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Decomposition with Mixed Model when Trend-Cycle Component is Exponential

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ABSTRACT

This paper discusses the decomposition of an observed time series data which admits the mixed model when trend-cycle component is exponential. Once the estimates of the trend-cycle and seasonal components, are obtained, estimates of the residuals or irregular component can be obtained from the observed series either by successive subtraction (for the Additive model) or by successive division (for the Multiplicative model) of the estimates of trend-cycle and seasonal components. However, for the mixed model, estimates of the residuals obtained either by successive subtraction from or by successive division of the observed series are contaminated by estimates of the estimates of trend-cycle and seasonal components. The purpose of this study is to show how estimates of residuals uncontaminated by the estimates of trend-cycle and seasonal components can be obtained. The Buys-Ballot procedure for time series decomposition was adopted in this study. The procedure is based on row, column and overall means and variances of the Buys-Ballot table. When trend-cycle component is exponential estimates of the trend-cycle component and seasonal indices are derived from these row, column and overall means of the Buys-Ballot table. Simulated series were used to illustrate results. Evaluation of the estimates of the residuals shows that the fitted models adequately describe the patterns in the simulated series. The proposed procedure has been recommended for decomposition of any observed series that admits the mixed model and exponential trend-cycle component.

Keywords*:* Mixed model; Exponential trend-cycle component; Buys-Ballot procedure; Time series decomposition; Contaminated residuals

1. Introduction

In descriptive time series analysis, the three decomposition models most commonly used are the Additive Model:

$$X_t = T_t + S_t + C_t + V_t \qquad 1$$

Multiplicative Model:

$$X_t = T_t \times S_t \times C_t \times v_t \qquad 2$$

and Mixed Model/Pseudo-Additive;

$$X_t = T_t \times S_t \times C_t + v_t \qquad 3$$

Where for time $t = 1, 2, ..., n$ X_t is the observed series, T_t is the trend (long term direction), S_t is the seasonal effect (systematic, calendar related movements), C_t is the cyclical (long term oscillations or swings about the trend) and irregular (v_t) (unsystematic, short term fluctuations) (Kendal and Ord, 1990; Chatfield, 2004). If short period of time are involved, the cyclical component is superimposed into the trend (Chatfield, 2004) and the observed time series $\left(X_t,\ t = 1, 2, ..., n\right)$ can be decomposed into the trend-cycle component (M_t), seasonal component (S_t) and the irregular/residual component (v_t) Therefore, the decomposition models are restated as

Additive Model:

$$X_t = M_t + S_t + v_t \qquad 4$$

Multiplicative Model:

$$X_t = M_t \times S_t \times v_t \qquad 5$$

and Mixed Model Pseudo-Additive/;

$$X_t = M_t \times S_t + v_t \qquad 6$$

It is always assumed that the seasonal effect when it exists has period s, that is, it repeats after s time periods.

$$S_{t+s} = S_t, \text{ for all } t \qquad 7$$

For Equation (4), it is convenient to make the further assumption that the sum of the seasonal components over a complete period is zero.

$$\sum_{j=1}^{s} S_{t+j} = 0 \qquad 8$$

Similarly, for Equations (5) and (6), the convenient variant assumption is that the sum of the seasonal components over a complete period is s.

$$\sum_{j=1}^{s} S_{t+j} = s \qquad 9$$

It is also assumed that the irregular component e_t is the Gaussian $N(0, \sigma_1^2)$ white noise for Equations (4) and (6), while for Equation (5), e_t is the Gaussian $N(1, \sigma_2^2)$ white noise. And for Equations (4) through (6) it is assumed that $Cov(e_t, e_{t+k}) = 0, \forall\ k \neq 0$.

As far as the descriptive method of decomposition is concerned, the traditional practice is to estimate and isolate the time series components available in the observed series in so far as possible. The first step will usually be to estimate and eliminate M_t for each time period from the actual data either by subtraction, for Equation (4) or division, for Equations (5). The de-trended series is obtained as $X_t - \hat{M}_t$ for Equation (4) or $X_t / \hat{M}_t$ for Equations (5) and (6). The seasonal effect is obtained by estimating the average of the de-trended series at each season. The de-trended, de-seasonalized series is obtained as $X_t - \hat{M}_t - \hat{S}_t$ for Equation (1.4) or $X_t / (\hat{M}_t\ \hat{S}_t)$ for Equation (5) and (6). This gives the residual or irregular component. Having fitted a model to a time series, one often wants to see if the residuals are purely random. For detailed discussion of residual analysis, see Box *et al.* (1994), Ljung and Box (1978).

From the foregoing it is clear that the steps of decomposition outlined above may not be applicable to the mixed model. The residual estimate can neither be derived by subtracting the estimates of the Trend-Cycle and Seasonal components from the observed series, as in the additive model or by division as in the multiplicative model. If we obtain and eliminate M_t for each time period from the actual data by subtraction for Equation (5) the de-trended series obtained is

$$X_t - \hat{M}_t = M_t S_t + e_t - \hat{M}_t. \qquad 10$$

The seasonal effect obtained by estimating the average of the de-trended series at each season may not be completely free from the trend effect. The de-trended, de-seasonalized series (the residual or irregular component) obtained as

$$\hat{e}_t = X_t - \hat{M}_t - \hat{S}_t = M_t S_t + e_t - \hat{M}_t - \hat{S}_j \qquad 11$$

may also not be free from trend and seasonal effect.

Following the division procedure, the de-trended series obtained is for Equations (5).

$$X_t / \hat{M}_t = \frac{M_t S_t}{\hat{M}_t} + \frac{e_t}{\hat{M}_t} \qquad 12$$

And the residual or irregular component (the de-trended, de-seasonalized series) is

$$\hat{e}_t = \frac{X_t}{\hat{M}_t \hat{S}_t} = \frac{M_t S_t}{\hat{M}_t \hat{S}_t} + \frac{e_t}{\hat{M}_t \hat{S}_t} \qquad 13$$

So in either case (subtraction or division), the residual is not free from trend and seasonal effect. That is, neither subtraction nor division yields the desired residual. This is perhaps, why many analysts avoid using the mixed model, even when it provides the best fit to a study data. This makes very necessary to provide a procedure for decomposition of an observed time series data (and obtain the residuals) when the appropriate model is the mixed model. Therefore, the ultimate objective of this study is to develop a procedure for decomposition of a series that admits the mixed model when trend-cycle component is Exponential. Specifically, the study (i) obtained estimates of the trend parameters and seasonal indices independently from the row, column and overall means and the variances of the observed series arranged in Buys-Ballot table, (ii) proposed the decomposition model using the estimates and (iii) illustrated the procedure using empirical examples.

2. Methodology

The estimates of trend parameters and seasonal indices are derived from the row, column and overall means and variances of the observed series arranged in Buys-Ballot table. Following the ways of Iwueze and Nwogu (2004 and 2005), the row, column and overall means and variances obtained for the mixed model when trend-cycle component is Exponential are shown in Table 1 (Row, column and overall means and variances for the Mixed model when Trend-Cycle component is Exponential).

