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Joint Parameter Estimation in Measurement Errors and Non-Response for Sensitive variables

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ABSTRACT

In this paper, joint parameter estimation for mean and variance was considered in the presence of measurement errors and non-response for the Sensitive variable using auxiliary information. The properties of the suggested estimator(s) have been studied. Expressions for bias and mean square error up to first order of approximation are derived, and the theoretical characteristics of the suggested estimators are scrutinised. A numerical study is carried out to observe the performance of the proposed estimators. The scope of this work is to create better estimators that can effectively manage both kinds of mistakes at the same time, especially in randomised response settings where biasing sensitive data is common. Applications to actual data and simulation studies are used to evaluate the estimators' effectiveness. When both measurement error and non-response are present, the results show that the suggested estimators outperform the traditional estimators.

Keywords: Sensitive Variable; RRT; Measurement Error; Non-response; Joint Parameters

1. Introduction

It is very decisive to collect the evidence and the statistics in survey research. There are a few possible concerns faced by researchers during research. Above all, the participants didn't answer the query due to multifaceted problems, such as investigating opinion poll was not accurately deliberated, even the investigation was not correctly done, the statements were too personal to respond, and finally, training was not imparted to the team involved in the survey.

For the last many decades, interest has developed among statisticians to estimate the parameters of interest in the existence of measurement errors (ME). In sampling surveys, estimators' properties established on statistics frequently presume that the interpretations are the approved measurements of the studied distinctiveness (Singh & Sharma, 2015), though this postulation is not satisfactory in various usages and facts are not pure in the presence of measurement errors, like reporting and calculating errors. The result becomes invalid in the presence of ME that has been intended with no ME. In the case of small ME, it can be ignored; in the case of experiential statistics, that is supported by the statistical inferences stayed as suitable. Nowadays, online surveys are widely used with the aid of e-mail and social network sites. Virtual surveys are cost-effective, but the probability of NR is also elevated (Tiwari et al., 2022)

To improve the estimation of population parameters for sensitive variables, particularly in the presence of non-response and measurement error, recent advances in survey sampling have been made. In order to create logarithmic and exponential-type estimators, Choudhary (2023) proposed applying the Optional Randomised Response Technique (ORRT) under stratified sequential sampling. When applied to sensitive variable estimation, ORRT models considerably reduce bias and mean square error (MSE), according to the paper Using ORRT Models for Mean Estimation under Non-response and Measurement Errors in Stratified Successive Sampling. Azeem et al. (2024) addressed the necessity for an accurate variance estimate in An Efficient Estimator of Population Variance of a Sensitive Variable with a New Randomised Response Technique by creating a novel scrambling-based randomised response technique. Their approach is more accurate than existing models and uses auxiliary data to improve the efficiency of variance estimation. Efficient Estimation of Population Variance of a Sensitive Variable employing a New Scrambling Response Model by Saleem et al. (2023) employed a similar scrambling-response methodology to create a generalised estimator employing two auxiliary variables. In the presence of measurement errors and partial non-responses, their results showed a considerable reduction in MSE, offering a reliable method for handling sensitive variables. These recent innovations collectively reinforce the methodological foundations for more accurate and efficient estimation in sensitive survey data processing.

The researchers sometimes wrongly calculated that they gathered data without any error during the investigation; nevertheless, this is not persistently perfect. When assumption about phenomena is debased, succeeding estimators may malfunction. The sampling error has frequently occurred. In addition to this, another occurring error is the measurement errors. Such error types during computing frequently happened during the collection and recording of data, against the data research phase. These errors upset the accuracy of the participants' information. Generally, such errors happened during the questionnaire use, chosen method of information gathering, lack of training for the team about the conduct of the investigation, and the proposed or unplanned mistakes of the participant. There are various ways to minimise such errors, it can removing them during pre-testing of tools, and an appropriate instructional guidebook for the survey group and by twice entering the statistics with caution by the comparison of excels sheets.

A model was introduced by Hansen and Hurwitz (1946) that was used to minimise the NR and Cochran (1977) calculated the ratio and regression estimations. It's modified by Khare and Srivastava (1997), Okafor and Lee (2000) in the ratio and regression estimator under NR. Some other noticeable investigations on the estimator under NR are done by Singh and Kumar (2008), Kreuter et al. (2010); Unal and Kadilar (2019), Bii et al. (2020).

Meijer & Wansbeek (2000); Bound, Brown & Mathiowetz (2001); Hausman, (2001); Srivastava & Shalabh, (2001); Manisha & Singh, (2002); Singh & Karpe, (2009); Kumar et al., (2011); Shukla et al., (2012) all used different experimental facts to measure ME. In survey information, there are many vital sources of ME, as given by Shalabh (1997), Sud and Srivastava (2000). The assets of many population parameter estimations with ME were studied by Kumar, Singh, and Smarandache (2011), and Sharma and Singh (2013), Cochran (1968); Fuller (1995); Biemer et al., (2011); Shukla et al. (2012) and Zahid & Shabbir (2019). Ahmad & Yaseen *(2024) proposed the generalised log-type class of estimators for estimating population means in stratified sampling under non-response and measurement error. Zahid et al. (2022) suggested a generalised class of estimators for sensitive variables in the presence of measurement error and non-response. Although their focus was on estimating the population mean, this work builds on their methodology by jointly estimating the mean and variance for sensitive variables under similar conditions.* However, the estimation of variance and other population parameters is ignored by their estimator. The current study in this paper extends their methodology by simultaneously estimating the variance and mean of the sensitive variable under the same measurement error and non-response framework. The proposed estimators preserve the efficiency gains for mean estimation while offering additional variance estimation expressions and performance measures. The recommended estimators also outperform the mean estimator of Zahid et al. in terms of higher percent relative efficiency (PRE) and lower MSE, particularly as non-response levels rise, according to comparison studies based on real and simulated datasets. Therefore, our joint estimating method provides a more comprehensive inference strategy where both central tendency and variability are significant in sensitive survey data.

