed a method for solving the fuzzy SP problem by using possibility theory to evaluate the possibility level of each arc. To address the difficulties that arise from this approach, Keshavarz and Khorram [8] transformed the fuzzy SP problem into a bi-level problem and proposed an effective solution based on the parametric shortest path method.

Deng et al. [3] enhanced the classical Dijkstra algorithm to solve fuzzy shortest path problems by incorporating the graded mean integration representation of fuzzy numbers. In addition, several researchers have employed heuristic approaches to identify the shortest path in networks where arc costs are characterized by fuzzy values. Despite these developments, traditional fuzzy set theory represents only the degree of membership and does not explicitly include the degree of non-membership. In such cases, the non-membership degree is generally considered as the complement of the membership value.

To overcome this limitation, Atanassov [5] introduced the concept of intuitionistic fuzzy sets (IFS), which simultaneously considers both membership and non-membership degrees. Within this framework, the sum of these two degrees is constrained to be less than or equal to one, allowing for a degree of hesitation or uncertainty. Inspired by this concept, many researchers have investigated shortest path problems in networks with intuitionistic fuzzy arc costs under the IFS framework. In this context, Mukherjee studied the shortest path problem in an intuitionistic fuzzy environment and proposed approaches to address the associated uncertainties.

Geetharamani and Jayagowri [4] introduced a new approach for solving the intuitionistic fuzzy shortest path (IFSP) problem by applying an intuitionistic fuzzy shortest path length method together with a similarity measure. A. Nagoor Gani and M. Mohammed Jabarulla [1] developed a technique for identifying the intuitionistic fuzzy shortest path in network structures. Karunambigai M. G., Ranagasamy P., Atanassov K., and Palaniappan N. [7] proposed a method based on intuitionistic fuzzy graphs to determine the shortest paths in networks with uncertainty. Lazim Abdullah and Pinxin Goh [10] presented a decision-making framework using Pythagorean fuzzy sets and demonstrated its application in solid waste management problems. L. Sujatha and Hyacinta J. D. [11] studied the shortest path problem in networks where the edge weights are represented by intuitionistic fuzzy values. Furthermore, Asim Basha, Mohammed Jabarulla, and Said Broumi [12] applied Pythagorean fuzzy triangular numbers to solve the shortest path problem in uncertain network environments.

First addressing neutrosophy in 1995, Smarandache [20] defined neutrosophic set theory, one of the most significant new computational formulas, between 1999 and 2005. Data that is imprecise, partial, uncertain, or distorted cannot be handled by fuzzy sets or intuitionistic fuzzy sets. As extensions of fuzzy sets and intuitionistic fuzzy sets, Smarandache created the concepts of neutrosophic sets and neutrosophic logic. An effective method for resolving the time-dependent shortest path problem in a Fermeatean Neutrosophic environment is discussed by K. Vidhya, A. Saraswathi, and Said Broumi [6]. In order to create shortest path solutions in a network, M. Parimala, Broumi, K. Prakash, et al. [15] employed the Bellman-Ford algorithm. R.R. Yager[16] established the Pythagorean membership grades for decision making. X. Zhang and Z. Xu [21] investigate the extension of TOPSIS to multiple criteria decision making using Pythagorean fuzzy sets.

S. Broumi, A. Bakali, M. Talea, F. Smarandache, and L. Vladareanu [17] created the Shortest path issue using trapezoidal fuzzy neutrosophic information. S. Broumi, M. Talea, A. Bakali, and F. Smarandache [18] define us in terms of Single Valued Neutrosophic Graphs. Broumi et al. [19] provide a new approach for finding neutrosophic shortest paths using interval-valued neutrosophic numbers, which is validated with numerical examples and compared to current methods.

Finally, Ajay and Chellamani covered Pythagorean Neutrosophic fuzzy graphs [2]. M. Asim Basha, M. Mohammed Jabarulla, and Said Broumi [13] devised an approach for determining the shortest path between nodes based on the Neutrosophic Pythagorean fuzzy triangular number. We chose the singularly uncommon case known as a neutrosophic Pythagorean set, in which truth and falsehood are completely independent of indeterminacy, from among the three probable dependence situations in a neutrosophic set.

$0 \leq \mu_1(v_i')^2 + \beta_1(v_i')^2 + \sigma_1(v_i')^2 \leq 2$ are the dependencies. Next, we investigate neutrosophic pythagorean graphs and apply neutrosophic pythagorean sets to fuzzy graphs. In order to choose the destination nodes, the Neutrosophic Pythagorean Fuzzy Shortest Path Problem is defined in this work using triangular numbers and a rating function.