Measure	Estimate
$\bar{X}_{i\cdot}$	$bC_3 e^{cs(i-1)} + \bar{v}_{i\cdot}$
$\bar{X}_{\cdot j}$	$\frac{b}{m}\left(\frac{1-e^{cn}}{1-e^{cs}}\right) e^{cj} S_j + \bar{v}_{\cdot j}$
$\bar{X}_{\cdot\cdot}$	$\frac{bC_3}{m}\left(\frac{1-e^{cn}}{1-e^{cs}}\right) + \bar{v}_{\cdot\cdot}$
$\sigma^2_{i\cdot}$	$\sigma^2 + \frac{s}{s-1}\left(b^2 \sigma^2_{e^{cj}S_j}\right) e^{2cs(i-1)}$
$\sigma^2_{j\cdot}$	$\sigma^2 + \frac{b^2}{m-1}\left[\left(\frac{1-e^{2cn}}{1-e^{2cs}}\right) - \frac{1}{m}\left(\frac{1-e^{cn}}{1-e^{cs}}\right)^2\right] e^{2cj} S_j^2$
$\sigma^2_{x\cdot}$	$\sigma^2 + \frac{b^2 s}{n-1}\left\{C_3^2\left[\left(\frac{1-e^{2cn}}{1-e^{2cs}}\right) - \frac{1}{m}\left(\frac{1-e^{cn}}{1-e^{cs}}\right)^2\right] + \left(\frac{1-e^{2cn}}{1-e^{2cs}}\right)\sigma^2_{e^{cj}S_j}\right\}$

Table 1: *Estimate of trend parameters and seasonal indices (Iwueze and Nwogu 2004 and 2005)*

Estimates of Trend parameters and Seasonal indices

From Table 1, the row mean is

$$\bar{X}_{i\cdot} = (bC_3)e^{cs(i-1)} + \bar{v}_{i\cdot} \cong (bC_3)e^{cs(i-1)}$$

Similarly

$$\bar{X}_{(i+1)\cdot} \cong (bC_3)e^{cs(i)} \qquad 14$$

$$\frac{\bar{X}_{(i+1)\cdot}}{\bar{X}_{i\cdot}} = \frac{(bC_3)e^{cs(i)}}{(bC_3)e^{cs(i-1)}} = e^{cs}$$

$$e^{cs} = \frac{\bar{X}_{(i+1)\cdot}}{\bar{X}_{i\cdot}} \qquad 15$$

$$cs = \text{Ln}\left(\frac{\bar{X}_{(i+1)\cdot}}{\bar{X}_{i\cdot}}\right) = \text{Ln}\left(\bar{X}_{(i+1)\cdot}\right) - \text{Ln}\left(\bar{X}_{i\cdot}\right)$$

$$\hat{c}_i = \frac{1}{s}\text{Ln}\left(\frac{\bar{X}_{(i+1)\cdot}}{\bar{X}_{i\cdot}}\right) = \frac{1}{s}\left[\text{Ln}\left(\bar{X}_{(i+1)\cdot}\right) - \text{Ln}\left(\bar{X}_{i\cdot}\right)\right],\ i = 1, 2, \ldots, m-1 \qquad 16$$

$$\hat{c} = \frac{1}{s(m-1)}\sum_{i=1}^{m-1}\left[\text{Ln}\left(\bar{X}_{(i+1)\cdot}\right) - \text{Ln}\left(\bar{X}_{i\cdot}\right)\right],$$

$$= \frac{1}{s(m-1)}\left[\text{Ln}\left(\bar{X}_{2\cdot}\right) - \text{Ln}\left(\bar{X}_{1\cdot}\right) + \text{Ln}\left(\bar{X}_{3\cdot}\right) - \text{Ln}\left(\bar{X}_{2\cdot}\right) + \ldots + \text{Ln}\left(\bar{X}_{m\cdot}\right) - \text{Ln}\left(\bar{X}_{(m-1)\cdot}\right)\right]$$

$$= \frac{1}{s(m-1)}\left[\text{Ln}\left(\bar{X}_{m\cdot}\right) - \text{Ln}\left(\bar{X}_{1\cdot}\right)\right]$$

$$= \frac{1}{n-s}\left[\text{Ln}\left(\bar{X}_{m\cdot}\right) - \text{Ln}\left(\bar{X}_{1\cdot}\right)\right] \qquad 17a$$

$$= \frac{1}{n-s}\text{Ln}\left(\frac{\bar{X}_{m\cdot}}{\bar{X}_{1\cdot}}\right) \qquad 17b$$

From the column mean

$$\bar{X}._j = \frac{b}{m}\left(\frac{1-e^{cn}}{1-e^{cs}}\right) e^{cj} S_j + \bar{v}._j \cong \frac{b}{m}\left(\frac{1-e^{cn}}{1-e^{cs}}\right) e^{cj} S_j$$

$$\bar{X}._j \cong \frac{b}{m}\left(\frac{1-e^{cn}}{1-e^{cs}}\right) e^{cj} S_j = B e^{cj} S_j$$

$$B\, S_j = \frac{\bar{X}._j}{e^{cj}} \qquad 18$$

where

$$B = \frac{b}{m}\left(\frac{1-e^{cn}}{1-e^{cs}}\right) \qquad 19$$

From (18)

$$B \sum_{j=1}^{s} S_j = \sum_{j=1}^{s} \frac{\bar{X}._j}{e^{cj}}$$

For the mixed model, the sum of the seasonal effect over a complete cycle is s (the seasonal period). That is, $\sum_{j=1}^{s} S_j = s$

Hence,

$$B\, s = \sum_{j=1}^{s} \frac{\bar{X}._j}{e^{cj}} \text{ or}$$

$$B = \frac{1}{s}\sum_{j=1}^{s} \frac{\bar{X}._j}{e^{cj}} \qquad 20$$

Substituting the B into (18) we have the expression for Sj as

$$\hat{S}_j = \frac{1}{B}\,\frac{\bar{X}._j}{e^{\hat{c}j}} = \frac{1}{\frac{1}{s}\sum_{j=1}^{s} \frac{\bar{X}._j}{e^{\hat{c}j}}}\,\frac{\bar{X}._j}{e^{\hat{c}j}}$$

$$= \frac{\bar{X}._j / e^{\hat{c}j}}{\frac{1}{s}\sum_{j=1}^{s}\left(\bar{X}._j / e^{\hat{c}j}\right)} \qquad 21$$

From (19) and (20)

$$B = \frac{b}{m}\left(\frac{1-e^{cn}}{1-e^{cs}}\right) = \frac{1}{s}\sum_{j=1}^{s} \frac{\bar{X}._j}{e^{cj}}$$

Hence,

$$\hat{b} = \frac{m}{s}\left(\frac{1-e^{cs}}{1-e^{cn}}\right)\sum_{j=1}^{s} \frac{\bar{X}._j}{e^{cj}}$$

$$= \frac{m}{s}\left(\frac{1-e^{\hat{c}s}}{1-e^{\hat{c}n}}\right)\sum_{j=1}^{s}\left(\bar{X}._j / e^{\hat{c}j}\right) \qquad 22$$

The summary of estimates of trend parameters and seasonal indices for the mixed model when trend-cycle component is exponential is given in Table 2 (Parameter estimates when trend-cycle component is exponential).

