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Interval-Valued Neutrosophic Models Based on Superhypergraphs

Takaaki Fujita [1*]

[1]Independent Researcher, Tokyo, Japan.
* Corresponding Email: Takaaki.fujita060@gmail.com

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ABSTRACT

Graph theory provides a fundamental framework for modeling relationships via vertices and edges. Hypergraphs extend this framework by allowing hyperedges to connect multiple vertices simultaneously, while superhypergraphs further generalize hypergraphs through iterated powerset constructions, thereby capturing complex hierarchical dependencies. This paper introduces and investigates the interval-valued neutrosophic superhypergraph, a new structure that unifies interval-valued neutrosophic hypergraphs with neutrosophic superhypergraphs. By assigning an interval-valued neutrosophic triple to each vertex at every hierarchical level, the proposed model represents truth, indeterminacy, and falsity degrees within multilevel relations. We also discuss several engineering applications of interval-valued neutrosophic superhypergraphs.

Keywords: HyperGraph; Neutrosophic Graph; SuperHyperGraph; Interval-valued Neutrosophic Hypergraph; Interval-valued Neutrosophic Superhypergraph

1 Introduction

1.1 Graph, Hypergraph, and Superhypergraph

Networks are commonly modeled by graphs, in which entities are represented by vertices and pairwise interactions by edges [1]. Yet, many phenomena involve relations among more than two entities at the same time, and such multiway interactions are not naturally captured by ordinary graphs. This motivates the use of hypergraphs. A hypergraph extends a graph by allowing an edge (a hyperedge) to connect any nonempty subset of the vertex set, thereby providing a direct formalism for higher-order relationships [2, 3, 4].

When the relationships to be modeled are not only multiway but also explicitly hierarchical or multilayered, standard hypergraphs may still be inadequate or cumbersome to use. To accommodate nested structure, the notion of a SuperHyperGraph has been proposed. It is defined through iterated powerset constructions, enabling one to encode connectivity patterns that are recursive, nested, and hierarchical [5]. This line of research has attracted substantial recent attention [5].

Graphs and hypergraphs provide intuitive abstractions for complex systems and support numerous applications in artificial intelligence, network science, data mining, informatics, chemistry, physics, and related fields [6]. By explicitly supporting multi-level relations, SuperHyperGraphs offer a flexible framework for modeling and analyzing the intricate structures that arise in real-world networks. For the reader's convenience, Figure 1 schematically compares a graph, a hypergraph, and a SuperHyperGraph. A graph represents pairwise relations, a hypergraph allows one edge to connect several vertices simultaneously, and a SuperHyperGraph further accommodates nested or hierarchical higher-order objects.

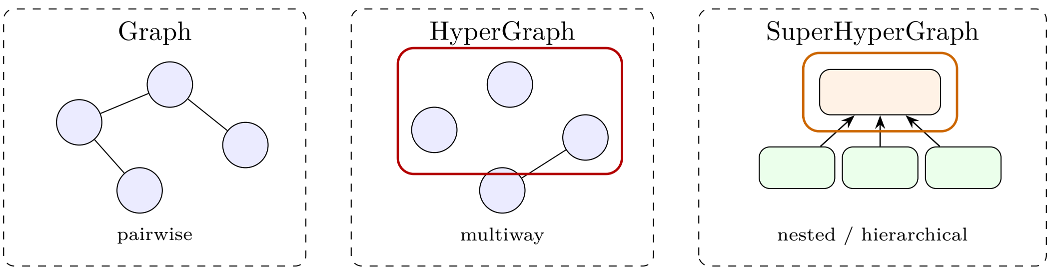

Figure 1: Schematic comparison of a graph, a hypergraph, and a SuperHyperGraph.

1.2 Uncertain Graphs: Fuzzy and Neutrosophic Variants

Uncertainty is intrinsic to many real-world systems and therefore should be modeled explicitly. To this end, a variety of set-based formalisms have been developed, including fuzzy sets [7], intuitionistic fuzzy sets [8], neutrosophic sets [9, 10], soft sets [11, 12], hypersoft sets [13], rough sets [14], and plithogenic sets [15, 16]. These frameworks have subsequently been transferred to graph theory, producing network models in which uncertainty is embedded in the vertex and/or edge descriptions. Notable examples include fuzzy graphs [17], intuitionistic fuzzy graphs [18], vague graphs [19], neutrosophic graphs [20, 21], plithogenic graphs [22, 23], soft graphs [24], hyper-soft graphs [25], and rough graphs [26].

In this paper, we focus on neutrosophic graphs and interval-valued neutrosophic graphs. Neutrosophic graphs assign to each vertex and edge three degrees—truth, indeterminacy, and falsity—each taking values in $[0, 1]$, to model uncertainty precisely. Interval-valued neutrosophic graphs assign to each vertex and edge interval-valued truth, indeterminacy, and falsity degrees in $[0, 1]$, capturing uncertainty ranges and vagueness. These notions have also been extended to hypergraphs and have been actively studied in the literature. For reference, Table 1 presents a comparison between neutrosophic graphs and interval-valued neutrosophic graphs.

Aspect	Neutrosophic Graph (NG)	Interval-Valued Neutrosophic Graph (IVNG)
Underlying structure	A (usually finite, simple) graph $G = (V, E)$.	A (usually finite, simple) graph $G = (V, E)$.
Vertex labeling	Each $v \in V$ has a triple $\big(T_V(v), I_V(v), F_V(v)\big) \in [0, 1]^3$.	Each $v \in V$ has an interval triple $\big([T_V^L(v), T_V^U(v)], [I_V^L(v), I_V^U(v)], [F_V^L(v), F_V^U(v)]\big)$.
Edge labeling	Each $e \in E$ has a triple $\big(T_E(e), I_E(e), F_E(e)\big) \in [0, 1]^3$.	Each $e \in E$ has an interval triple $\big([T_E^L(e), T_E^U(e)], [I_E^L(e), I_E^U(e)], [F_E^L(e), F_E^U(e)]\big)$.
Basic constraints	$0 \le T, I, F \le 1$ and typically $0 \le T + I + F \le 3$.	$0 \le T^L \le T^U \le 1$, etc., and typically $T^L + I^L + F^L \le 3$, $T^U + I^U + F^U \le 3$.
Uncertainty granularity	Point-valued assessment (single estimate).	Bounded assessment (ranges), capturing measurement noise / expert disagreement.
Expressiveness vs. simplicity	Simpler model; fewer parameters (3 values per vertex/edge).	More expressive; more parameters (6 values per vertex/edge).
Relation between models	—	IVNG generalizes NG: NG is obtained when $T^L = T^U$, $I^L = I^U$, $F^L = F^U$ for all vertices/edges.
Typical use cases	When neutrosophic degrees can be specified confidently.	When only interval bounds are credible (imprecise data, expert intervals).