2. Methodology

Definition 2.1

Pythagorean fuzzy graph (PFG) is a structure representing a non-empty set that incorporates the concepts of Pythagorean fuzzy sets, allowing for enhanced representation of uncertainty and imprecision in graph theory X' is a combination in $\mathrm{G} = (\widetilde{\mathrm{P}}', \widetilde{\mathrm{Q}}')$ with $\widetilde{\mathrm{P}}'$ a PFSet on X' and $\widetilde{\mathrm{Q}}'$ a PF Relation on X' is,

$$\mu_{\widetilde{Q}'}(uv) \leq \mu_{\widetilde{P}'}(u) \wedge \mu_{\widetilde{P}'}(v)$$
$$\lambda_{\widetilde{Q}'}(uv) \leq \lambda_{\widetilde{P}'}(u) \vee \lambda_{\widetilde{P}'}(v)$$

where $\mu_{\widetilde{Q}'}: X' \times X' \to [0,1]$ and $\lambda_{\widetilde{Q}'}: X' \times X' \to [0,1]$ represent the member and non-member functions of $\widetilde{Q}'$, respectively, with $0 \leq \left(\mu_{\widetilde{Q}'}(u)\right)^2 + \left(\lambda_{\widetilde{Q}'}(u)\right)^2 \leq 1, \ \forall\, u, v \in E$.

Definition 2.2

A Neutrosophic Pythagorean set is defined with truth and falsity acting as dependent Neutrosophic components within a non-empty universe.

$$E = \{(e, \mu_E(e), \beta_E(e), \sigma_E(e)) : e \in U\}, \text{where}$$

$\mu_E(e), \beta_E(e), \sigma_E(e) \in [0,1]$, $0 \leq (\mu_E(e))^2, (\beta_E(e))^2, (\sigma_E(e))^2 \leq 2, \forall\, e \in U$, $\mu_E(e), \beta_E(e), \sigma_E(e)$ are the degrees of member, indeterminacy and non-member respectively. Here $\mu_E(e)$ and $\sigma_E(e)$ are dependent and $\beta_E(e)$ is independent.

Definition 2.3

A NPFG is $G' = (V', E')$, where $V' = \{v_1', v_2', \ldots, v_n'\}$ such that μ_1', β_1' and σ_1' from $V' \to [0,1]$ with $0 \le \mu_1(v_i')^2 + \beta_1(v_i')^2 + \sigma_1(v_i')^2 \le 2, \forall\, v_i' \in V'$ Indicates the relationships of member, indeterminacy, and non-member functions in a corresponding manner and $E' \subseteq V' \times V'$ where μ_2, β_2, σ_2 from $V' \times V' \to [0,1]$ such that,

$$\mu_2\left(v_i' v_j'\right) \le \mu_1(v_i') \wedge \mu_1(v_j')$$
$$\beta_2\left(v_i' v_j'\right) \le \beta_1(v_i') \wedge \beta_1(v_j')$$
$$\sigma_2\left(v_i' v_j'\right) \le \sigma_1(v_i') \vee \sigma_1(v_j')$$

With $0 \le \left(\mu_2\left(v_i' v_j'\right)\right)^2 + \left(\beta_2\left(v_i' v_j'\right)\right)^2 + \left(\sigma_2\left(v_i' v_j'\right)\right)^2 \le 2, \forall\, \left(v_i' v_j'\right) \in E.$

3. Algebraic Operations and Neutrosophic Pythagorean Triangular Fuzzy Numbers

A useful notation is provided by the Neutrosophic Pythagorean triangular fuzzy values.

$$\text{for } \widetilde{Z}' = \langle (\tilde{p}, \tilde{q}, \tilde{r}), (\tilde{s}, \tilde{t}, \tilde{u}), (\widetilde{w}, \tilde{x}, \tilde{y}) \rangle$$

where, $$\left(\mu_{\tilde{P}_1}{}'(v')\right)^2, \left(\mu_{\tilde{P}_2}{}'(v')\right)^2, \left(\mu_{\tilde{P}_3}{}'(v')\right)^2 = \left(\tilde{\imath}, \tilde{\jmath}, \tilde{k}\right),$$
$$\left(\beta_{\tilde{P}_1}{}'(v')\right)^2, \left(\beta_{\tilde{P}_2}{}'(v')\right)^2, \left(\beta_{\tilde{P}_3}{}'(v')\right)^2 = (\widetilde{m}, \tilde{n}, \tilde{o}),$$
$$\left(\sigma_{\tilde{P}_1}{}'(v')\right)^2, \left(\sigma_{\tilde{P}_2}{}'(v')\right)^2, \left(\sigma_{\tilde{P}_3}{}'(v')\right)^2 = (\tilde{q}, \tilde{r}, \tilde{s}).$$