Parameter	Estimate
b	$\frac{m}{s}\left(\frac{1-e^{\hat{c}s}}{1-e^{\hat{c}n}}\right)\sum_{j=1}^{s}\left(\bar{X}._j / e^{\hat{c}j}\right)$ or $\frac{1}{(m-1)C_3}\sum_{i=1}^{m-1} \bar{X}_{(i+1)}.e^{-\hat{c}si}$
c	$\frac{1}{(n-s)}\,\text{Ln}\left(\frac{\bar{X}_m.}{\bar{X}_1.}\right)$ or $\frac{1}{(n-s)}\left[\text{Ln}\left(\bar{X}_m.\right) - \text{Ln}\left(\bar{X}_1.\right)\right]$
$\hat{S}_j$	$\frac{\bar{X}._j / e^{cj}}{\frac{1}{s}\left[\sum_{j=1}^{s}\left(\bar{X}._j / e^{cj}\right)\right]}$

Table 2: *Characterization of mixed models for time series decomposition quadratic and exponential (Nzenwa, 2024)*

$$C_3 = \frac{1}{s}\sum_{j=1}^{s}\left(e^{cj} S_j\right),\ \sigma^2_{e^{cj}S_j} = \frac{1}{s}\sum_{j=1}^{s}\left(e^{cj} S_j - C_3\right)^2$$

Having obtained the parameter estimates, the estimates of the series at time t is obtained as

$$\hat{X}_t = \hat{M}_t \hat{S}_t \quad (23)$$

$$\hat{X}_{(i-1)s+j} = \hat{M}_{(i-1)s+j} \hat{S}_{(i-1)s+j} \quad (24)$$

where

$$\hat{M}_t = \hat{b} e^{\hat{c} t} \quad (25)$$

$$\hat{M}_{(i-1)s+j} = \hat{b} e^{\hat{c}[(i-1)s+j]} \quad (26)$$

$\hat{S}_t$ is as defined in Table 2 and

$$\hat{e}_t = X_t - \hat{X}_t \quad (27)$$

$$\hat{e}_{(i-1)s+j} = X_{(i-1)s+j} - \hat{X}_{(i-1)s+j} \quad (28)$$

If the fitted model in (23) is adequate, then the $\hat{e}_t = \hat{e}_{(i-1)s+j}$ in (28) must satisfy the properties of a purely random process. The plots of the observed series and the mean, variance and the residual for the simulated series

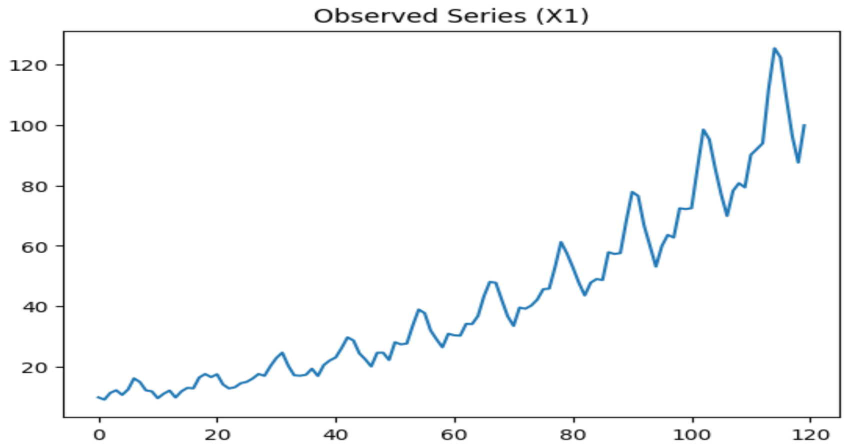

Figure 1: *The plot of the original series of contaminated decomposition*

Figure 1 above showed the observed series of contaminated decomposition which an exponential growth pattern with recurring seasonal fluctuations, confirming the presence of a mixed model structure.

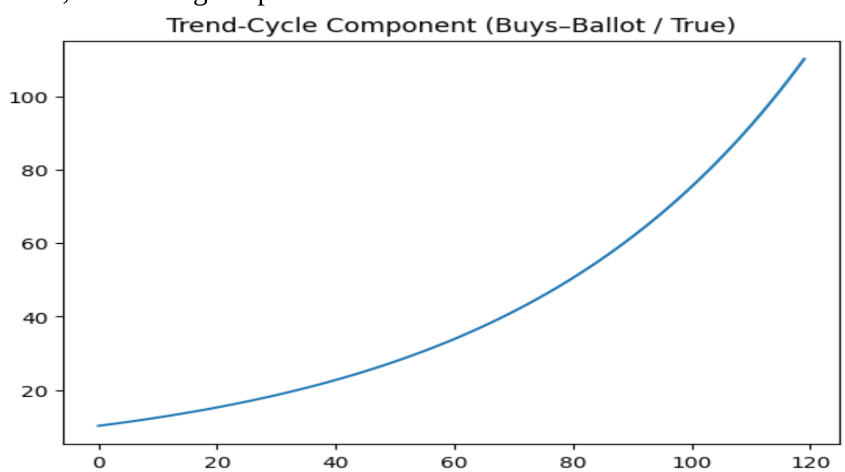

Figure 2*: The plot of the trend component of uncontaminated decomposition*

As shown in Figure 2 above the Buys–Ballot-based trend estimate closely follows the true exponential form, demonstrating accurate recovery of the trend-cycle component.

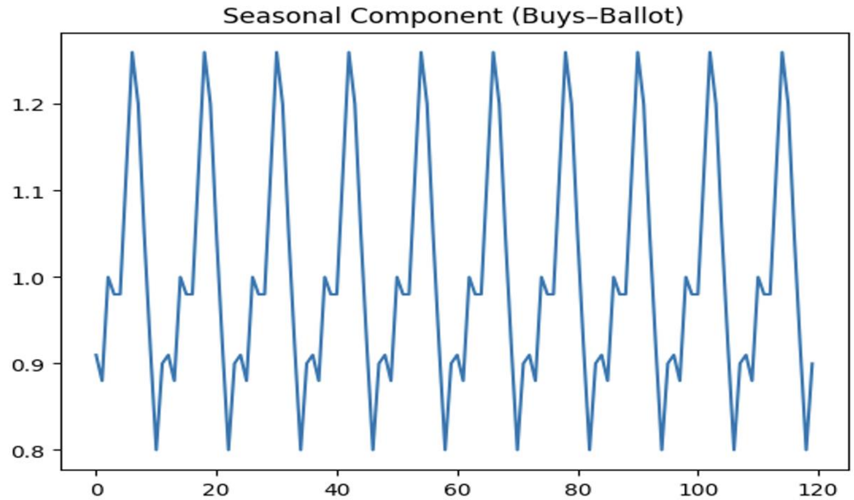

Figure 3:*The plot of the seasonal component of the uncontaminated decomposition*

As shown in figure 3 above the estimated seasonal indices exhibit stable periodic behavior, consistent with the theoretical seasonal structure imposed during simulation.