Con-temporarily, few of the researchers investigated the measurement error issue with non-response at the same time, such as Kumar et al. (2015); Singh and Sharma (2015); Azeem and Hanif (2017), Zahid & Shabbir (2018), Khalil et al. (2018), taking into consideration the issue of ME and non-response under stratified sampling techniques.

Through all the literature reviewed above, it is seen that a lot of work has been done on mean estimation with measurement errors and little with no response. There is no single work in the literature that deals with estimating sensitive variables for general parameters such as mean and variance, combining estimation in the presence of ME and NR. The main objective of this paper is to investigate the combined mean and variance parameter estimators in the presence of both NR and ME for sensitive variables.

2. Methodology

Let O = {O1, O2,..., ON} be a finite population of size N. Suppose that a sample of size n is drawn from O by using simple random sampling without replacement.

We assume that a population of size N consists of two mutually exclusive groups: N1(respondents) and N2 (non-respondents).

After selecting the sample, we assume that n1 units respond and n2 units do not respond.

We select a sub-sample of size k, (k= n2/g); g > 1 from the n2 non-responding units.

Let Y be the sensitive study variable, which is not observed directly, and X be a non-sensitive auxiliary variable which has positive correlation with Y. Let S be a scrambling variable which is independent of Y and X.

The respondent is asked to give a scrambled response for the study variable Y, given by Z = Y + S, and is asked to provide a true response for X. Let (xi, zi) be observed values instead of the true values (X_i, Z_i) for the i^{th} (i = 1.2 …, n) unit.

2.1 Notations:

The notations of the variables Z and X in the form of deviations are given as:

$$U_i = Z_i - \overline{Z} \qquad ; \qquad V_i = X_i - \overline{X},$$

$$\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i \qquad ; \qquad \overline{Z} = \frac{1}{n}\sum_{i=1}^{n} Z_i,$$

Be unbiased estimators of the population mean $\overline{X}$ and $\overline{Z}$ respectively but $S_X{}^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2$& $S_Z{}^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(Z_i - \overline{Z}\right)^2$ are not unbiased estimators of $(\sigma_X{}^2, \sigma_Z{}^2)$ respectively, the expected values of $S_X{}^2$ and $S_Z{}^2$ in the presence of M.E. are:

$$E(S_X{}^2) = \sigma_X{}^2 + \sigma_V{}^2 \qquad ; \qquad E(S_Z{}^2) = \sigma_Z{}^2 + \sigma_U{}^2 \,,$$

And for non-response:

$$E(S_{X'}{}^2) = \sigma_{X'}{}^2 + \sigma_{V'}{}^2; \qquad E(S_{Z'}{}^2) = \sigma_{Z'}{}^2 + \sigma_{U'}{}^2 \,,$$

When $\sigma_U{}^2$ is known, then $\quad \hat{\sigma}_X{}^2 = S_X{}^2 - \sigma_V{}^2 > 0 \,,$

$\sigma_U{}^2$ is known, then $\quad \hat{\sigma}_Z{}^2 = S_Z{}^2 - \sigma_U{}^2 > 0 \,,$

For non-response:

When $\sigma_{U'}{}^2$ is known, then the unbiased estimator of $\sigma_{X'}{}^2$ is:

$$\hat{\sigma}_{X'}{}^2 = S_{X'}{}^2 - \sigma_{V'}{}^2 > 0 \,,$$

When $\sigma_{U'}{}^2$known, then the unbiased estimator of is $\sigma_{Z'}{}^2$:

$$\hat{\sigma}_{Z'}{}^2 = S_{Z'}{}^2 - \sigma_{U'}{}^2 > 0 \,,$$

2.2. Let the mean and variances of the study variable and auxiliary variables be given as:

$$\bar{z} = \bar{Z}(1 + \epsilon_\circ) \qquad ; \qquad S_z{}^2 = S_Z{}^2(1 + \epsilon_1)$$

$$\bar{x} = \bar{X}(1 + \epsilon_2) \qquad ; \qquad S_x{}^2 = S_X{}^2(1 + \epsilon_3)$$

$$E(\epsilon_\circ) = E(\epsilon_1) = E(\epsilon_2) = E(\epsilon_3) = 0 \,,$$

$$E(\epsilon_\circ{}^2) = \frac{C_Z{}^2}{n}\left[1 + \frac{S_U{}^2}{S_Z{}^2}\right] + \frac{w_2(k-1)}{n} C_{Z'}\left[1 + \frac{S_{U'}{}^2}{S_{Z'}{}^2}\right],$$