Definition 3.1

A Pythagorean triangular fuzzy number that is neutrosophic

$\widetilde{Z}' = \left\langle \left(\tilde{\imath}, \tilde{\jmath}, \tilde{k}\right), (\widetilde{m}, \tilde{n}, \tilde{o}), (\tilde{q}, \tilde{r}, \tilde{s}) \right\rangle$ is said to be zero NPTFNs iff,

$$\left(\tilde{\imath}, \tilde{\jmath}, \tilde{k}\right) = (0,0,0);\ (\widetilde{m}, \tilde{n}, \tilde{o}) = (1,1,1); (\tilde{q}, \tilde{r}, \tilde{s}) = (1,1,1)$$

Definition 3.2

Let $\widetilde{Z}' = \widetilde{Z_1'}, \widetilde{Z_2'}$, then $\widetilde{Z_1'} = \left\langle \left(\tilde{\imath}_1, \tilde{\jmath}_1, \tilde{k}_1\right), (\widetilde{m}_1, \tilde{n}_1, \tilde{o}_1), (\tilde{q}_1, \tilde{r}_1, \tilde{s}_1) \right\rangle$ and $\widetilde{Z_2'} = \left\langle \left(\tilde{\imath}_2, \tilde{\jmath}_2, \tilde{k}_2\right), (\widetilde{m}_2, \tilde{n}_2, \tilde{o}_2), (\tilde{q}_2, \tilde{r}_2, \tilde{s}_2) \right\rangle$ two Neutrosophic Pythagorean Triangular Fuzzy Values (NPTFV) in a set of R and $\lambda > 0$ are the operating guidelines. The operating guidelines are then,

$$(i)\ \widetilde{Z_1'} \oplus \widetilde{Z_2'} = \left\langle \begin{array}{c} \sqrt{(\tilde{\imath}_1^2 + \tilde{\imath}_2^2 - \tilde{\imath}_1^2\tilde{\imath}_2^2)}, \sqrt{(\tilde{\jmath}_1^2 + \tilde{\jmath}_2^2 - \tilde{\jmath}_1^2\tilde{\jmath}_2^2)}, \sqrt{\left(\tilde{k}_1^2 + \tilde{k}_2^2 - \tilde{k}_1^2\tilde{k}_2^2\right)},, \\ (\widetilde{m_1}\widetilde{m_2}, \tilde{n}_1\tilde{n}_2, \tilde{o}_1\tilde{o}_2), (\widetilde{q_1}\widetilde{q_2}, \tilde{r}_1\tilde{r}_2, \tilde{s}_1\tilde{s}_2) \end{array} \right\rangle$$

$$(ii)\ \widetilde{Z_1'} \otimes \widetilde{Z_2'} = \left\langle \begin{array}{c} \sqrt{\left(\widetilde{m_1}^2 + \widetilde{m_2}^2 - \widetilde{m_1}^2\widetilde{m_2}^2\right)}, \sqrt{\left(\widetilde{n_1}^2 + \widetilde{n_2}^2 - \widetilde{n_1}^2\widetilde{n_2}^2\right)}, \\ \sqrt{\left(\widetilde{o_1}^2 + \widetilde{o_2}^2 - \widetilde{o_1}^2\widetilde{o_2}^2\right)}, \\ \sqrt{\left(\widetilde{q_1}^2 + \widetilde{q_2}^2 - \widetilde{q_1}^2\widetilde{q_2}^2\right)}, \sqrt{\left(\widetilde{r_1}^2 + \widetilde{r_2}^2 - \widetilde{r_1}^2\widetilde{r_2}^2\right)}, \\ \sqrt{\left(\widetilde{s_1}^2 + \widetilde{s_2}^2 - \widetilde{s_1}^2\widetilde{s_2}^2\right)}, \left(\widetilde{\imath_1}\widetilde{\imath_2}, \widetilde{\jmath_1}\widetilde{\jmath_2}, \widetilde{k_1}\widetilde{k_2}\right) \end{array} \right\rangle$$

where $\lambda > 0$.