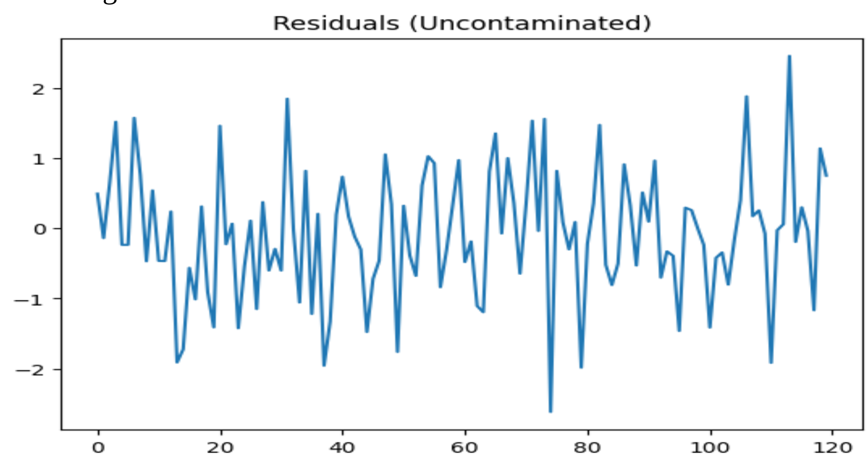

Figure 4: The plot of the residuals of the uncontaminated

As shown in Figure 4 above the residuals fluctuate randomly around zero with no discernible structure, confirming that the proposed method successfully isolates the irregular component.

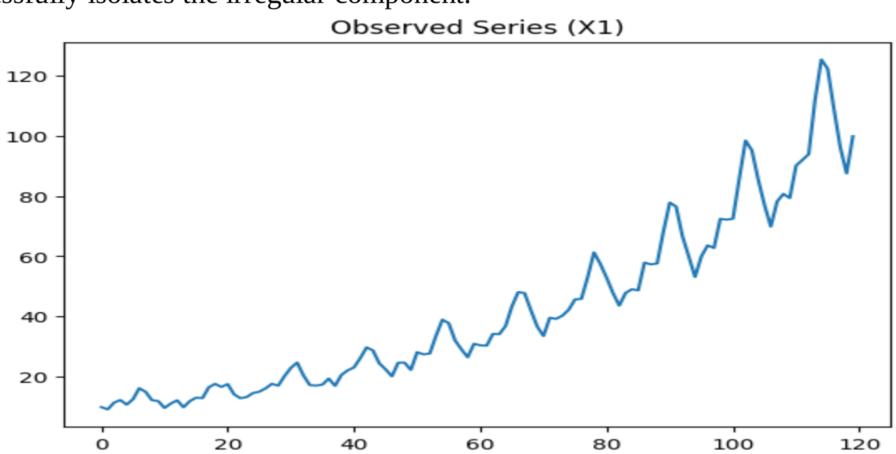

Figure 5: *The plot of the observed series for control*

As shown in figure 5 above the same simulated series is used to facilitate direct comparison between the traditional and proposed procedures.

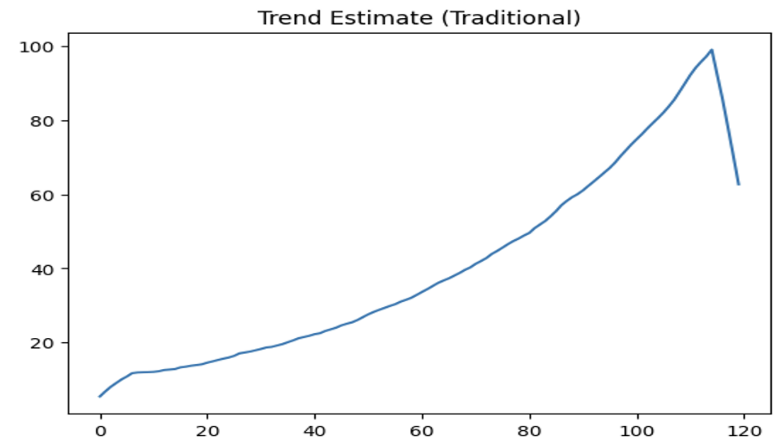

Figure 6: *The trend of the estimated trend contaminated*

As shown in figure 6 above the traditional trend estimate deviates from the true exponential pattern, particularly near the boundaries, indicating that seasonal effects are not completely removed during estimation.

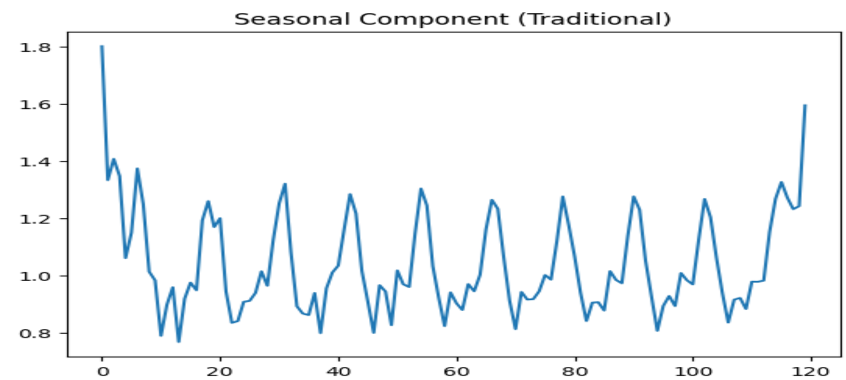

Figure 7: *The plot of the seasonal component contaminated*

As shown in figure 7 above the estimated seasonal component displays instability across cycles, suggesting contamination by trend and irregular components.