$$E(\epsilon_1{}^2) = \frac{\delta_{40}-1}{n} + \frac{w_2(k-1)}{n}(\delta_{40} - 1) \,,$$

$$E(\epsilon_2{}^2) = \frac{C_X{}^2}{n}\left[1 + \frac{S_V{}^2}{S_X{}^2}\right] + \frac{w_2(k-1)}{n} C_{X'}\left[1 + \frac{S_{V'}{}^2}{S_{X'}{}^2}\right],$$

$$E(\epsilon_3{}^2) = \frac{\delta_{40}-1}{n} + \frac{w_2(k-1)}{n}(\delta_{40} - 1) \,,$$

$$E(\epsilon_\circ \epsilon_1) = \frac{C_Z}{n}(\delta_{30}) + \frac{C_{Z'}}{n}\delta_{30} w_2(k-1) \,,$$

$$E(\epsilon_\circ \epsilon_2) = \rho_{ZX}\frac{C_Z C_X}{n} + \frac{w_2(k-1)}{n}\rho_{Z'X'} C_{Z'} C_{X'} \,,$$

$$E(\in_\circ \in_3) = \frac{C_Z}{n}(\delta_{12}) + \frac{w_2(k-1)}{n}\delta_{12}C_{Z'}\ ,$$

$$E(\in_1 \in_2) = \frac{C_X}{n}(\delta_{21}) + \frac{w_2(k-1)}{n}\delta_{21}C_{X'}\ ,$$

$$E(\in_1 \in_3) = \frac{\delta_{22}-1}{n} + \frac{w_2(k-1)}{n}(\delta_{22}-1)\ ,$$

$$E(\in_2 \in_3) = \frac{\delta_{03}}{n}C_X + \frac{w_2(k-1)}{n}\delta_{03}C_{X'}\ .$$

Conventional Estimator:

The general form of the parameter for a sensitive variable can be considered as

$$\hat{t}_{(a,b)} = \overline{Z}^{a}(\sigma_Z^2 - \sigma_S^2)^{b/2} \qquad (1)$$

Expressing eq (1) in terms of $\in$'s

$$= \overline{Z}^{a}\sigma_Y^{b}(1+\in_\circ)^a(1+R_{ZY}\in_1)^{b/2} \qquad (2)$$

$$= t_{(a,b)}(1+\in_\circ)^a(1+R_{ZY}\in_1)^{b/2}$$

$$\hat{t}_{(a,b)} - t_{(a,b)} = t_{(a,b)}\left[a\in_\circ + \frac{a(a-1)}{2}\in_\circ^2 + \frac{b}{2}\in_1 R_{ZY} + \frac{b(b-2)}{8}R_{ZY}^2\in_1^2 + \frac{ab}{2}\in_\circ\in_1 R_{ZY}\right] \qquad (3)$$

Taking the squaring and expectation of eq. (3) on both sides and neglecting the terms of $\in$ having higher order powers, i.e. three or more powers, we have

$$\text{MSE}\left(\hat{t}_{(a,b)}\right) = t_{(a,b)}^2\left[a^2\cdot E(\in_\circ)^2 + abR_{ZY}E(\in_\circ\in_1) + \frac{b^2}{4}R_{ZY}^2E(\in_1)^2\right] \qquad (4)$$

$$= t_{(a,b)}^2\left[a^2\left\{\frac{C_Z^2}{n}\left(1+\frac{S_U^2}{S_Z^2}\right) + w_2\frac{k-1}{n}C_{\acute{Z}}^2\left(1+\frac{S_{U'}^2}{S_{Z'}^2}\right)\right\} + abR_{ZY}\left\{\delta_{30}\frac{C_Z}{n} + \frac{w_2(k-1)}{n}\delta_{30}C_{Z'}\right\}\right.$$

$$\left. + \frac{b^2}{4}R_{ZY}^2\left\{\frac{(\delta_{40}-1)}{n} + \frac{w_2(k-1)}{n}(\delta_{40}-1)\right\}\right] \qquad (5)$$

$$= \frac{t_{(a,b)}^2}{n}\left[\left\{a^2C_Z^2\left(1+\frac{S_U^2}{S_Z^2}\right) + abR_{ZY}\delta_{30}C_{Z'} + \frac{b^2}{4}R_{ZY}^2 + abR_{ZY}(\delta_{40}-1)\right\} + A\left\{a^2C_{Z'}^2\left(1+\frac{S_{U'}^2}{S_{Z'}^2}\right) + \right.\right.$$

$$\left.\left. abR_{ZY}\delta_{30}C_{Z'} + \frac{b^2}{4}R_{ZY}^2(\delta_{40}-1)\right\}\right] \qquad (6)$$

In case the measurement error is zero

$$\text{MSE}\left(\hat{t}'_{(a,b)}\right) = \frac{t^2_{(a,b)}}{n}\left[f_1(a,b) + Af'_1(a,b)\right] \qquad (7)$$

3. Proposed Estimators

We propose joint parameter estimation on the mean and variance for ratio and exponential ratio in the presence of ME and NR for the sensitive study variable.

3.1. Properties of Estimators

In this, we estimate the expressions of mean square error (MSE) of the proposed estimators in the presence of ME and NR for the sensitive study variable.