The score function S and accuracy function H are used to evaluate and compare the grades of NPTFS, which enables the ranking of different NPTFS values and the selection of the best alternative.

Definition 3.3

Let $\widetilde{Z_1'} = \left\langle \left(\tilde{\imath}_1, \tilde{\jmath}_1, \tilde{k}_1\right), (\widetilde{m}_1, \tilde{n}_1, \tilde{o}_1), (\tilde{q}_1, \tilde{r}_1, \tilde{s}_1) \right\rangle$ be a NPTFV then, the function $\overline{S'}(\widetilde{A'}_1)$ and an function $\overline{H'}(\widetilde{A'}_1)$ of NPTFV are defined as follows.

$$(i)\overline{S'}(\widetilde{A'}_1) = \frac{1}{12}\left[8 + \left(\tilde{\imath}_1^2 + 2\tilde{\jmath}_1^2 + \tilde{k}_1^2\right) - (\widetilde{m}_1^2 + 2\tilde{n}_1^2 + \tilde{o}_1^2) - (\tilde{q}_1^2 + 2\tilde{r}_1^2 + \tilde{s}_1^2)\right]$$
$$(ii)\ \overline{H'}(\widetilde{A'}_1) = \frac{1}{4}\left[\left(\tilde{\imath}_1^2 + 2\tilde{\jmath}_1^2 + \tilde{k}_1^2\right) - (\tilde{q}_1^2 + 2\tilde{r}_1^2 + \tilde{s}_1^2)\right]$$

Following that, the comparisons among two NPTFVs should be made in the order of their relationships.

Definition 3.4

Let $\widetilde{Z_1'} = \left\langle \left(\tilde{\imath}_1, \tilde{\jmath}_1, \tilde{k}_1\right), (\widetilde{m}_1, \tilde{n}_1, \tilde{o}_1), (\tilde{q}_1, \tilde{r}_1, \tilde{s}_1) \right\rangle$ & $\widetilde{Z_2'} = \left\langle \left(\tilde{\imath}_2, \tilde{\jmath}_2, \tilde{k}_2\right), (\widetilde{m}_2, \tilde{n}_2, \tilde{o}_2), (\tilde{q}_2, \tilde{r}_2, \tilde{s}_2) \right\rangle$ be two NPTFVs in the set of real numbers. Then, we define a ranking method as follows.

(i) If $\overline{S'}(\widetilde{A'}_1) > \overline{S'}(\widetilde{A'}_2)$, then $\widetilde{A'}_1 > \widetilde{A'}_2$, that is, $\widetilde{A'}_1$ is sup to $\widetilde{A'}_2$, denoted by $\widetilde{A'}_1 > \widetilde{A'}_2$.

(ii) If $\overline{S'}\left(\widetilde{A'}_1\right) = \overline{S'}(\widetilde{A'}_2)$, and $\overline{H'}(\widetilde{A'}_1) > \overline{H'}(\widetilde{A'}_2)$ then $\widetilde{A'}_1 > \widetilde{A'}_2$, that is $\widetilde{A'}_1$ is sup to $\widetilde{A'}_2$, denoted by $\widetilde{A'}_1 > \widetilde{A'}_2$.

4. Based on Pythagorean numbers, to establish the shortest path

The following section provides a novel algorithmic method for solving PSP. With the SN, nod 1, and the DN, nod n, it is assumed that there are in fact n nod. The collection of all nodes containing a link to node i is indicated as $M_{P_{(i)}}$, and the Pythagorean length of nod i and j is indicated by f_{ij}.

4.1 Developing BELLMAN dynamical programming in mathematical

Consider an acyclic direct connecting graph $G = (V, E)$ from SN I and DN 'n' organized by logical ordering $(P_{ij};\ i < j)$. The forward pass calculation process can be used to find the shortest path using the Bellman dynamic programming language. This is how the effective Bellman programming technique is described.

$$l(i) = \begin{cases} 0, & i = 1 \\ \min_{i<j}\left[l(i) + f_{ij}\right], & \text{otherwise} \end{cases} ------\rightarrow (1)$$

where $l(i)$ is the distance of the SP nod that is i from the node SN 1 and f_{ij} is the weight of the direct edge P_{ij}.