Table 1: Concise comparison between neutrosophic graphs and interval-valued neutrosophic graphs

1.3 Motivation and Our Contributions

In view of the above, research on interval-valued neutrosophic sets and graphs, as well as on SuperHyperGraphs, is of considerable importance. However, graph-theoretic frameworks that systematically integrate these concepts remain insufficiently developed. To address this gap, this paper introduces and investigates the interval-valued neutrosophic superhypergraph, a novel structure that unifies interval-valued neutrosophic hypergraphs with neutrosophic superhypergraphs. By assigning interval-valued neutrosophic triples to each vertex at every hierarchical level, the proposed model is able to represent truth, indeterminacy, and falsity degrees within multilevel relationships.

It should also be noted that the present paper is deliberately restricted to the theoretical aspect of the subject. Accordingly, performance evaluation through computational experiments or application-oriented validation is left for future investigation by specialists in the relevant areas. It should further be kept in mind that the examples included in this paper are illustrative desk-based considerations intended only to clarify the proposed concepts.

2 Preliminaries

This section fixes notation and recalls the basic objects used throughout the paper.

2.1 SuperHyperGraphs

A hypergraph generalizes a graph by allowing an edge (a hyperedge) to join an arbitrary nonempty subset of vertices. A SuperHyperGraph (in the sense of iterated powersets) further allows vertices themselves to be higher–order, set–valued objects obtained by repeatedly applying the powerset operator; this is useful for modeling hierarchical or recursively defined relations (e.g., groups of groups) [5, 27].

Definition 2.1 (Base set). A base set is a fixed nonempty set S. All higher–order objects in this paper are built from S using standard set–theoretic constructions.

Definition 2.2 (Powerset). For a set S, the powerset of S is

$$\mathcal{P}(S) := \{ A : A \subseteq S \}.$$

The nonempty powerset is $\mathcal{P}(S) \setminus \{\varnothing\}$.

Definition 2.3 (Hypergraph). [3, 28] A (finite simple) hypergraph is a pair $H = (V, E)$ where V is a finite set (vertices) and

$$E \subseteq \mathcal{P}(V) \setminus \{\varnothing\}.$$

Elements of E are called hyperedges.

Definition 2.4 (Iterated powerset). [29] For a set X and an integer $k \geq 0$, define recursively

$$\mathcal{P}^0(X) := X, \qquad \mathcal{P}^{k+1}(X) := \mathcal{P}\big(\mathcal{P}^k(X)\big).$$

Likewise, define the iterated nonempty powerset by

$$\mathcal{P}^0_{\neq\varnothing}(X) := X, \qquad \mathcal{P}^{k+1}_{\neq\varnothing}(X) := \mathcal{P}\big(\mathcal{P}^k_{\neq\varnothing}(X)\big) \setminus \{\varnothing\}.$$

Definition 2.5 (n-SuperHyperGraph). [5] Let V_0 be a finite base set and let $n \geq 1$. An n-SuperHyperGraph is a pair

$$\mathrm{SuHG}^{(n)} = (V, E),$$

such that

$$V \subseteq \mathcal{P}^n(V_0), \qquad E \subseteq \mathcal{P}(V) \setminus \{\varnothing\}.$$

Elements of V are called n-supervertices and elements of E are called n-superedges.

Example 2.6 (A concrete 2-SuperHyperGraph). Let

$$V_0 = \{a, b, c\}$$

and take $n = 2$. Then $\mathcal{P}^1(V_0) = \mathcal{P}(V_0)$ and $\mathcal{P}^2(V_0) = \mathcal{P}(\mathcal{P}(V_0))$.

Define the following 2-supervertices (each is a set of subsets of V_0):

$$X_1 = \big\{\{a\}, \{a, b\}\big\}, \qquad X_2 = \big\{\{b, c\}\big\}, \qquad X_3 = \big\{\{c\}, \{a, c\}\big\}.$$

Clearly $X_1, X_2, X_3 \in \mathcal{P}^2(V_0)$. Set

$$V = \{X_1, X_2, X_3\} \subseteq \mathcal{P}^2(V_0).$$

Now define 2-superedges as nonempty subsets of V, for instance,

$$e_1 = \{X_1, X_2\}, \qquad e_2 = \{X_2, X_3\}, \qquad e_3 = \{X_1, X_2, X_3\}.$$

Then

$$E = \{e_1, e_2, e_3\} \subseteq \mathcal{P}(V) \setminus \{\varnothing\}.$$

Hence $\mathrm{SuHG}^{(2)} = (V, E)$ is a 2-SuperHyperGraph in the sense of Definition 2.5.

For reference, a diagrammatic illustration of this example is given in Figure 2.

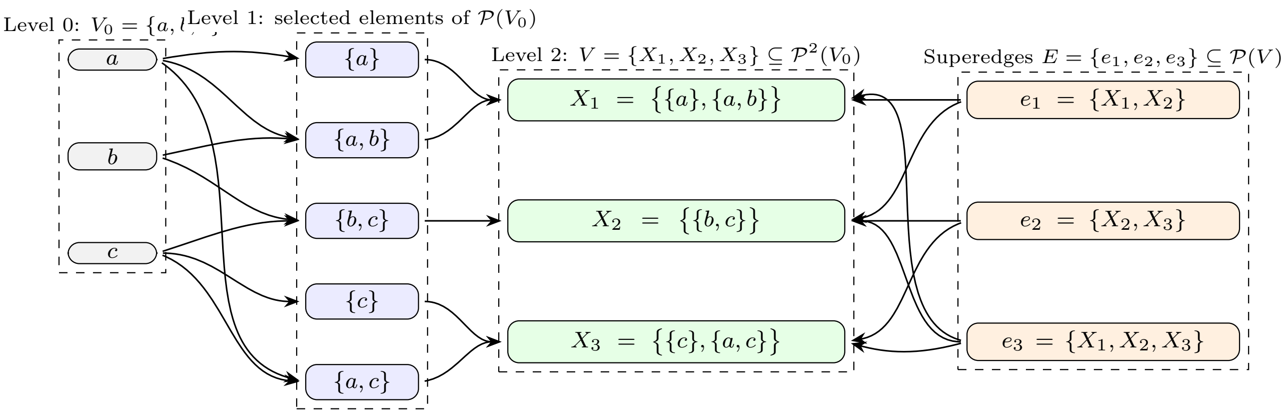

Figure 2: A diagrammatic view of the concrete 2-SuperHyperGraph in Example 2.6. Arrows from left to right indicate set membership across levels, while arrows from e_i to X_j indicate incidence $X_j \in e_i$.

2.2 Neutrosophic n-SuperHyperGraphs

A neutrosophic set assigns to each element three (generally independent) degrees: truth, indeterminacy, and falsity, each in $[0,1]$ [30, 31]. Neutrosophic graphs and hypergraphs label vertices and/or incidences by such triples; see, e.g., [20, 32, 33]. We recall a standard hypergraph version and then present a compatible extension to the n-SuperHyperGraph setting.