Ratio Estimator: The ratio estimator for general parameter estimation is given as:

$$t_r = \overline{Z}^a \left(\sigma_Z^2 - \sigma_S^2\right)^{b/2} \frac{\overline{X}}{\overline{x}} \qquad (8)$$

$$t_r - t_{(a,b)} = t_{(a,b)}\left[a \in_o + \frac{a(a-1)}{2} \in_o^2 + \frac{b}{2} \in_1 R_{ZY} + \frac{b(b-2)}{8} R_{YZ}^2 \in_1^2 + \frac{ab}{2} \in_o \in_1 R_{ZY} - \in_o \in_2 - \frac{b}{2} \in_1 \in_2 R_{ZY} + \in_2^2\right] \qquad (9)$$

Taking square and expectation and neglecting higher order powers i.e. three or more

$$\text{MSE}(t'_r) = t^2_{(a,b)}\left[a^2 \cdot E(\in_\circ)^2 + \frac{b^2}{4} E(\in_1)^2 R_{ZY}^2 + E(\in_2)^2 + abE(\in_\circ \in_1) R_{ZY} - 2aE(\in_\circ \in_1) - bE(\in_1 \in_2) R_{ZY}\right] \qquad (10)$$

$$= t^2_{(a,b)}\left[a^2\left\{\frac{C_Z^2}{n}\left(1 + \frac{S_U^2}{S_Z^2}\right) + \frac{w_2(k-1)}{n} C_{Z'}^2\left(1 + \frac{S_{U'}^2}{S_{Z'}^2}\right)\right\} + \frac{b^2}{4} R_{ZY}^2\left\{\frac{(\delta_{40}-1)}{n} + \frac{w_2(k-1)}{n}(\delta_{40} - 1)\right\} + \left\{\frac{C_X^2}{n}\left(1 + \frac{S_V^2}{S_X^2}\right) + \frac{w_2(k-1)}{n} C_{X'}^2\left(1 + \frac{S_{V'}^2}{S_{X'}^2}\right)\right\} + abR_{ZY}\left\{\delta_{30}\frac{C_Z}{n} + \frac{w_2(k-1)}{n}\delta_{30} C_{Z'}\right\} - 2a\left\{\rho_{ZX}\frac{C_Z}{n} C_X + \frac{w_2(k-1)}{n}\rho_{Z'X'} C_{Z'} C_{X'}\right\} - bR_{ZY}\left\{\delta_{21}\frac{C_X}{n} + \frac{w_2(k-1)}{n}\delta_{21} C_{X'}\right\}\right] \qquad (11)$$

$$= \frac{t^2_{(a,b)}}{n}\left[\left\{a^2 C_Z^2\left(1 + \frac{S_U^2}{S_Z^2}\right) + \frac{b^2}{4} R_{ZY}^2(\delta_{40} - 1) + C_X^2\left(1 + \frac{S_V^2}{S_X^2}\right) + abR_{ZY}\delta_{30} C_{Z'} - 2a\rho_{ZX} C_Z C_X - bR_{ZY}\delta_{21} C_X\right\} + w_2(k-1)\left\{a^2 C_{Z'}^2\left(1 + \frac{S_{U'}^2}{S_{Z'}^2}\right) + \frac{b^2}{4} R_{ZY}^2(\delta_{40} - 1) + C_{X'}^2\left(1 + \frac{S_{V'}^2}{S_{X'}^2}\right) + abR_{ZY}\delta_{30} C'_Z - 2a\rho_{Z'X'} C_{Z'} C_{X'} - bR_{ZY}\delta_{21} C_{X'}\right\}\right] \qquad (12)$$

Equation (12) can be written as

$$= \frac{t_{(a,b)}^2}{n}\left[\left\{a^2 C_Z^2\left(1+\frac{S_U^2}{S_Z^2}\right)+\frac{b^2}{4}R_{ZY}^2(\delta_{40}-1)+C_X^2\left(1+\frac{S_V^2}{S_X^2}\right)+abR_{ZY}\delta_{30}C_Z-2a\rho_{ZX}C_Z C_X-\right.\right.$$

$$\left.bR_{ZY}\delta_{21}C_X\right\}+A\left\{w_2(k-1)\left\{a^2 C_{Z'}^2\left(1+\frac{S_{U'}^2}{S_{Z'}^2}\right)+\frac{b^2}{4}R_{ZY}^2(\delta_{40}-1)+C_{X'}^2\left(1+\frac{S_{V'}^2}{S_{X'}^2}\right)+\right.\right.$$

$$\left.\left.\left.abR_{ZY}\delta_{30}C_Z'-2a\rho_{Z'X'}C_{Z'}C_{X'}-bR_{ZY}\delta_{21}C_{X'}\right\}\right\}\right] \quad (13)$$