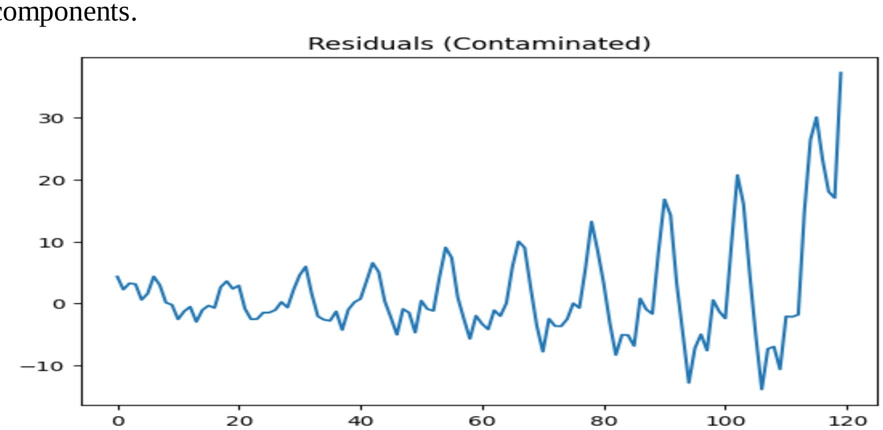

Figure 8: *The plot of the residual contaminated*

As shown in figure 8 above the residual component exhibits systematic oscillations and increasing variability, indicating that the traditional decomposition fails to isolate the irregular component under the mixed model.

Traditional (Contaminated) Decomposition

From the second set of plots:

- The trend estimate is distorted at the ends.
- The seasonal component is irregular and unstable.
- The residuals clearly retain structure (increasing amplitude and seasonal oscillations)
- This visually confirms contamination of residuals when traditional decomposition is applied to a mixed model.

Proposed (Uncontaminated) Decomposition.

From the first set of plots:

- The trend-cycle is smooth and exponential.

- The seasonal component is stable and periodic.
- The residuals fluctuate randomly around zero with constant variance.

This visually confirms that your Buys–Ballot-based method yields uncontaminated residuals.

3. Empirical Examples

The empirical examples consist of 106 data sets of 120 observations each simulated from $X_t = \left(be^{ct}\right) * S_t + e_t$, with $b = 10$, $c = 0.02$, $s = 12$, $m = 10$, $v_t \sim N(0,1)$ and S_j given in Table 3.

j	1	2	3	4	5	6	7	8	9	10	11	12
S_j	0.91	0.88	1.00	0.98	0.98	1.12	1.26	1.20	1.05	0.92	0.80	0.90

Table 3: *Seasonal (S_j) indices used in the simulation of the Mixed series*

The simulated series were obtained using the MINITAB software. The row means and standard deviations of the simulated series presented in Buys-Ballot table are shown in Appendix A while the corresponding column means and standard deviations are given Appendix B. Using these row and column means, estimates of the trend parameters and seasonal indices were obtained following the procedure outlined below.

Given m = 10, s = 12 and n = 120, n-s = 108 and m/s = 10/12 = 0.8333

1 First we obtain estimate of c from using Equation **(**17**)**

$$\frac{1}{(n-s)} \mathrm{Ln}\left(\frac{\bar{X}_{m\cdot}}{\bar{X}_{1\cdot}}\right),$$

where $\bar{X}_{m\cdot}$ is the mean of the m-th row and $\bar{X}_{1\cdot}$ is the mean of the first row.

2 Next, we obtain estimate of b from the column means using Equation (22)

$$\frac{m}{s}\left(\frac{1-e^{\hat{c}s}}{1-e^{\hat{c}n}}\right)\sum_{j=1}^{s}\left(\bar{X}_{\cdot j} / e^{\hat{c}j}\right)$$

3 The seasonal indices were computed using Equation (21)

Estimates of the trend parameters and seasonal indices obtained are shown in Table 4. while comparing them with the parameters used in the simulations. As Table 4 shows, the parameter estimates are quite close to the actual values used in the simulation both in magnitude and signs.

Having obtained the estimates of trend parameters and seasonal indices, the residuals are obtained using Equation (27) or (28) and evaluated for the properties of purely random process. The ACF, PACF and the basic statistics of the residuals from the fitted models given in Table 5 indicate that the fitted models adequately describe the patterns simulated series. More than 95 percent of the ACFs and PACFs lie within $\pm 2/\sqrt{n}$, (where n =120) while the basic statistics are, clearly, those of the purely random process.

Parameter	(Actual values)	2	3	5	6	7	9	10	11	12	13
c	0.0200	0.0200	0.0202	0.0200	0.0198	0.0198	0.0200	0.0200	0.0201	0.0199	0.0201
b	10.000	9.9952	9.8438	10.0035	10.1314	10.1817	9.9919	9.9591	9.9733	10.0584	9.9196
S1	0.9100	0.9172	0.9173	0.9095	0.9035	0.9145	0.9026	0.9048	0.9149	0.9071	0.9118
S2	0.8800	0.8665	0.8666	0.8702	0.8728	0.8818	0.8746	0.8788	0.8635	0.8861	0.8822
S3	1.0000	1.0041	1.0042	0.9985	0.9971	1.0006	1.0088	0.9986	1.0019	0.9956	0.9891
S4	0.9800	0.9763	0.9763	0.9875	0.9755	0.9803	0.9713	0.9763	0.9875	0.9762	0.9802
S5	0.9800	0.9873	0.9874	0.9848	0.9871	0.9814	0.9864	0.9805	0.9787	0.9719	0.9888
S6	1.1200	1.1286	1.1287	1.1218	1.1284	1.1190	1.1218	1.1255	1.1107	1.1308	1.1194
S7	1.2600	1.2569	1.2571	1.2546	1.2596	1.2454	1.2644	1.2544	1.2647	1.2548	1.2568
S8	1.2000	1.1928	1.1929	1.2021	1.1956	1.2049	1.2071	1.2064	1.1981	1.1977	1.2042
S9	1.0500	1.0561	1.0562	1.0451	1.0656	1.0500	1.0429	1.0463	1.0504	1.0517	1.0516
S10	0.9200	0.9171	0.9172	0.9366	0.9251	0.9296	0.9085	0.9168	0.9228	0.9266	0.9061
S11	*0.8000*	0.7937	0.7938	0.7938	0.7930	0.7981	0.8020	0.8038	0.8010	0.8038	0.8070
S12	*0.9000*	0.9033	0.9033	0.8956	0.8967	0.8944	0.9095	0.9078	0.9058	0.8977	0.9029

Table 4: *Parameter Estimates of the Mixed model when Trend-cycle component is Exponential*

Table 4 as shown above, shows the values of the estimated parameters of seasonal indices using the derived mixed model recover closely the actual vales of the seasonal indices**.**