4.2 Neutrosophic Pythagorean Bellman-Ford algorithm pseudo code:

```
nprank[s] ← 0
npdist[s] ← Empty NPTFN.
Add s → Q
For every nod i (exclude in s) in the NPG G
 rank[i] ← ∞
Add i → Q
For End
u ← s
While(Q is not emty)
get rid of Q′s vertex u
For each adj vertex v of vertex u
relaxed ← False
tem _ npdist[v] ←  npdist[u]  ⊕ edge _ weight (u, v)
temp _ nprank [v] ← rank of NP (temp _ npdist[v])
IF temp _ nprank[v]  < nprank[v] then
npdist[v] ← temp _ npdist[v]
nprank[v] ← temp _ nprank[v]
nprev[v] ← u
IF End
For End
Consequently, if relaxed is False
loop exit
IF End
u ← nod ∈ Q with a min rank  value
While End
For each arc(u, v) in PFG G do
If npdist[u]edge _ weight (u, v) (nprank[v] > rank _ of _ Pythagorean),
false return
If End
For End
The SP from SNs and DNs is represented by the NPTN npdist[u], which is a PN.
```

4.4 Network Terminology

In computer networking, Bellman-Ford is used to locate the shortest routes in routing tables, facilitating the effective transfer of data packets across networks. GPS navigation aids navigation apps and devices by using Bellman-Ford to determine the shortest or fastest paths between sites.

The Bellman Ford algorithm is defined to find the SP between any two points along the path because there are many routes and paths connecting them but only one needs to show the shortest distance. Think about this If you were to visualize India as a graph, each city or site would be a vertex, and the connections would be lines. To ensure that messages may be sent between the cities or sites successfully, the route aims to locate a NPTSP.

The Neutrosophic Pythagorean Triangular Fuzzy Shortest Path (NPTFSP) and recommended Bellman Ford Algorithm for NPTFSP cost are shown below.

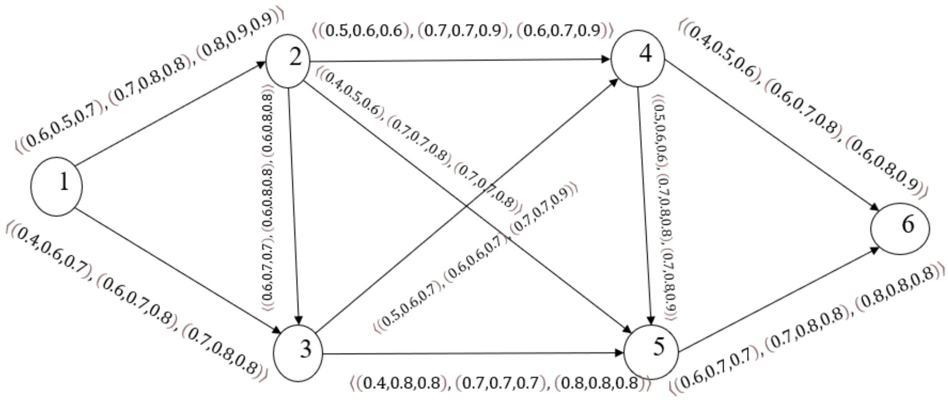

Figure 1: NPTFSP Network

Table 1: - The edges of NPTFNs.

Edges	Pythagorean distance
$1 \to 2$	$\langle(0.6,0.5,0.7),(0.7,0.8,0.8),(0.8,0.9,0.9)\rangle$
$1 \to 3$	$\langle(0.4,0.6,0.7),(0.6,0.7,0.8),(0.7,0.8,0.8)\rangle$
$2 \to 4$	$\langle(0.5,0.6,0.6),(0.7,0.7,0.9),(0.6,0.7,0.9)\rangle$
$2 \to 3$	$\langle(0.6,0.7,0.7),(0.6,0.8,0.8),(0.6,0.8,0.8)\rangle$
$2 \to 5$	$\langle(0.4,0.5,0.6),(0.7,0.7,0.8),(0.7,0.7,0.8)\rangle$
$3 \to 4$	$\langle(0.5,0.6,0.7),(0.6,0.6,0.7),(0.7,0.7,0.9)\rangle$
$3 \to 5$	$\langle(0.4,0.8,0.8),(0.7,0.7,0.7),(0.8,0.8,0.8)\rangle$
$4 \to 5$	$\langle(0.5,0.6,0.6),(0.7,0.8,0.8),(0.7,0.8,0.9)\rangle$
$4 \to 6$	$\langle(0.4,0.5,0.6),(0.6,0.7,0.8),(0.6,0.8,0.9)\rangle$
$5 \to 6$	$\langle(0.6,0.7,0.7),(0.7,0.8,0.8),(0.8,0.8,0.8)\rangle$