Definition 2.7 (Single-valued neutrosophic hypergraph). [33] Let $V = \{v_1, \ldots, v_N\}$ be a finite vertex set. A single-valued neutrosophic hyperedge on V is specified by three maps

$$T,\ I,\ F : V \to [0,1]$$

(its truth, indeterminacy, and falsity memberships), typically subject to

$$0 \le T(v) + I(v) + F(v) \le 3 \qquad (\forall\, v \in V).$$

Its support is

$$\mathrm{supp}(T, I, F) := \{\, v \in V :\ T(v) > 0 \text{ or } I(v) > 0 \text{ or } F(v) > 0 \,\}.$$

A single-valued neutrosophic hypergraph is a pair $H = (V, \{E_i\}_{i=1}^{M})$ where each E_i is a single-valued neutrosophic hyperedge on V and

$$V = \bigcup_{i=1}^{M} \mathrm{supp}(E_i).$$

Definition 2.8 (Neutrosophic n-SuperHyperGraph). Let V_0 be a finite ground set and let $n \ge 1$. A neutrosophic n-SuperHyperGraph is a tuple

$$\mathcal{N}\mathrm{SuHG}^{(n)} \;=\; \big(V, E,\ T_V, I_V, F_V,\ T_E, I_E, F_E\big),$$

where:

(i) (V, E) is an n-SuperHyperGraph in the sense of Definition 2.5;

(ii) $T_V, I_V, F_V : V \to [0,1]$ assign to each supervertex $v \in V$ its truth, indeterminacy, and falsity degrees, satisfying

$$0 \le T_V(v) + I_V(v) + F_V(v) \le 3 \qquad (\forall\, v \in V);$$

(iii) $T_E, I_E, F_E : E \times V \to [0,1]$ assign neutrosophic incidence degrees, satisfying

$$0 \le T_E(e, v) + I_E(e, v) + F_E(e, v) \le 3 \qquad (\forall\, e \in E,\ \forall\, v \in V);$$

(iv) (support compatibility) if $v \notin e$ then

$$T_E(e, v) = I_E(e, v) = F_E(e, v) = 0;$$

equivalently, the support of the neutrosophic incidence profile of e is contained in the crisp superedge e;

(v) (containment constraints) for all $e \in E$ and $v \in V$,

$$T_E(e, v) \le T_V(v), \qquad I_E(e, v) \le I_V(v), \qquad F_E(e, v) \le F_V(v).$$

Example 2.9 (A neutrosophic 2-SuperHyperGraph for a corporate team structure). Let

$$V_0 = \{\mathrm{Ayano}, \mathrm{Syohei}, \mathrm{Kazuya}, \mathrm{Dave}\}.$$

Level 1 (teams). Define three teams (subsets of V_0):

$$t_1 = \{\mathrm{Ayano}, \mathrm{Syohei}\}, \qquad t_2 = \{\mathrm{Syohei}, \mathrm{Kazuya}\}, \qquad t_3 = \{\mathrm{Kazuya}, \mathrm{Dave}\}.$$

Level 2 (divisions as 2-supervertices). Let

$$v_1 = \{t_1, t_2\}, \qquad v_2 = \{t_2, t_3\}.$$

Then $v_1, v_2 \in \mathcal{P}^2(V_0) = \mathcal{P}(\mathcal{P}(V_0))$, and we set

$$V = \{v_1, v_2\} \subseteq \mathcal{P}^2(V_0).$$

Projects as 2-superedges. Define

$$e_1 = \{v_1, v_2\}, \qquad e_2 = \{v_1\},$$

so $E = \{e_1, e_2\} \subseteq \mathcal{P}(V) \setminus \{\varnothing\}$. Thus $\mathrm{SuHG}^{(2)} = (V, E)$ is a 2-SuperHyperGraph.

Neutrosophic memberships on supervertices. Let $T_V, I_V, F_V : V \to [0,1]$ be given by

$$(T_V(v_1), I_V(v_1), F_V(v_1)) = (0.90, 0.05, 0.05), \qquad (T_V(v_2), I_V(v_2), F_V(v_2)) = (0.80, 0.10, 0.10).$$

Neutrosophic memberships on incidences. Define $T_E, I_E, F_E : E \times V \to [0,1]$ by specifying the nonzero values

$$(T_E(e_1, v_1), I_E(e_1, v_1), F_E(e_1, v_1)) = (0.85, 0.10, 0.05),$$

$$(T_E(e_1, v_2), I_E(e_1, v_2), F_E(e_1, v_2)) = (0.75, 0.15, 0.10),$$

$$(T_E(e_2, v_1), I_E(e_2, v_1), F_E(e_2, v_1)) = (0.88, 0.07, 0.05),$$

and setting $T_E(e, v) = I_E(e, v) = F_E(e, v) = 0$ for all remaining pairs (e, v). Note that these values are introduced here as illustrative hypothetical assignments for the purpose of definition and explanation, and are not computed from empirical data or a separate estimation procedure.

Then the support compatibility condition holds automatically (no memberships are assigned outside the crisp edge), and the containment constraints are immediate from the chosen values, e.g., $T_E(e_1, v_2) = 0.75 \le 0.80 = T_V(v_2)$ and similarly for I and F.

Hence

$$\big(V, E, T_V, I_V, F_V, T_E, I_E, F_E\big)$$

is a neutrosophic 2-SuperHyperGraph modeling (i) divisions as groups of teams and (ii) uncertainty in project–division participation.

For reference, a schematic overview of this example is provided in Figure 3.

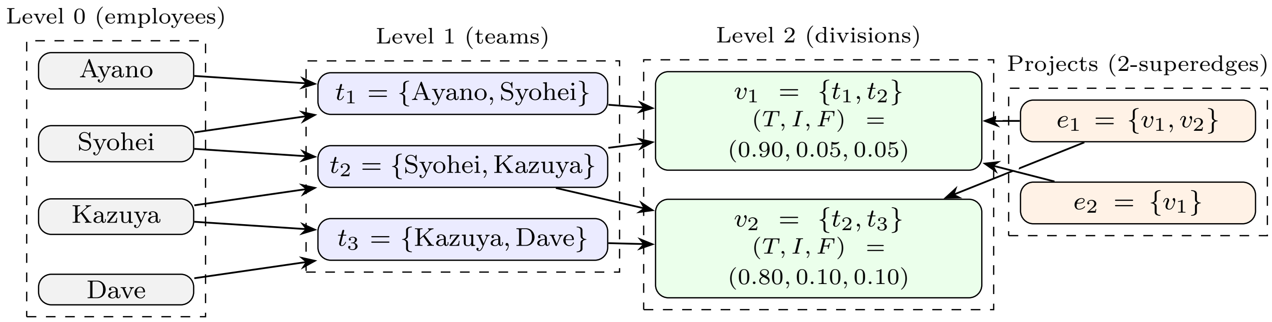

Figure 3: A neutrosophic 2-SuperHyperGraph for a corporate team structure. Solid arrows indicate hierarchical membership. Labels on project–division links are incidence triples (T, I, F).