In case measurement error is zero

$$= \frac{t_{(a,b)}^2}{n}\left[a^2 C_Z^2+\frac{b^2}{4}R_{ZY}^2(\delta_{40}-1)+abR_{ZY}\delta_{30}C_Z+C_X^2-2a\rho_{ZX}C_Z C_X-bR_{ZY}\delta_{21}C_X+A\left\{a^2 C_{Z'}^2+\right.\right.$$

$$\left.\left.\frac{b^2}{4}R_{ZY}^2(\delta_{40}-1)+C_{X'}^2+abR_{ZY}\delta_{30}C_{Z'}-2a\rho_{Z'X'}C_{Z'}C_{X'}-bR_{ZY}\delta_{21}C_{X'}\right\}\right] \quad (14)$$

$$= \frac{t_{(a,b)}^2}{n}[f_1(a,b)+C_X\{C_X-(2a\rho_{ZX}C_Z+b\delta_{21}R_{ZY})\}$$

$$+A\{f_1'(a,b)+\{C_{X'}-(2a\rho_{Z'X'}C_{Z'}+b\delta_{21}R_{ZY})\}\}]$$

$$= \frac{t_{(a,b)}^2}{n}\left[f_1(a,b)+C_X\left\{C_X-\left(f_3(a,b)\right)\right\}+A\left\{f_1'(a,b)+C_X\{C_X-f_3'(a,b)\}\right\}\right] \quad (15)$$

Where; $f_3(a,b)=2a\rho_{ZX}C_X+b\delta_{21}R_{ZY}$

Exponential Ratio: The exponential ratio estimator for parameter estimation in the presence of measurement error and non-response for sensitive study variables is given as:

$$\hat{t}_{cr}=\overline{Z}^{a}(\sigma_Z^2-\sigma_S^2)^{b/2}\, exp\left[\frac{\overline{X}-\overline{x}}{\overline{X}+\overline{x}}\right] \quad (16)$$

$$= t_{(a,b)}\left[1+a\in_\circ+\frac{a(a-1)}{2}\in_\circ^2+\frac{b}{2}\in_1 R_{ZY}+\frac{b(b-2)}{8}R_{ZY}^2\in_1^2+\frac{ab}{2}\in_\circ\in_1 R_{ZY}\right]exp\left[\frac{\overline{X}-\overline{x}}{\overline{X}+\overline{x}}\right] \quad (17)$$

$$\hat{t}_{cr}-t_{(a,b)}=t_{(a,b)}\left[a\in_\circ+\frac{a(a-1)}{2}\in_\circ^2+\frac{b}{2}\in_1 R_{ZY}+\frac{b(b-2)}{8}R_{ZY}^2\in_1^2+\frac{ab}{2}\in_\circ\in_1 R_{ZY}=-\frac{\in_2}{2}-\frac{a}{2}\in_\circ\in_2-\frac{b}{4}\in_1\in_2 R_{ZY}+3\frac{\in_2^2}{8}\right] \quad (18)$$

Taking square and expectation and neglecting higher order powers, i.e. three or more

$$\text{MSE}(\hat{t}_{cr}')=t_{(a,b)}^2\left[a\cdot E(\in_\circ)^2+\frac{b^2}{4}R_{ZY}^2E(\in_1)^2+\frac{E(\in_1)^2}{4}+abR_{ZY}E(\in_\circ\in_1)-aE(\in_\circ\in_2)-\frac{b}{2}R_{ZY}E(\in_1\in_2)\right] \quad (19)$$

$$= t_{(a,b)}^2\left[a^2\left\{\frac{C_Z^2}{n}\left(1+\frac{S_U^2}{S_Z^2}\right)+w_2\frac{k-1}{n}C_{Z'}^2\left(1+\frac{S_{U'}^2}{S_{Z'}^2}\right)\right\}+\frac{b^2}{4}R_{ZY}^2\left\{\frac{(\delta_{40}-1)}{n}+\frac{w_2(k-1)}{n}(\delta_{40}-1)\right\}+\right.$$

$$\frac{1}{4}\left\{\frac{C_X^2}{n}\left(1+\frac{S_V^2}{S_X^2}\right)+\frac{w_2(k-1)}{n}C_{X'}^2\left(1+\frac{S_{V'}^2}{S_{X'}^2}\right)\right\}+abR_{ZY}\left\{\delta_{30}\frac{C_Z}{n}+\frac{w_2(k-1)}{n}\delta_{30}C_{Z'}\right\}-a\left\{\frac{\rho_{ZX}C_Z C_X}{n}+\right.$$

$$\left.\left.\frac{w_2(k-1)}{n}\rho_{Z'X'}C_{Z'}C_{X'}\right\}-\frac{b}{2}R_{ZY}\left\{\delta_{21}\frac{C_X}{n}+\frac{w_2(k-1)}{n}\delta_{21}C_{X'}\right\}\right] \quad (20)$$

Equation (20) can be written as

$$= \frac{t_{(a,b)}^2}{n}\left[\left\{a^2 C_Z^2\left(1+\frac{S_U^2}{S_Z^2}\right)+\frac{b^2}{4}R_{ZY}^2(\delta_{40}-1)+\frac{C_X^2}{4}\left(1+\frac{S_V^2}{S_X^2}\right)+abR_{ZY}\delta_{30}C_Z\right.\right.$$

$$\left.-a\rho_{ZX}C_Z C_X-\frac{b}{2}R_{ZY}\delta_{21}C_X\right\}+w_2(k-1)\left\{a^2C_{Z'}^2\left(1+\frac{S_{U'}^2}{S_{Z'}^2}\right)+\frac{b^2}{4}R_{ZY}^2(\delta_{40}-1)+\frac{C_{X'}^2}{4}\left(1+\frac{S_{V'}^2}{S_{X'}^2}\right)\right.$$