4.5 Illustrative Example:

This section is based on a numerical issue that has been modified to demonstrate how the suggested algorithm might be used.

$$l(1) = 0,$$

$$l(2) = \min_{i<2}\{l(1) + p_{12}\} = \min\{\langle[0,0,0],[1,1,1],[1,1,1]\rangle \oplus \langle(0.6,0.5,0.7),(0.7,0.8,0.8),(0.8,0.9,0.9)\rangle\}$$

$$l(2) = \langle(0.6,0.5,0.7),(0.7,0.8,0.8),(0.8,0.9,0.9)\rangle,$$

$$l(3) = \min_{i<3}\{l(1) + p_{13}, l(2) + p_{23}\}$$

$$= \min\left\{\begin{matrix}\langle[0,0,0],[1,1,1],[1,1,1]\rangle \oplus \langle(0.4,0.6,0.7),(0.6,0.7,0.8),(0.7,0.8,0.8)\rangle, \\ \langle(0.6,0.5,0.7),(0.7,0.8,0.8),(0.8,0.9,0.9)\rangle \oplus \langle(0.6,0.7,0.7),(0.6,0.8,0.8),(0.6,0.8,0.8)\rangle\end{matrix}\right\}$$

$$= \min\left\{\begin{matrix}\langle(0.4,0.6,0.7),(0.6,0.7,0.8),(0.7,0.8,0.8)\rangle = 0.415, \\ \langle[0.534,0.688,0.830],[0.3,0.36,0.42]\rangle = 0.615\end{matrix}\right\}$$

$$l(3) = \langle(0.4,0.6,0.7),(0.6,0.7,0.8),(0.7,0.8,0.8)\rangle,$$

$$l(4) = \min_{i<4}\{l(2) + p_{24}, l(3) + p_{34}\}$$

$$= \min\left\{\begin{matrix}\langle(0.6,0.5,0.7),(0.7,0.8,0.8),(0.8,0.9,0.9)\rangle \oplus \langle(0.5,0.6,0.6),(0.7,0.7,0.9),(0.6,0.7,0.9)\rangle, \\ \langle(0.4,0.6,0.7),(0.6,0.7,0.8),(0.7,0.8,0.8)\rangle \oplus \langle(0.5,0.6,0.7),(0.6,0.6,0.7),(0.7,0.7,0.9)\rangle\end{matrix}\right\}$$

$$= \min\left\{\begin{matrix}\langle[0.7211,0.7211,0.8207],[0.49,0.56,0.72],[0.48,0.63,0.81]\rangle = 0.5973, \\ \langle[0.6083,0.7684,0.8602],[0.36,0.42,0.56],[0.49,0.56,0.72]\rangle = 0.67575\end{matrix}\right\}$$

$$l(4) = \langle[0.7211,0.7211,0.8207],[0.49,0.56,0.72],[0.48,0.63,0.81]\rangle,$$

$$l(5) = \min_{i<5}\{l(2) + p_{25}, l(3) + p_{35}, l(4) + p_{45}\}$$

$$= \min\left\{\begin{matrix}\langle(0.6,0.5,0.7),(0.7,0.8,0.8),(0.8,0.9,0.9)\rangle \\ \oplus \\ \langle(0.4,0.5,0.6),(0.7,0.7,0.8),(0.7,0.7,0.8)\rangle, \\ \langle(0.4,0.6,0.7),(0.6,0.7,0.8),(0.7,0.8,0.8)\rangle \\ \oplus \\ \langle(0.4,0.8,0.8),(0.7,0.7,0.7),(0.8,0.8,0.8)\rangle, \\ \langle[0.7211,0.7211,0.8207],[0.49,0.56,0.72],[0.48,0.63,0.81]\rangle \\ \oplus \\ \langle(0.5,0.6,0.6),(0.7,0.8,0.8),(0.7,0.8,0.9)\rangle\end{matrix}\right\}$$