2.3 Interval-valued Neutrosophic Hypergraph

An interval-valued neutrosophic set (IVNS) assigns to every element three closed subintervals of $[0, 1]$—one each for truth, indeterminacy, and falsity[34, 35]. The use of intervals, rather than single numbers, enables a richer representation of uncertainty and vagueness. An interval-valued neutrosophic graph (IVNG) extends a classical graph by equipping every vertex and every edge with an IVNS triple [36, 37]. Thus both objects and relationships are described through interval-valued truth, indeterminacy, and falsity degrees. An interval-valued neutrosophic hypergraph is a generalized hypergraph in which each hyperedge maps vertices to interval-valued neutrosophic membership functions [38, 39]. These functions assign, for each vertex, three intervals representing degrees of truth, indeterminacy, and falsity, respectively.

Definition 2.10 (Interval-valued Neutrosophic Hypergraph). (cf.[38, 39]) Let $X = \{x_1, x_2, \ldots, x_n\}$ be a finite set of vertices, and let $\mathrm{IVNS}(X)$ denote the collection of all interval-valued neutrosophic sets on X, where each

$$A(x) \;=\; \big([T_L(x), T_U(x)],\ [I_L(x), I_U(x)],\ [F_L(x), F_U(x)]\big)$$

satisfies

$$0 \le T_L(x) \le T_U(x) \le 1, \quad 0 \le I_L(x) \le I_U(x) \le 1, \quad 0 \le F_L(x) \le F_U(x) \le 1.$$

An interval-valued neutrosophic hypergraph is an ordered pair

$$\mathcal{H} = (X, E),$$

where

$$E = \{E_1,\ E_2,\ \ldots,\ E_m\} \;\subseteq\; \mathrm{IVNS}(X)$$

satisfies

1. Nontriviality: each $E_j \ne O = ([0, 0], [0, 0], [0, 0])$ for $j = 1, 2, \ldots, m$;
2. Coverage: $\displaystyle\bigcup_{j=1}^{m} \{\, x \in X :\ E_j(x) \ne O \} \;=\; X.$

Here each $E_j(x)$ gives the interval-valued neutrosophic membership of the vertex x in the hyperedge E_j.

Example 2.11 (Interval-valued Neutrosophic Hypergraph for E-commerce Product Classification). Let

$$X = \{\text{Laptop},\ \text{Smartphone},\ \text{Tablet},\ \text{Smartwatch}\}.$$

We define three interval-valued neutrosophic hyperedges corresponding to product categories:

1. Portable Devices (E_1)

$$E_1(\text{Laptop}) = \big([0.90, 1.00],\ [0.00, 0.05],\ [0.00, 0.05]\big),$$
$$E_1(\text{Smartphone}) = \big([0.85, 0.95],\ [0.05, 0.10],\ [0.00, 0.05]\big),$$

$$E_1(\text{Tablet}) = \big([0.80, 0.90],\ [0.05, 0.15],\ [0.00, 0.05]\big),$$
$$E_1(\text{Smartwatch}) = \big([0.75, 0.85],\ [0.10, 0.20],\ [0.00, 0.05]\big).$$

2. High-End Products (E_2)

$$E_2(\text{Laptop}) = \big([0.70, 0.85],\ [0.10, 0.15],\ [0.00, 0.10]\big),$$
$$E_2(\text{Smartphone}) = \big([0.65, 0.80],\ [0.10, 0.20],\ [0.00, 0.10]\big),$$
$$E_2(\text{Tablet}) = \big([0.60, 0.75],\ [0.15, 0.25],\ [0.00, 0.10]\big),$$
$$E_2(\text{Smartwatch}) = \big([0.55, 0.70],\ [0.20, 0.30],\ [0.00, 0.10]\big).$$

3. Budget-Friendly (E_3)

$$E_3(\text{Laptop}) = \big([0.40, 0.55],\ [0.20, 0.30],\ [0.10, 0.20]\big),$$
$$E_3(\text{Smartphone}) = \big([0.60, 0.75],\ [0.10, 0.20],\ [0.05, 0.15]\big),$$
$$E_3(\text{Tablet}) = \big([0.65, 0.80],\ [0.05, 0.15],\ [0.00, 0.10]\big),$$
$$E_3(\text{Smartwatch}) = \big([0.50, 0.65],\ [0.15, 0.25],\ [0.05, 0.15]\big).$$

One verifies easily that for each $j = 1, 2, 3$ and each $x \in X$:

$$0 \le T_L(x) \le T_U(x) \le 1, \quad 0 \le I_L(x) \le I_U(x) \le 1, \quad 0 \le F_L(x) \le F_U(x) \le 1,$$

and

$$T_L(x) + I_L(x) + F_L(x) \le 3, \quad T_U(x) + I_U(x) + F_U(x) \le 3.$$

Moreover no E_j is identically $[0, 0]$, and $\bigcup_{j=1}^{3}\{x \in X : E_j(x) \neq O\} = X$, so $\mathcal{H} = (X, \{E_1, E_2, E_3\})$ is a valid interval-valued neutrosophic hypergraph modeling uncertain category memberships in an e-commerce recommendation system.

3 Main Results: Interval-Valued Neutrosophic n-SuperHyperGraphs

As a main contribution of this paper, we introduce an interval-valued neutrosophic enrichment of n-SuperHyperGraphs and record several basic structural facts.

Definition 3.1 (Interval-valued neutrosophic n-SuperHyperGraph). Let V_0 be a finite ground set and let $n \ge 1$. Recall the iterated powersets

$$\mathcal{P}^0(V_0) := V_0, \qquad \mathcal{P}^{k+1}(V_0) := \mathcal{P}\big(\mathcal{P}^k(V_0)\big) \quad (k \ge 0).$$

An interval-valued neutrosophic n-SuperHyperGraph is a tuple

$$\mathcal{H} \ = \ \big(V, E,\, T_V^L, T_V^U,\, I_V^L, I_V^U,\, F_V^L, F_V^U,\, T_E^L, T_E^U,\, I_E^L, I_E^U,\, F_E^L, F_E^U\big),$$

where:

(i) $V \subseteq \mathcal{P}^n(V_0)$ is the set of n-supervertices;

(ii) $E \subseteq \mathcal{P}(V) \setminus \{\varnothing\}$ is the set of n-superedges;

(iii) $T_V^L, T_V^U, I_V^L, I_V^U, F_V^L, F_V^U : V \to [0, 1]$ define, for each $v \in V$, intervals

$$\mathbf{T}_V(v) := [T_V^L(v), T_V^U(v)], \quad \mathbf{I}_V(v) := [I_V^L(v), I_V^U(v)], \quad \mathbf{F}_V(v) := [F_V^L(v), F_V^U(v)];$$

(iv) $T_E^L, T_E^U, I_E^L, I_E^U, F_E^L, F_E^U : E \times V \to [0, 1]$ define, for each $(e, v) \in E \times V$, intervals

$$\mathbf{T}_E(e, v) := [T_E^L(e, v), T_E^U(e, v)], \quad \mathbf{I}_E(e, v) := [I_E^L(e, v), I_E^U(e, v)], \quad \mathbf{F}_E(e, v) := [F_E^L(e, v), F_E^U(e, v)].$$