$$\left.\left.+abR_{ZY}\delta_{30}C_{Z'}-a\rho_{Z'X'}C_{Z'}C_{X'}-\frac{b}{2}R_{ZY}\delta_{21}C_{X'}\right\}\right] \quad (21)$$

$$= \frac{t_{(a,b)}^2}{n}\left[\left\{a^2 C_Z^2\left(1+\frac{S_U^2}{S_Z^2}\right)+\frac{b^2}{4}R_{ZY}^2(\delta_{40}-1)+ab\delta_{30}R_{ZY}C_Z+\frac{C_X^2}{4}\left(1+\frac{S_V^2}{S_X^2}\right)\right.\right.$$

$$\left.-a\rho_{ZX}C_Z C_X-\frac{b}{2}R_{ZY}\delta_{21}C_X\right\}+w_2(k-1)\left\{a^2C_{Z'}^2\left(1+\frac{S_{U'}^2}{S_{Z'}^2}\right)+\frac{b^2}{4}R_{ZY}^2(\delta_{40}-1)+abR_{ZY}\delta_{30}C_{Z'}\right.$$

$$\left.\left.+\frac{C_{X'}^2}{4}\left(1+\frac{S_{V'}^2}{S_{X'}^2}\right)-a\rho_{Z'X'}C_{Z'}C_{X'}-\frac{b}{2}R_{ZY}\delta_{21}C_{X'}\right\}\right] \quad (22)$$

In case the measurement error is zero

$$= \frac{t_{(a,b)}^2}{n}\left[\left\{a^2C_Z^2+\frac{b^2}{4}R_{ZY}^2(\delta_{40}-1)+ab\delta_{30}R_{ZY}C_Z+\frac{C_X^2}{4}-a\rho_{ZX}C_ZC_X-\frac{b}{2}R_{ZY}\delta_{21}C_X\right\}+A\left\{a^2C_{Z'}^2+\right.\right.$$

$$\left.\left.\frac{b^2}{4}R_{ZY}^2(\delta_{40}-1)+ab\delta_{30}R_{ZY}C_{Z'}+\frac{C_{X'}^2}{4}-a\rho_{Z'X'}C_{Z'}C_{X'}-\frac{b}{2}R_{ZY}\delta_{21}C_{X'}\right\}\right] \quad (23)$$

$$= \frac{t_{(a,b)}^2}{n}\left[f_1(a,b)+\frac{C_X}{2}\left\{\frac{C_X}{2}-(2a\rho_{ZX}C_Z+b\delta_{21}R_{ZY})\right\}\right.$$

$$\left.+A\left\{f_1'(a,b)+\frac{C_{X'}}{2}\left\{\frac{C_{X'}}{2}-(2a\rho_{Z'X'}C_{Z'}+b\delta_{21}R_{ZY})\right\}\right\}\right]$$

$$= \frac{t_{(a,b)}^2}{n}\left[f_1(a,b)+\frac{C_X}{2}\left(\frac{C_X}{2}-f_3(a,b)\right)+A\left\{f_1'(a,b)+\frac{C_{X'}}{2}\left(\frac{C_{X'}}{2}-f_3'(a,b)\right)\right\}\right]. \quad (24)$$

Table 3.1:
Different types of proposed Ratio Estimators

a	b	R	Estimator	Mean Square Error
1	0	1	$\overline{Z}\cdot\left(\frac{\overline{X}}{\overline{\overline{x}}}\right)$	$\frac{t_{(a.b)}^2}{n}\left[\{C_Z^2+C_X^2-2\rho_{ZX}C_ZC_X\}+A\{C_{Z'}^2+C_{X'}^2-2\rho_{Z'X'}C_{Z'}C_{X'}\}\right]$
0	1	1	$(\sigma_Z^2-\sigma_S^2)^{1/2}\cdot\left(\frac{\overline{X}}{\overline{\overline{x}}}\right)$	$\frac{t_{(a.b)}^2}{n}\left[\left\{\frac{(\delta_{40}-1)}{4}+C_X^2-\delta_{21}C_X\right\}+A\left\{\frac{(\delta_{40}-1)}{4}+C_{X'}^2-\delta_{21}C_{X'}\right\}\right]$
1	1	1	$\overline{Z}\cdot(\sigma_Z^2-\sigma_S^2)^{1/2}\cdot\left(\frac{\overline{X}}{\overline{\overline{x}}}\right)$	$\frac{t_{(a.b)}^2}{n}\left[\left\{C2_Z+\frac{(\delta_{40}-1)}{4}+C_Z\delta_{30}+C2_X-2\rho_{ZX}C_ZC_X-\delta_{21}C_X\right\}+A\left\{C2_{Z'}+\frac{(\delta_{40}-1)}{4}+\delta_{30}C_{Z'}+C2_{X'}-2\rho_{Z'X'}C_{Z'}C_{X'}-\delta_{21}C_{X'}\right\}\right]$