Lag	X1		X2		X3		X4		X5	
	ACF	PACF	ACF	PACF	ACF	PACF	ACF	PACF	ACF	PACF
1	0.08	0.08	0.08	0.08	-0.03	-0.03	0.08	0.08	0.02	0.02
2	-0.04	-0.05	0.22	0.22	-0.05	-0.05	-0.09	-0.10	-0.01	-0.01
3	-0.29	-0.28	-0.06	-0.09	-0.08	-0.08	0.07	0.08	-0.12	-0.12
4	-0.03	0.02	-0.04	-0.08	0.01	0.01	0.11	0.09	0.14	0.15
5	0.03	0.01	-0.06	-0.01	0.02	0.01	-0.02	-0.02	0.14	0.14
6	0.08	0.00	0.08	0.11	-0.05	-0.06	0.01	0.02	-0.02	-0.04
7	0.02	0.01	0.09	0.09	0.06	0.06	0.12	0.10	0.03	0.07
8	-0.04	-0.03	-0.07	-0.15	-0.05	-0.05	-0.13	-0.16	0.11	0.12
9	-0.01	0.02	0.03	0.01	0.04	0.03	0.03	0.09	0.01	-0.05
10	0.03	0.03	-0.01	0.07	-0.06	-0.06	-0.09	-0.15	-0.02	-0.02
11	-0.09	-0.13	0.15	0.16	0.02	0.01	-0.10	-0.08	-0.10	-0.08
12	-0.06	-0.05	0.13	0.09	-0.05	-0.06	-0.01	0.02	-0.01	-0.06
13	0.01	0.03	0.08	-0.05	0.05	0.05	0.20	0.19	0.11	0.09
14	0.15	0.10	0.18	0.17	0.07	0.07	-0.01	-0.04	-0.03	-0.05
15	0.04	-0.01	-0.15	-0.15	-0.14	-0.13	-0.08	0.03	-0.12	-0.12
16	0.02	0.03	0.13	0.12	-0.09	-0.10	0.07	0.00	-0.06	0.00
17	-0.12	-0.05	-0.04	0.02	0.01	0.01	0.03	0.02	0.06	0.04
18	-0.12	-0.09	0.16	0.09	0.05	0.01	-0.04	-0.06	0.00	-0.05
19	-0.06	-0.05	-0.10	-0.13	-0.01	0.00	0.10	0.12	-0.09	-0.04
20	0.00	-0.07	0.14	0.10	0.00	0.00	0.09	-0.01	0.17	0.25
21	0.14	0.11	-0.07	0.00	0.22	0.23	0.00	0.07	-0.01	-0.07
22	0.02	-0.02	-0.04	-0.08	-0.06	-0.05	-0.03	-0.07	0.02	0.00
23	0.09	0.09	-0.02	-0.07	-0.06	-0.05	0.01	0.04	-0.18	-0.08
24	-0.07	-0.01	0.04	0.07	0.01	0.05	-0.09	-0.11	-0.06	-0.11
25	0.06	0.11	-0.08	-0.16	0.07	0.05	0.12	0.20	0.11	0.07
26	-0.17	-0.17	-0.05	-0.04	0.06	0.05	0.10	-0.06	-0.02	-0.07
27	0.04	0.04	0.03	0.03	-0.11	-0.11	-0.08	0.02	0.11	0.12
28	0.03	0.04	0.11	0.18	0.11	0.12	0.03	0.03	0.07	0.17
29	0.12	0.03	-0.11	-0.23	-0.06	-0.01	-0.05	-0.09	0.00	-0.02
30	0.10	0.12	0.00	-0.15	0.06	0.03	0.02	0.02	0.00	-0.02
Mean	-0.004		-0.002		0.042		0.042		-0.081	
StDev	0.916		1.015		0.857		0.934		0.957	
Skewness	-0.100		-0.200		0.120		-0.080		0.020	
Kurtosis	-0.440		-0.550		0.010		0.030		-0.480	
Min	-2.145		-2.598		-2.155		-2.300		-2.287	
Max	2.017		2.061		2.170		2.460		2.250	
Median	-0.017		0.076		-0.045		0.110		-0.080	

Table 5: *ACF AND PACF of Estimated Error terms*

Lag	X6		X7		X8		X9		X10	
	ACF	PACF	ACF	PACF	ACF	PACF	ACF	PACF	ACF	PACF
1	0.08	0.08	0.03	0.03	-0.02	-0.02	0.11	0.11	0.00	0.00
2	0.00	-0.01	-0.01	-0.01	0.24	0.24	0.08	0.07	-0.11	-0.11
3	-0.09	-0.09	-0.01	-0.01	-0.02	-0.01	0.10	0.08	0.06	0.05
4	-0.05	-0.03	0.19	0.19	-0.05	-0.11	0.17	0.15	0.02	0.01
5	-0.02	-0.02	-0.07	-0.08	0.08	0.09	0.00	-0.04	-0.02	-0.01
6	-0.06	-0.06	-0.17	-0.17	0.11	0.16	0.08	0.06	0.04	0.04
7	-0.01	-0.01	0.02	0.04	0.10	0.06	0.06	0.02	0.01	0.00
8	0.00	0.00	-0.05	-0.09	0.08	0.01	0.07	0.03	-0.11	-0.10
9	0.18	0.17	-0.15	-0.14	0.13	0.13	0.08	0.07	0.02	0.02
10	0.09	0.06	-0.22	-0.16	-0.04	-0.03	0.09	0.05	0.03	0.01
11	-0.09	-0.11	0.00	-0.03	0.20	0.14	0.18	0.16	0.04	0.05
12	-0.05	-0.01	0.07	0.07	-0.13	-0.12	-0.09	-0.17	-0.14	-0.14
13	-0.20	-0.18	-0.07	-0.04	0.05	-0.05	0.05	0.03	-0.03	-0.03
14	-0.06	-0.04	0.06	0.10	0.08	0.13	0.02	-0.02	0.00	-0.03
15	-0.09	-0.09	0.05	-0.01	-0.11	-0.14	0.02	-0.02	0.00	0.01
16	0.18	0.19	0.07	-0.02	0.12	0.01	0.05	0.09	-0.04	-0.05
17	-0.09	-0.15	-0.02	0.00	0.12	0.20	0.10	0.04	-0.11	-0.12

18	0.06	0.04	0.14	0.10	0.04	0.01	-0.08	-0.11	0.09	0.10
19	0.07	0.04	0.16	0.11	0.21	0.14	0.04	0.03	0.02	0.01
20	0.03	0.03	0.01	-0.01	-0.07	-0.11	0.10	0.07	0.00	0.01
21	0.08	0.10	-0.02	0.00	0.13	0.14	-0.13	-0.19	0.01	0.00
22	0.01	0.08	-0.01	-0.01	-0.04	-0.04	0.06	0.12	-0.09	-0.11
23	-0.05	-0.02	0.04	0.01	0.06	0.01	0.01	0.01	-0.09	-0.07
24	-0.10	-0.10	-0.11	-0.03	-0.05	-0.09	-0.07	-0.14	-0.03	-0.09
25	-0.02	-0.06	-0.03	0.03	0.14	0.06	-0.16	-0.10	0.12	0.09
26	0.07	0.03	0.13	0.17	-0.05	0.00	-0.10	-0.13	-0.19	-0.20
27	0.05	0.07	0.02	0.06	0.15	0.02	0.06	0.12	0.02	0.06
28	0.01	-0.08	-0.02	0.08	0.17	0.15	-0.03	0.00	-0.03	-0.10
29	-0.03	0.03	-0.13	-0.09	-0.04	0.01	-0.05	0.05	0.05	0.06
30	0.03	-0.03	0.00	-0.11	0.12	-0.07	0.07	0.07	-0.03	-0.05
Mean	-0.008		0.005		-0.114		-0.003		0.043	
StDev	0.885		0.754		0.932		0.906		0.869	
Skewness	-0.210		0.340		-0.080		0.600		-0.360	
Kurtosis	-0.340		-0.460		-0.150		0.350		0.000	
Min	-2.335		-1.598		-2.396		-1.875		-2.265	
Max	1.820		1.684		2.315		2.695		2.038	
Median	-0.021		-0.039		-0.105		-0.101		0.076	

Table 5: *Continued*

From Table 5 above is the result of the autocorrelation functions and partial autocorrelation function from the estimates of the means parameters of the proposed mixed model.