These data are required to satisfy, for all $v \in V$ and all $(e, v) \in E \times V$,

$$\begin{aligned}
&0 \le T_V^L(v) \le T_V^U(v) \le 1, \quad 0 \le I_V^L(v) \le I_V^U(v) \le 1, \quad 0 \le F_V^L(v) \le F_V^U(v) \le 1,\\
&0 \le T_E^L(e, v) \le T_E^U(e, v) \le 1, \quad 0 \le I_E^L(e, v) \le I_E^U(e, v) \le 1, \quad 0 \le F_E^L(e, v) \le F_E^U(e, v) \le 1,\\
&\text{(support compatibility)} \qquad v \notin e \implies T_E^L(e, v) = T_E^U(e, v) = I_E^L(e, v) = I_E^U(e, v) = F_E^L(e, v) = F_E^U(e, v) = 0,\\
&\text{(containment constraints)} \qquad T_E^L(e, v) \le T_V^L(v), \quad T_E^U(e, v) \le T_V^U(v),\\
&\qquad\qquad\qquad I_E^L(e, v) \le I_V^L(v), \quad I_E^U(e, v) \le I_V^U(v),\\
&\qquad\qquad\qquad F_E^L(e, v) \le F_V^L(v), \quad F_E^U(e, v) \le F_V^U(v).
\end{aligned}$$

Remark 3.2 (Optional nontriviality and coverage conditions). Definition 3.1 does not force the structure to be covering or nontrivial. In many applications, one additionally imposes:

(a) Vertex nontriviality: for every $v \in V$, $T_V^U(v) + I_V^U(v) + F_V^U(v) > 0$.

(b) Edge nontriviality: for every $e \in E$, there exists $v \in e$ with $T_E^U(e, v) + I_E^U(e, v) + F_E^U(e, v) > 0$.

(c) Coverage: for every $v \in V$, there exists $e \in E$ with $v \in e$ and $T_E^U(e, v) + I_E^U(e, v) + F_E^U(e, v) > 0$.

When needed, we will state these as explicit assumptions.

Example 3.3 (Interval-valued neutrosophic 2-SuperHyperGraph for a corporate organization). Let

$$V_0 = \{\text{Ayano}, \text{Syohei}, \text{Kazuya}, \text{Dave}\} \qquad (n = 2).$$

Define level-1 teams in $\mathcal{P}(V_0)$:

$$t_1 = \{\text{Ayano}, \text{Syohei}\}, \qquad t_2 = \{\text{Syohei}, \text{Kazuya}\}, \qquad t_3 = \{\text{Kazuya}, \text{Dave}\}.$$

Define 2-supervertices (divisions) in $\mathcal{P}^2(V_0) = \mathcal{P}(\mathcal{P}(V_0))$:

$$v_1 = \{t_1, t_2\}, \qquad v_2 = \{t_2, t_3\},$$

and set $V = \{v_1, v_2\} \subseteq \mathcal{P}^2(V_0)$. Let the projects (superedges) be subsets of V:

$$e_1 = \{v_1, v_2\}, \qquad e_2 = \{v_2\}, \qquad \text{so } E = \{e_1, e_2\} \subseteq \mathcal{P}(V) \setminus \{\varnothing\}.$$

Assign interval-valued neutrosophic memberships to supervertices:

$$\begin{aligned}
&\mathbf{T}_V(v_1) = [0.80, 0.90], \quad \mathbf{I}_V(v_1) = [0.05, 0.10], \quad \mathbf{F}_V(v_1) = [0.00, 0.10],\\
&\mathbf{T}_V(v_2) = [0.60, 0.75], \quad \mathbf{I}_V(v_2) = [0.10, 0.20], \quad \mathbf{F}_V(v_2) = [0.05, 0.15].
\end{aligned}$$

For incidences, set the nonzero values (and set all remaining (e, v) to $[0, 0]$):

$$\begin{aligned}
&\mathbf{T}_E(e_1, v_1) = [0.70, 0.85], \quad \mathbf{I}_E(e_1, v_1) = [0.05, 0.10], \quad \mathbf{F}_E(e_1, v_1) = [0.00, 0.05],\\
&\mathbf{T}_E(e_1, v_2) = [0.55, 0.70], \quad \mathbf{I}_E(e_1, v_2) = [0.10, 0.15], \quad \mathbf{F}_E(e_1, v_2) = [0.05, 0.10],\\
&\mathbf{T}_E(e_2, v_2) = [0.58, 0.72], \quad \mathbf{I}_E(e_2, v_2) = [0.10, 0.18], \quad \mathbf{F}_E(e_2, v_2) = [0.05, 0.10].
\end{aligned}$$

Then support compatibility holds because $\mathbf{T}_E(e, v) = \mathbf{I}_E(e, v) = \mathbf{F}_E(e, v) = [0, 0]$ whenever $v \notin e$, and the containment constraints hold componentwise, e.g.,

$$T_E^U(e_1, v_2) = 0.70 \le 0.75 = T_V^U(v_2), \qquad F_E^L(e_2, v_2) = 0.05 \le 0.05 = F_V^L(v_2),$$

and similarly for the remaining components. Hence $\mathcal{H}$ is an interval-valued neutrosophic 2-SuperHyperGraph.

Example 3.4 (A "team-of-teams-of-teams" interval-valued neutrosophic 3-SuperHyperGraph). Let $V_0 = \{\text{Ayano}, \text{Syohei}, \text{Kazuya}\}$ and $n = 3$. Define teams $t_1 = \{\text{Ayano}, \text{Syohei}\}$, $t_2 = \{\text{Syohei}, \text{Kazuya}\}$ in $\mathcal{P}(V_0)$, divisions $g_1 = \{t_1\}$, $g_2 = \{t_2\}$ in $\mathcal{P}^2(V_0)$, and clusters (supervertices) in $\mathcal{P}^3(V_0)$:

$$v_1 = \{g_1, g_2\}, \qquad v_2 = \{g_2\}.$$

Let $V = \{v_1, v_2\}$ and define superedges

$$e_1 = \{v_1, v_2\}, \qquad e_2 = \{v_1\}, \qquad E = \{e_1, e_2\}.$$

Assign supervertex intervals:

$$\begin{aligned}
&\mathbf{T}_V(v_1) = [0.85, 0.95], \quad \mathbf{I}_V(v_1) = [0.02, 0.10], \quad \mathbf{F}_V(v_1) = [0.00, 0.05],\\
&\mathbf{T}_V(v_2) = [0.60, 0.75], \quad \mathbf{I}_V(v_2) = [0.10, 0.20], \quad \mathbf{F}_V(v_2) = [0.00, 0.10].
\end{aligned}$$

Assign incidence intervals (and set all other (e, v) to $[0, 0]$):

$$\begin{aligned}
&\mathbf{T}_E(e_1, v_1) = [0.80, 0.92], \quad \mathbf{I}_E(e_1, v_1) = [0.05, 0.12], \quad \mathbf{F}_E(e_1, v_1) = [0.00, 0.05],\\
&\mathbf{T}_E(e_1, v_2) = [0.55, 0.70], \quad \mathbf{I}_E(e_1, v_2) = [0.10, 0.18], \quad \mathbf{F}_E(e_1, v_2) = [0.05, 0.10],\\
&\mathbf{T}_E(e_2, v_1) = [0.83, 0.90], \quad \mathbf{I}_E(e_2, v_1) = [0.03, 0.08], \quad \mathbf{F}_E(e_2, v_1) = [0.00, 0.04].
\end{aligned}$$

All axioms in Definition 3.1 are satisfied, hence $\mathcal{H}$ is an interval-valued neutrosophic 3-SuperHyperGraph.