Table 3.2:
Different types of proposed Exponential Ratio Estimator

a	b	R	Estimator	MSE
1	0	1	$\overline{Z}\cdot exp\left(\frac{\overline{X}-\overline{x}}{\overline{\overline{X}}+\overline{x}}\right)$	$\frac{t_{(a.b)}^2}{n}\left[\left\{C_Z^2+\frac{C_X^2}{4}-\rho_{ZX}C_ZC_X\right\}+A\left\{C_{Z'}^2+\frac{C_{X'}^2}{4}-\rho_{Z'X'}C_{Z'}C_{X'}\right\}\right]$

0	1	1	$(\sigma_Z^2-\sigma_S^2)^{1/2}\cdot exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)$	$\frac{t_{(a.b)}^2}{n}\left[\left\{\frac{(\delta_{40}-1)}{4}+\frac{C_X^2}{4}-\frac{1}{2}C_X\delta_{21}\right\}+A\left\{\frac{(\delta_{40}-1)}{4}+\frac{C_{X'}^2}{4}-\frac{1}{2}C_{X'}\delta_{21}\right\}\right]$
1	1	1	$\bar{Z}\cdot(\sigma_Z^2-\sigma_S^2)^{1/2}\cdot exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)$	$\frac{t_{(a.b)}^2}{n}\left[\left\{C2_Z+\frac{(\delta_{40}-1)}{4}+\delta_{30}C_Z+\frac{C_X^2}{4}-\rho_{ZX}C_ZC_X-\frac{1}{2}C_X\delta_{21}\right\}+A\left\{C2_{Z'}+\frac{(\delta_{40}-1)}{4}R_{ZY}^2+\delta_{30}R_{ZY}C_{Z'}+\frac{C_{X'}^2}{4}-\rho_{Z'X'}C_{Z'}C_{X'}-\frac{1}{2}C_{X'}\delta_{21}\right\}\right]$

4. Empirical Study

In this section, we compare the performance of the proposed estimators using simulated and real data sets.

4.1 Simulation data sets

The simulated populations are given as:

Population 1. X = N(5, 10), Y = X + N(0, 1), y = Y + N(1, 3), x = X + N(1, 3), N = 5000;

Population 2. X = N(5, 10), Y = X + N(0, 1), y = Y + N(2, 3), x = X + N(2, 3), N = 5000,

$S\sim N(0,0.1\sigma_x)$ and $Z=Y+S$.

Table 4.1:
PRE on 10% Non-response rate on Population-I

Population size		**Estimator**	**1/k**		
N1	**N2**		**1/2**	**1/3**	**1/4**
4500	500	$\hat{t}$	100	100	100
		t_r	396.05	384.23	313.75
		t_{er}	229.47	226.80	206.39
4250	750	$\hat{t}$	100	100	100
		t_r	260.10	318.26	526.33
		t_{er}	186.96	207.73	257.98
4000	1000	$\hat{t}$	100	100	100
		t_r	361.28	345.19	302.06
		t_{er}	220.17	215.86	202.46

The Percent Relative Efficiency (PRE) of three estimators at a 10% non-response rate is shown in the above table. With 4500–4000 respondents (N1) and 500–1000 non-respondents (N2), the total population size is 5000. The baseline is the standard estimator $\hat{t}$ which has a PRE of 100 in every setting. With PRE values ranging from 260.10 to 526.33, the response-adjusted estimator t_r performs noticeably better and exhibits much higher efficiency. Interestingly, at 1/k = ¼, the maximum efficiency for $\hat{t}$ is found when N1 = 4250 and N2 = 750. With PREs ranging from 186.96 to 257.98, the error-adjusted estimator t_{er} likewise outperforms the base estimator. Both t_r and t_{er} perform better than the standard estimator overall, with t_r being the most effective of all the combinations.

Table 4.2:
PRE on 20% Non-response rate on Population-I

Population size		Estimator	1/k		
N1	**N2**		**1/2**	**1/3**	**1/4**
4500	500	$\hat{t}$	100	100	100
		t_r	317.53	426.28	347.59
		t_{er}	207.47	237.86	217.66
4250	750	$\hat{t}$	100	100	100
		t_r	351.45	287.75	302.52
		t_{er}	217.69	198.03	203.40
4000	1000	$\hat{t}$	100	100	100
		t_r	401.71	385.17	409.73
		t_{er}	231.26	227.71	234.65

The above table displays the PRE of the same three estimators with a 20% non-response rate. Compared to Table 4.1, there are more non-respondents in this instance, and the adjustment effect is more noticeable. Again, the PRE of 100 is maintained by the baseline estimator. $\hat{t}$. The response-adjusted estimator t_r shows notable efficiency benefits, especially for 1/k = 1/3 and 1/4, with PRE values as high as 426.28 and 409.73. These results demonstrate the great efficacy of the t_r estimator even in the presence of increased non-response. Additionally, the error-adjusted estimator t_{er} regularly perform better than the basic estimator, with PRE values ranging from 198.03 to 234.65. These findings indicate that while both adjusted estimators continue to perform better as non-response increases, t_r's relative efficiency improves more sharply, proving its robustness in situations with moderate non-response.