4. Summary, Recommendation and Conclusion

In summary, this paper discusses decomposition of an observed series in which the pattern is best described by the mixed model when the trend-cycle component is exponential. The purpose is to show how estimates of residuals uncontaminated by the estimates of trend-cycle and seasonal components can be obtained. Estimates of the residuals or irregular component uncontaminated by estimates of the trend-cycle and seasonal components are obtained by successive subtraction for the Additive model and by successive division for the Multiplicative model. However, for the mixed model, estimates of the residuals obtained either by successive subtraction from or by successive division of the observed series are contaminated by estimates of the estimates of trend-cycle and seasonal components. In most cases, analysts uncertain of what to do are constrained resort to successive subtraction or successive division and end up with inadequate model or appeal to probability model, abandoning the descriptive time series even when it provides the best fit. Therefore, the ultimate objective of the study is to provide model for fitting adequate model for an observed time series data when the appropriate decomposition model is the mixed model and trend-cycle component is exponential.

The method adopted in this study is the Buys-Ballot procedure for time series decomposition The procedure is based on row, column and overall means and variances of the Buys-Ballot table. For details of the Buys-Ballot procedure, see Iwueze and Nwogu (2004, 2005 & 2014), Iwueze and Ohakwe (2004). When trend-cycle component is exponential, estimates of the trend-cycle component and seasonal indices are derived independently from the row, column and overall means of the Buys-Ballot table. Simulated series were used to illustrate results.

The results from the simulated series indicate that the proposed procedure recovered almost precisely, the parameter values used in the simulations. Evaluation of the ACF and PACF of estimates of the residuals (estimates of the irregular component) show that they lie within the 95% interval, $\pm 2/\sqrt{n}$ (where n =120). The basic statistics of the residuals also satisfy the properties of the purely random process. These indicate that the fitted models adequately describe the patterns in the simulated series.

The visual comparison clearly demonstrates that the traditional decomposition method yields contaminated residuals when applied to a mixed model with exponential trend-cycle component. In contrast, the Buys–Ballot-based procedure effectively separates the trend, seasonal, and irregular components, producing residuals consistent with a purely random process.

Therefore, the proposed model has been recommended for decomposition of any observed time series data which admits the mixed model when trend-cycle component is exponential.

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Declaration

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Appendix (A-B)

Appendix A: Row Means and Standard Deviations of the Mixed Model

	1		2		3		4		5		6		7		8		9		10	
Row	X1i	Z1i	X2i	Z2i	X3i	Z3i	X4i	Z4i	X5i	Z5i	X6i	Z6i	X7i	Z7i	X8i	Z8i	X9i	Z9i	X10i	Z10i
1	11.08	1.98	11.39	2.26	11.21	1.99	11.56	2.16	11.44	2.21	11.60	2.31	11.69	1.88	11.19	1.98	11.35	2.06	11.36	1.65
2	14.20	2.01	14.21	2.24	14.90	2.04	14.67	2.52	14.39	2.56	14.44	2.39	13.96	2.63	14.67	2.69	14.53	1.57	14.43	2.62
3	18.67	3.03	18.33	2.88	18.84	3.25	17.91	2.65	18.27	2.82	18.34	3.36	18.81	3.08	18.51	2.57	18.26	3.42	18.41	3.15
4	23.09	3.49	23.05	3.35	23.19	3.61	23.40	3.77	23.37	3.44	23.71	3.85	23.41	3.43	23.37	3.54	23.24	3.83	23.18	3.65
5	30.16	4.62	30.06	4.45	29.69	5.01	30.09	4.40	29.81	4.61	29.72	4.74	29.80	4.24	29.94	4.32	29.91	4.67	29.49	4.37
6	37.78	5.99	37.88	5.98	37.73	6.39	37.75	5.43	38.07	5.41	37.62	5.68	37.92	5.24	38.43	6.07	38.37	5.40	37.92	5.84
7	48.22	6.94	48.17	7.72	48.15	7.45	48.24	7.44	48.06	7.58	48.07	7.11	47.91	7.52	48.18	7.26	48.10	7.83	48.03	6.92
8	61.14	9.34	61.57	9.70	61.39	9.17	60.91	9.54	61.45	9.53	61.01	9.50	61.11	9.39	61.08	9.20	61.00	9.22	61.49	9.23
9	77.93	11.94	77.74	11.44	77.81	11.60	77.54	11.61	78.05	11.97	77.60	11.55	77.92	11.62	77.68	12.73	77.74	12.41	77.17	11.99
10	98.92	14.57	99.00	14.89	99.06	14.70	98.64	15.11	99.32	14.79	98.57	15.27	98.93	14.69	99.09	14.90	98.77	15.11	98.57	15.13
$\bar{\bar{X}}$																				
STD																				

Appendix A continue

	11		12		13		14		15		16		17		18		19		20	
Row	X11i	Z11i	X12i	Z12i	X13i	Z13i	X14i	Z14i	X15i	Z15i	X16i	Z16i	X17i	Z17i	X18i	Z18i	X19i	Z19i	Z20i	Z20i
1	11.34	2.10	11.57	2.09	11.31	2.42	11.12	2.42	11.24	2.02	11.22	1.92	11.67	2.09	11.38	2.17	11.12	1.95	11.15	1.49
2	14.68	2.27	14.10	2.55	14.60	2.20	14.41	2.57	14.34	1.98	14.82	2.23	14.84	2.13	14.45	2.41	14.66	2.02	14.80	2.29
3	18.61	2.71	18.38	2.73	18.47	2.70	18.59	2.82	18.37	3.01	18.52	2.57	18.33	2.98	18.57	2.84	19.05	2.54	18.25	3.53
4	23.60	4.09	23.57	3.08	23.17	3.39	23.61	3.68	23.20	3.97	23.51	3.59	23.37	3.05	24.04	3.68	23.21	4.30	23.66	3.85
5	30.11	4.27	29.68	4.37	29.62	4.99	29.80	4.46	30.01	4.67	29.76	4.14	29.84	4.77	29.59	3.79	29.49	4.84	29.69	4.60
6	38.06	5.98	37.56	5.87	38.30	6.08	38.03	5.75	38.36	6.09	37.80	6.27	37.94	5.57	38.33	6.04	37.61	6.04	37.33	6.26
7	48.29	8.02	47.81	6.90	48.37	7.29	48.39	7.75	48.16	7.54	48.19	7.39	47.73	7.79	48.40	7.31	48.44	7.15	48.46	7.40
8	61.32	8.86	60.73	9.33	61.12	8.80	61.63	9.74	61.02	8.74	60.94	9.23	61.11	9.37	61.84	9.58	61.30	9.47	60.86	8.94
9	77.93	11.88	77.97	11.77	77.89	11.75	77.99	12.03	77.76	11.70	78.07	12.35	78.02	11.68	77.71	11.79	77.70	12.17	77.48	11.93
10	98.99	15.11	99.27	15.40	99.30	14.94	98.97	14.94	98.95	15.12	98.83	15.53	98.95	14.68	98.64	15.04	99.07	14.84	98.71	15.01
$\bar{\bar{X}}$																				
Std																				