Proposition 3.5 (Forgetting memberships yields a crisp n-SuperHyperGraph). If $\mathcal{H}$ is an interval-valued neutrosophic n-SuperHyperGraph, then (V, E) is an n-SuperHyperGraph.

Proof. By Definition 3.1, we have $V \subseteq \mathcal{P}^n(V_0)$ and $E \subseteq \mathcal{P}(V) \setminus \{\varnothing\}$, which is precisely the set-theoretic condition for an n-SuperHyperGraph. □

Theorem 3.6 (Collapse of intervals: reduction to the single-valued neutrosophic case). Let $\mathcal{H}$ be an interval-valued neutrosophic n-SuperHyperGraph. Assume that all endpoints coincide, i.e., for every $v \in V$ and $(e, v) \in E \times V$,

$$T_V^L(v) = T_V^U(v), \quad I_V^L(v) = I_V^U(v), \quad F_V^L(v) = F_V^U(v),$$

and

$$T_E^L(e, v) = T_E^U(e, v), \quad I_E^L(e, v) = I_E^U(e, v), \quad F_E^L(e, v) = F_E^U(e, v).$$

Then $\mathcal{H}$ canonically determines a single-valued neutrosophic n-SuperHyperGraph by identifying each interval $[x, x]$ with x.

Proof. Define scalar functions $T_V(v) := T_V^L(v)$ (and similarly I_V, F_V, T_E, I_E, F_E). All inequalities in Definition 3.1 immediately reduce to the corresponding scalar inequalities, including support compatibility ($v \notin e \Rightarrow T_E(e, v) = I_E(e, v) = F_E(e, v) = 0$) and componentwise containment ($T_E(e, v) \le T_V(v)$, etc.). □

Theorem 3.7 (Reduction to interval-valued neutrosophic hypergraphs when $n = 1$). Let $n = 1$ and let $\mathcal{H}$ be an interval-valued neutrosophic 1-SuperHyperGraph on V_0. Assume that the supervertex set is the set of singletons

$$V = \{\{x\} : x \in V_0\} \subseteq \mathcal{P}(V_0),$$

and identify $x \in V_0$ with $\{x\} \in V$. Then $\mathcal{H}$ induces an interval-valued neutrosophic hypergraph on the vertex set V_0, with hyperedges $\widehat{e} \subseteq V_0$ corresponding to $e \in E \subseteq \mathcal{P}(V)$ via

$$\widehat{e} := \{\, x \in V_0 : \{x\} \in e \,\},$$

and interval-valued incidence memberships given by

$$\mathbf{T}_{\widehat{e}}(x) := \mathbf{T}_E(e, \{x\}), \qquad \mathbf{I}_{\widehat{e}}(x) := \mathbf{I}_E(e, \{x\}), \qquad \mathbf{F}_{\widehat{e}}(x) := \mathbf{F}_E(e, \{x\}).$$

Proof. Since $E \subseteq \mathcal{P}(V) \setminus \{\varnothing\}$, each $e \in E$ corresponds bijectively to a nonempty subset $\widehat{e} \subseteq V_0$. Support compatibility in Definition 3.1 yields $[0, 0]$ memberships outside $\widehat{e}$, and all endpoint constraints carry over verbatim under the identification $x \leftrightarrow \{x\}$. □

Theorem 3.8 (Intersection and union operations (membership-wise)). Let

$$\mathcal{H}_1 = (V, E_1, \dots), \qquad \mathcal{H}_2 = (V, E_2, \dots)$$

be interval-valued neutrosophic n-SuperHyperGraphs on the same supervertex set V, and assume their supervertex membership functions coincide (i.e., the same $T^L_V, T^U_V, \dots, F^U_V$ are used for both).

(i) Define the intersection $\mathcal{H}_\cap = (V, E_1 \cap E_2, \dots)$ by setting, for $e \in E_1 \cap E_2$ and $v \in V$,

$$T^L_\cap(e, v) := \min\{T^L_{E_1}(e, v), T^L_{E_2}(e, v)\}, \qquad T^U_\cap(e, v) := \min\{T^U_{E_1}(e, v), T^U_{E_2}(e, v)\},$$

and analogously for $I^L_\cap, I^U_\cap, F^L_\cap, F^U_\cap$. Then $\mathcal{H}_\cap$ is an interval-valued neutrosophic n-SuperHyperGraph.

(ii) Define the union $\mathcal{H}_\cup = (V, E_1 \cup E_2, \dots)$ as follows. For each $e \in E_1 \cup E_2$ and $v \in V$, set

$$T^L_\cup(e, v) := \begin{cases} T^L_{E_1}(e, v), & e \in E_1 \setminus E_2, \\ T^L_{E_2}(e, v), & e \in E_2 \setminus E_1, \\ \max\{T^L_{E_1}(e, v), T^L_{E_2}(e, v)\}, & e \in E_1 \cap E_2, \end{cases}$$

and likewise for $T^U_\cup, I^L_\cup, I^U_\cup, F^L_\cup, F^U_\cup$. Then $\mathcal{H}_\cup$ is an interval-valued neutrosophic n-SuperHyperGraph. Moreover, if both inputs satisfy the coverage condition in Remark 3.2(c), then $\mathcal{H}_\cup$ is also covering.

Proof. We verify the axioms of Definition 3.1. For (i), the pointwise minimum of two valid endpoints remains in $[0, 1]$ and preserves the inequality $L \le U$. Support compatibility is preserved because both $\mathcal{H}_1$ and $\mathcal{H}_2$ assign $[0, 0]$ outside e. Containment holds since $\min\{\cdot, \cdot\} \le$ each argument, hence, e.g.,

$$T^U_\cap(e, v) \le T^U_{E_i}(e, v) \le T^U_V(v) \qquad (i = 1, 2),$$

and similarly for the other components and lower endpoints.

For (ii), the piecewise definition copies valid data on edges appearing in exactly one structure, and uses pointwise maxima on common edges. Maxima preserve $L \le U$ and keep endpoints in $[0, 1]$. Containment holds because $\max\{\cdot, \cdot\} \le T^U_V(v)$ whenever each argument is $\le T^U_V(v)$, and likewise for lower endpoints and for I and F. Coverage for the union follows immediately: if a vertex v is covered in $\mathcal{H}_1$ (or $\mathcal{H}_2$), then it remains covered by the same witnessing edge in $E_1 \cup E_2$. □

Remark 3.9 (On "intersection closure" under coverage). Even if $\mathcal{H}_1$ and $\mathcal{H}_2$ are both covering (Remark 3.2(c)), their intersection $\mathcal{H}_\cap$ need not be covering, because an element of V might be covered in $\mathcal{H}_1$ by edges not present in $\mathcal{H}_2$, and vice versa. Accordingly, Theorem 3.8(i) is stated for the core axioms of Definition 3.1 without asserting coverage.