$$= \min\begin{Bmatrix} \langle(0.68,0.6614,0.8207),(0.49,0.56,0.64),(0.56,0.63,0.72)\rangle = 0.5924 \\ \langle(0.5426,0.87727,0.9035),(0.42,0.49,0.56),(0.56,0.64,0.72)\rangle = 0.9991 \\ \langle(0.7999,0.8323,0.8894),(0.343,0.448,0.576),(0.336,0.504,0.729)\rangle = 0.7345 \end{Bmatrix}$$

$$l(5) = \langle(0.68,0.6614,0.8207),(0.49,0.56,0.64),(0.56,0.63,0.72)\rangle$$

$$l(6) = \min_{i<6}\{l(4) + p_{46}, l(5) + p_{56}\}$$

$$= \min\begin{Bmatrix} \langle[0.7211,0.7211,0.8207],[0.49,0.56,0.72],[0.48,0.63,0.81]\rangle \\ \oplus \\ \langle(0.4,0.5,0.6),(0.6,0.7,0.8),(0.6,0.8,0.9)\rangle, \\ \langle(0.68,0.6614,0.8207),(0.49,0.56,0.64),(0.56,0.63,0.72)\rangle \\ \oplus \\ \langle(0.6,0.7,0.7),(0.7,0.8,0.8),(0.8,0.8,0.8)\rangle \end{Bmatrix}$$

$$= \min\begin{Bmatrix} \langle[0.7725,0.7999,0.8894],[0.294,0.392,0.576],[0.288,0.504,0.729]\rangle = 0.7349, \\ \langle[0.8098,0.8445,0.9129],[0.343,0.448,0.514],[0.448,0.504,0.576]\rangle = 07578 \end{Bmatrix}$$

$$l(6) = \langle[0.7725,0.7999,0.8894],[0.294,0.392,0.576],[0.288,0.504,0.729]\rangle,$$

Table 2: Minimum path of the NPTFSP

Edges	Minimum path of the NPTFSP
1 → 2 → 4 → 6	**1.7472**
1 → 2 → 5 → 6	1.7652
1→ 2 → 3 → 5 → 6	2.4564
1 → 2 → 3 → 4 → 6	2.4451
1→ 2 → 4 → 5 → 6	2.5046
1 → 3 → 4 → 6	1.8164
1 → 3 → 5 → 6	1.8277
1 → 3 → 5 → 6	2.5689

Thus,

$$l(6) = l(4) \oplus p_{46}$$
$$= l(2) \oplus p_{24} \oplus p_{46}$$
$$= l(1) \oplus p_{12} \oplus p_{24} \oplus p_{46}$$
$$l(6) = p_{12} \oplus p_{24} \oplus p_{46}.$$

Hence, the path $P(1,2,4,6)$ is determined to be the shortest path, with the path direction given by 1→2→4→6.

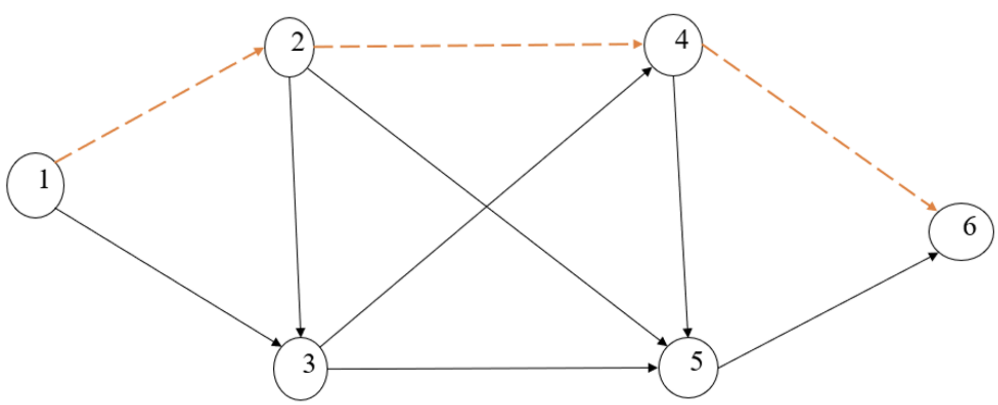

Figure 2: TPFSP Network

5. Conclusion

This paper presents a method for solving shortest path problems in networks with neutrosophic Pythagorean triangular fuzzy edge weights. A pseudo-data example is included to demonstrate the procedure. Furthermore, the Bellman–Ford algorithm is adapted within a neutrosophic Pythagorean trapezoidal framework to solve the corresponding shortest path problem.

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Declarations

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