Appendix B: The Column Means and Standard Deviation of Mixed Model Exponential

	1		2		3		4		5		6		7		8		9		10	
Col	X1j	Z1j	X2j	Z2j	X3j	Z3j	X4j	Z4j	X5j	Z5j	X6j	Z6j	X7j	Z7j	X8j	Z8j	X9j	Z9j	X10j	Z10j
1	34.46	23.13	34.56	23.81	34.29	24.09	34.26	23.94	34.33	24.49	34.01	23.76	34.50	23.78	34.53	23.67	34.00	24.07	33.97	23.79
2	34.39	23.59	33.31	23.18	33.69	23.46	34.00	23.11	33.51	23.48	33.51	23.65	33.93	23.06	33.94	23.92	33.61	23.23	33.66	23.37
3	39.32	27.86	39.38	27.27	39.21	27.22	39.47	27.05	39.23	27.69	39.05	27.20	39.27	27.61	39.00	27.32	39.55	26.87	39.02	27.20
4	39.49	27.83	39.06	26.99	39.56	27.34	39.04	27.01	39.58	27.32	38.97	27.13	39.24	27.39	39.27	26.90	38.85	26.93	38.92	27.14
5	39.64	28.43	40.30	28.14	39.46	28.24	39.81	27.39	40.27	27.28	40.22	27.87	40.07	27.30	40.05	27.67	40.25	27.85	39.88	27.47
6	46.20	32.80	47.00	32.50	46.60	32.60	46.40	32.40	46.80	32.10	46.90	32.30	46.60	32.30	46.50	32.60	46.70	32.60	46.70	32.70
7	53.60	36.80	53.40	37.20	53.30	37.10	53.90	37.30	53.40	37.30	53.40	37.20	52.90	37.00	53.70	37.40	53.70	37.40	53.10	36.80
8	52.00	36.00	51.70	35.80	52.40	35.70	51.90	35.70	52.20	36.30	51.70	35.40	52.20	35.90	52.50	36.30	52.30	36.50	52.10	36.00
9	46.40	32.10	46.70	32.30	46.70	31.80	46.20	32.10	46.30	32.60	47.00	32.00	46.40	32.10	46.50	32.30	46.10	31.80	46.10	32.40
10	41.61	28.71	41.37	29.25	41.97	28.57	41.23	28.59	42.33	29.28	41.62	28.34	41.90	28.89	41.45	28.10	40.97	28.78	41.21	28.51
11	36.44	25.32	36.53	26.15	36.72	25.48	36.79	25.76	36.60	25.59	36.39	25.25	36.69	26.16	36.69	25.27	36.90	25.40	36.86	25.13
12	41.93	29.09	42.41	29.26	42.53	29.43	41.85	29.12	42.13	29.27	41.97	29.03	41.94	29.35	42.41	29.46	42.69	28.92	42.47	29.21
$\bar{\bar{X}}$	42.12	42.14	42.2	42.07	42.22	42.07	42.15	42.21	42.13	42.00										
Std	28.85	28.89	28.81	28.7	28.95	28.69	28.77	28.83	28.81	28.72										

Appendix B continues

	11		12		13		14		15		16		17		18		19		20	
Col	X11j	Z11j	X12j	Z12j	X13j	Z13j	X14j	Z14j	X15j	Z15j	X16j	Z16j	X17j	Z17j	X18j	Z18j	X19j	Z19j	X20j	Z20j
1	34.57	23.78	34.13	23.70	34.40	23.91	34.27	23.57	34.83	23.43	34.44	23.64	33.94	23.60	34.76	23.58	34.25	23.66	33.73	24.00
2	33.29	23.72	34.01	23.49	33.96	23.48	33.96	23.16	33.68	23.76	33.85	23.06	33.59	23.48	33.93	23.18	33.84	23.43	33.96	23.38
3	39.41	27.43	38.98	26.86	38.85	27.47	38.74	27.62	39.22	27.31	39.11	27.06	39.72	27.49	39.43	27.22	39.14	26.80	39.17	26.59
4	39.63	27.31	38.99	27.31	39.28	27.53	39.05	26.71	38.65	27.08	38.83	26.73	39.13	27.74	39.26	27.33	39.54	27.19	39.29	27.39
5	40.07	27.51	39.60	27.67	40.43	27.52	40.03	27.90	40.17	27.62	40.17	27.15	40.14	26.93	39.73	28.24	40.03	27.93	39.93	27.50
6	46.40	32.20	47.00	32.80	46.70	32.50	47.30	32.70	46.70	32.40	46.30	32.60	46.50	32.10	47.20	32.20	46.70	32.00	46.30	32.40
7	53.90	37.00	53.20	37.50	53.50	37.10	53.70	37.10	53.80	37.00	53.80	37.30	53.50	36.80	53.60	37.20	53.60	37.30	53.80	36.60
8	52.10	36.30	51.80	36.20	52.30	35.60	52.40	36.20	51.90	36.00	52.30	37.00	51.50	36.10	52.10	36.10	52.20	36.30	51.60	36.20
9	46.60	32.40	46.40	32.10	46.60	32.80	46.50	32.20	46.50	32.50	46.40	32.20	46.40	32.40	46.60	32.30	46.70	32.30	46.80	32.50
10	41.77	28.61	41.70	28.69	40.97	29.14	41.85	29.31	41.11	28.99	41.84	29.08	41.83	29.10	41.58	28.22	41.34	29.34	41.66	29.00
11	36.99	25.23	36.90	26.09	37.23	25.69	36.90	26.49	36.74	25.90	36.87	25.89	37.23	25.31	36.79	25.60	36.44	26.31	36.46	25.28
12	42.68	29.23	42.04	29.43	42.50	29.31	42.26	29.01	42.46	29.21	42.08	28.90	42.62	29.33	42.56	28.93	42.24	28.64	41.73	28.91
$\bar{\bar{X}}$	42.29	42.06	42.22	42.25	42.14	42.17	42.18	42.29	42.17	42.04										
Std	28.81	28.89	28.89	28.92	28.83	28.83	28.76	28.75	28.85	28.73										