For reference, a comparison of neutrosophic n-SuperHyperGraphs and interval-valued neutrosophic n-SuperHyperGraphs is provided in Table 2.

4 Additional Example: Some Applications in Engineering

In this section, we discuss several engineering applications of interval-valued neutrosophic n-SuperHyperGraphs. As illustrated by these examples, interval-valued neutrosophic n-SuperHyperGraphs may be applied across a wide range of engineering domains. It should be noted, however, that these examples are only preliminary desk-based application considerations. Further validation, including computational experiments and other application-oriented investigations, may be required in future studies where appropriate.

Example 4.1 (Interval-valued neutrosophic 2-SuperHyperGraph for cyber–physical fault diagnosis). Consider a cyber–physical system (CPS) consisting of sensors, actuators, and controllers:

$$V_0 = \{\mathrm{Temp}, \mathrm{Press}, \mathrm{Vib}, \mathrm{Valve}, \mathrm{Motor}, \mathrm{PLC}\}.$$

We build a 2-SuperHyperGraph in which level-1 objects represent local fault-signatures (groups of components that co-vary under a fault), and level-2 objects represent subsystems (groups of fault-signatures).

Table 2: Concise comparison of neutrosophic n-SuperHyperGraphs and interval-valued neutrosophic n-SuperHyperGraphs

Aspect	Neutrosophic n-SuperHyperGraph	Interval-Valued Neutrosophic n-SuperHyperGraph
Underlying crisp structure	(V, E) with $V \subseteq \mathcal{P}^n(V_0)$ and $E \subseteq \mathcal{P}(V) \setminus \{\varnothing\}$.	Same underlying (V, E) (typically), with the same domain constraints.
Vertex memberships	Scalar triples $(T_V(v), I_V(v), F_V(v)) \in [0, 1]^3$.	Interval triples $\big([T_V^L(v), T_V^U(v)],\ [I_V^L(v), I_V^U(v)],\ [F_V^L(v), F_V^U(v)]\big)$.
Incidence / edge–vertex memberships	Scalar triples $(T_E(e, v), I_E(e, v), F_E(e, v)) \in [0, 1]^3$ (often defined for $(e, v) \in E \times V$).	Interval triples $\big([T_E^L(e, v), T_E^U(e, v)],\ [I_E^L(e, v), I_E^U(e, v)],\ [F_E^L(e, v), F_E^U(e, v)]\big)$.
Support compatibility (typical)	Usually enforced as $v \notin e \Rightarrow (T_E, I_E, F_E) = (0, 0, 0)$.	Analogously, $v \notin e \Rightarrow$ all incidence intervals equal $[0, 0]$.
Containment constraint (typical)	Incidence bounded by vertex degrees, e.g. $T_E(e, v) \le T_V(v)$ (and similarly for I, F).	Endpointwise containment, e.g. $T_E^L(e, v) \le T_V^L(v)$ and $T_E^U(e, v) \le T_V^U(v)$ (and similarly for I, F).
Expressiveness	Models uncertainty with a single (point) estimate per membership.	Models uncertainty with lower/upper bounds (epistemic uncertainty, measurement variability).
Specialization / reduction	Recovers crisp n-SuperHyperGraphs by ignoring memberships.	Recovers neutrosophic n-SuperHyperGraphs when all intervals collapse: $L = U$.

Level 1 (fault-signatures). Let

$$s_1 = \{\mathrm{Temp}, \mathrm{Press}\}, \qquad s_2 = \{\mathrm{Vib}, \mathrm{Motor}\}, \qquad s_3 = \{\mathrm{Valve}, \mathrm{Press}\},$$

so $s_i \in \mathcal{P}(V_0)$.

Level 2 (subsystems as 2-supervertices). Define

$$v_1 = \{s_1, s_3\} \quad \text{(hydraulic/thermal loop)}, \qquad v_2 = \{s_2\} \quad \text{(drive train)},$$

and set $V = \{v_1, v_2\} \subseteq \mathcal{P}^2(V_0)$.

System-level fault modes as 2-superedges. Let

$$e_1 = \{v_1, v_2\} \quad \text{(cascading fault)}, \qquad e_2 = \{v_1\} \quad \text{(loop fault only)},$$

so $E = \{e_1, e_2\} \subseteq \mathcal{P}(V) \setminus \{\varnothing\}$.

Interval-valued neutrosophic memberships. Interpret $\mathbf{T}_V(v) = [T_V^L(v), T_V^U(v)]$ as confidence that the subsystem is implicated, $\mathbf{I}_V(v)$ as indeterminacy due to noise/missing data, and $\mathbf{F}_V(v)$ as confidence that the subsystem is not implicated. Assign

$$\mathbf{T}_V(v_1) = [0.75, 0.90], \quad \mathbf{I}_V(v_1) = [0.05, 0.15], \quad \mathbf{F}_V(v_1) = [0.00, 0.10],$$
$$\mathbf{T}_V(v_2) = [0.55, 0.75], \quad \mathbf{I}_V(v_2) = [0.10, 0.25], \quad \mathbf{F}_V(v_2) = [0.05, 0.20].$$

For incidence memberships, interpret $\mathbf{T}_E(e, v)$ as degree to which fault mode e engages subsystem v. Set the nonzero values

$$\mathbf{T}_E(e_1, v_1) = [0.70, 0.85], \quad \mathbf{I}_E(e_1, v_1) = [0.05, 0.15], \quad \mathbf{F}_E(e_1, v_1) = [0.00, 0.10],$$
$$\mathbf{T}_E(e_1, v_2) = [0.50, 0.70], \quad \mathbf{I}_E(e_1, v_2) = [0.10, 0.22], \quad \mathbf{F}_E(e_1, v_2) = [0.05, 0.20],$$
$$\mathbf{T}_E(e_2, v_1) = [0.72, 0.82], \quad \mathbf{I}_E(e_2, v_1) = [0.06, 0.14], \quad \mathbf{F}_E(e_2, v_1) = [0.00, 0.08],$$

and set $\mathbf{T}_E(e, v) = \mathbf{I}_E(e, v) = \mathbf{F}_E(e, v) = [0, 0]$ for all remaining pairs (e, v) (in particular, for $v \notin e$).

All endpoint constraints $0 \le L \le U \le 1$ are satisfied. Moreover, the containment constraints of Definition 3.1 hold componentwise, e.g.,

$$T_E^U(e_1, v_2) = 0.70 \le 0.75 = T_V^U(v_2), \qquad I_E^U(e_1, v_2) = 0.22 \le 0.25 = I_V^U(v_2).$$

Therefore,

$$\mathcal{H} = \big(V, E,\ T_V^L, T_V^U,\ I_V^L, I_V^U,\ F_V^L, F_V^U,\ T_E^L, T_E^U,\ I_E^L, I_E^U,\ F_E^L, F_E^U\big)$$

is an interval-valued neutrosophic 2-SuperHyperGraph modeling hierarchical fault diagnosis under uncertainty.

Example 4.2 (Interval-valued neutrosophic 3-SuperHyperGraph for multi-level requirements traceability). Let a safety-critical embedded system have the following engineering artifacts:

$$V_0 = \{\mathrm{R1}, \mathrm{R2}, \mathrm{R3}, \mathrm{D1}, \mathrm{D2}, \mathrm{T1}, \mathrm{T2}\},$$

where Ri are requirements, Dj are design modules, and Tk are verification tests. We model traceability and coverage across three abstraction layers: (i) artifact bundles, (ii) feature packages, and (iii) release scopes.

Level 1 (bundles). Define bundles in $\mathcal{P}(V_0)$:

$$b_1 = \{\mathrm{R1}, \mathrm{D1}, \mathrm{T1}\} \quad \text{(feature A bundle)}, \qquad b_2 = \{\mathrm{R2}, \mathrm{D2}, \mathrm{T2}\} \quad \text{(feature B bundle)},$$

$$b_3 = \{\mathrm{R3}, \mathrm{D1}\} \quad \text{(shared module bundle)}.$$

Level 2 (feature packages). Let

$$g_1 = \{b_1, b_3\} \quad \text{(package A+shared)}, \qquad g_2 = \{b_2\} \quad \text{(package B)},$$

so $g_i \in \mathcal{P}^2(V_0)$.

Level 3 (release scopes as 3-supervertices). Define

$$v_1 = \{g_1, g_2\} \quad \text{(release Rel-1)}, \qquad v_2 = \{g_1\} \quad \text{(hotfix scope)},$$

and set $V = \{v_1, v_2\} \subseteq \mathcal{P}^3(V_0)$.

Change-impact clusters as 3-superedges. Let

$$e_1 = \{v_1, v_2\} \quad \text{(common-impact cluster)}, \qquad e_2 = \{v_1\} \quad \text{(release-only impact)},$$

so $E = \{e_1, e_2\} \subseteq \mathcal{P}(V) \setminus \{\varnothing\}$.

Interval-valued neutrosophic memberships. Interpret $\mathbf{T}_V(v)$ as confidence that the scope v is sufficiently traceable/covered, $\mathbf{I}_V(v)$ as indeterminacy due to incomplete evidence, and $\mathbf{F}_V(v)$ as confidence of non-coverage. Assign

$$\begin{aligned} \mathbf{T}_V(v_1) &= [0.70, 0.88], \quad \mathbf{I}_V(v_1) = [0.08, 0.20], \quad \mathbf{F}_V(v_1) = [0.02, 0.12], \\ \mathbf{T}_V(v_2) &= [0.62, 0.80], \quad \mathbf{I}_V(v_2) = [0.10, 0.25], \quad \mathbf{F}_V(v_2) = [0.03, 0.15]. \end{aligned}$$

For incidence memberships (how strongly an impact cluster engages a scope), set

$$\begin{aligned} \mathbf{T}_E(e_1, v_1) &= [0.68, 0.84], \quad \mathbf{I}_E(e_1, v_1) = [0.10, 0.18], \quad \mathbf{F}_E(e_1, v_1) = [0.02, 0.10], \\ \mathbf{T}_E(e_1, v_2) &= [0.60, 0.78], \quad \mathbf{I}_E(e_1, v_2) = [0.12, 0.22], \quad \mathbf{F}_E(e_1, v_2) = [0.03, 0.15], \\ \mathbf{T}_E(e_2, v_1) &= [0.72, 0.82], \quad \mathbf{I}_E(e_2, v_1) = [0.09, 0.16], \quad \mathbf{F}_E(e_2, v_1) = [0.02, 0.10], \end{aligned}$$

and set $[0, 0]$ for all remaining pairs (e, v), in particular for $v \notin e$.

Then the endpoint constraints $0 \le L \le U \le 1$ hold. Support compatibility is satisfied by construction. Containment holds componentwise, e.g.,

$$T_E^U(e_1, v_2) = 0.78 \le 0.80 = T_V^U(v_2), \qquad F_E^U(e_1, v_2) = 0.15 \le 0.15 = F_V^U(v_2).$$

Hence this data define an interval-valued neutrosophic 3-SuperHyperGraph that encodes multi-level requirements traceability and uncertainty in release impact assessment.

5 Conclusion and Future Works

This paper introduced and investigated the interval-valued neutrosophic superhypergraph, a novel construct that synthesizes interval-valued neutrosophic hypergraphs with neutrosophic superhypergraphs. By assigning interval-valued truth, indeterminacy, and falsity degrees to hierarchical supervertices and superedges, the proposed model captures multilevel uncertainty in a more expressive manner than existing graph- and hypergraph-based frameworks. In particular, it provides a unified setting for representing higher-order relations together with interval-based neutrosophic information, which is useful when both hierarchical structure and uncertain assessments must be handled simultaneously.

Several directions remain open for future research. From a theoretical viewpoint, it would be meaningful to develop basic structural notions and invariants for interval-valued neutrosophic superhypergraphs, such as substructures, paths, connectivity, regularity-type notions, and matrix-based representations. It would also be natural to study corresponding operations, including union, intersection, product-type constructions, and homomorphism-like mappings, together with their algebraic and order-theoretic properties. In addition, several important extensions may be explored. In particular, incorporating HyperNeutrosophic Sets, QuadriPartitioned [40] and PentaPartitioned Neutrosophic Sets [41], Plithogenic Sets[42], and Rough[14], as well as Soft Sets[12], could further enrich the representational power and applicability of superhypergraph-based models. Such generalizations may lead to broader modeling frameworks for decision support, knowledge representation, pattern analysis, and complex network analysis under heterogeneous and uncertain information. It would also be worthwhile to investigate concrete case-oriented formulations in engineering and informatics, where multi-level interactions and interval-valued uncertainty naturally arise.

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Data Availability

This research is purely theoretical, involving no data collection or analysis. We encourage future researchers to pursue empirical investigations to further develop and validate the concepts introduced here.

Ethical Approval

As this research is entirely theoretical in nature and does not involve human participants or animal subjects, no ethical approval is required. I have not engaged in any unethical conduct.

Conflicts of Interest

The authors confirm that there are no conflicts of interest related to the research or its publication.

On the Use of AI

LLM-based software was used for language polishing (e.g., grammar and clarity checks), formatting verification, and assistance in diagnosing potential LaTeX compilation errors. It was not used to produce results or in any manner that would compromise research integrity.

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Takaaki Fujita

He was born in Japan. He is currently working as a System Engineer while also engaging in independent research in his spare time. He enjoys exploring a wide range of research topics. His research interests include Graph Theory, Combinatorics, Fuzzy Sets, Neutrosophic Sets, Hyperstructures, and SuperHyperstructures. He has developed numerous novel concepts in areas such as Fuzzy Sets, Neutrosophic Sets, Hypergraphs, and SuperHypergraphs.