Table 4.3:
PRE on 30% Non-response rate on Population-I

Population size		Estimator	1/k		
N1	N2		1/2	1/3	1/4
4500	500	$\hat{t}$	100	100	100
		t_r	317.53	426.28	347.59
		t_{er}	207.47	237.86	217.66
4250	750	$\hat{t}$	100	100	100
		t_r	447.09	981.39	507.20
		t_{er}	242.18	312.99	256.82
4000	1000	$\hat{t}$	100	100	100
		t_r	270.27	465.46	384.76
		t_{er}	191.23	247.41	229.16

The following table looks at the scenario with the highest non-response rate (30%), where there are 1500 non-respondents (N2). Even in this challenging situation, the typical estimator $\hat{t}$ maintains a fixed PRE of 100. However, when 1/k = 1/3 and N1 = 4250, the response-adjusted estimator t_{er} exhibits exceptionally high efficiency, with PRE values reaching 981.39. This implies that the t_r estimator gets even more efficient when non-response increases, particularly when a good 1/k ratio is used. While t_r still performs better overall, the error-adjusted estimator t_{er} still performs well, with a PRE of up to 312.99. Remarkably, the efficiency of both adjusted estimators climbs rather than falls with increasing non-response, suggesting that suitable adjustment methods such as t_r and t_{er} may benefit from optimal weighting schemes in addition to being robust to non-response. Overall, this table shows the performance of adjusted estimators in extreme non-response scenarios.

Table 4.4:
PRE on 10% Non-response rate on Population- II

Population size		Estimator	1/k		
N1	N2		1/2	1/3	1/4
4500	500	$\hat{t}$	100	100	100
		t_r	452.26	438.06	438.69
		t_{er}	242.73	239.78	240.32
4250	750	$\hat{t}$	100	100	100
		t_r	482.57	329.99	731.17
		t_{er}	248.93	211.26	288.13

		$\hat{t}$	100	100	100
4000	1000	t_r	427.10	313.51	490.06
		t_{er}	236.98	206.04	251.16

This table presents the Percent Relative Efficiency (PRE) of different estimators for three population size configurations under a 10% non-response rate. Three estimators are examined at three levels of 1/k (¼, ⅓, ¼): the basic estimator ($\hat{t}$), the response-based estimator (t_r), and the extended response estimator (t_{er}). According to the results, the PRE of $\hat{t}$ remains at 100 for all population sizes, acting as a baseline. The efficiency of the estimators t_{er} and t_r usually increases as 1/k grows. For example, in the circumstances of N1=4250 and N2=750, the PRE of t_r sharply increases from 329.99 at 1/k=1/3 to 731.17 at 1/4, indicating that efficiency grows with more stratification. Similar trends are observed in different sample configurations, demonstrating that advanced estimators outperform the basic estimator in 10% non-response conditions.

Table 4.5:
PRE on 20% Non-response rate on Population- II

Population size		**Estimator**	**1/k**		
N1	**N2**		**1/2**	**1/3**	**1/4**
		$\hat{t}$	100	100	100
4500	500	t_r	411.61	981.17	508.64
		t_{er}	233.59	311.90	255.60
		$\hat{t}$	100	100	100
4250	750	t_r	360.61	1598	443.37
		t_{er}	220.28	343.72	242.91
		$\hat{t}$	100	100	100
4000	1000	t_r	365.87	381.77	365.87
		t_{er}	221.77	226.80	241.18

This table displays the changes in estimator efficiency under a higher 20% non-response rate. In all cases, $\hat{t}$ remains at 100, just like in the previous table. A notable trend is that the efficiency of t_r and t_{er} more efficient than in Table 4.4. When N1=4250 and N2=750, for example, the PRE of t_r rises to 1598 at 1/k=1/3. Compared to the 329.99 reported under the 10% rate, this is substantially greater. This implies that when the non-response rate increases, the benefit of using complex estimators such as t_r increases. The extended response estimator t_{er} likewise shows a discernible improvement, albeit with values that are relatively lower than those of t_r , even though both estimators gain from stratification and consider non-response. This suggests that t_r may be more susceptible to increases in the non-response rate.

Table 4.6:
PRE on 30% Non-response rate on Population- II

Population size		Estimator	1/k		
N1	N2		1/2	1/3	1/4
4500	500	$\hat{t}$	100	100	100
		t_r	367.04	377.25	509.36
		t_{er}	222.48	226.04	257.33
4250	750	$\hat{t}$	100	100	100
		t_r	434.95	755.88	691.41
		t_{er}	239.52	292.45	285.59
4000	1000	$\hat{t}$	100	100	100
		t_r	493.60	1061.56	436.69
		t_{er}	251.76	319.25	242.15

At a 30% non-response rate, the differences in efficiency become even more apparent. Again, $\hat{t}$ serves as a baseline, and PRE is set to 100. However, the estimators t_r and t_{er} show significant increases. Specifically, when N1=4000 and N2=1000, the PRE of t_r soars to 1061.56 at 1/k =1/3, indicating how well this estimator handles high non-response. Its robustness is further demonstrated by the fact that the readings for t_{er} continuously remain over 200 in almost every arrangement. As the non-response rate increases, especially in more stratified populations, the results show that both response-based and extended response estimators perform better than the basic estimator

4.2 Application to a real data set

For this analysis, we are taking the real data from Rosner (2015). The description of the data is given as:

Variable	Mean	St.Dev
Forced expiratory volume (Y)	2.63	0.86
Age(X)	9.93	2.95
Smoke (S)	0.10	0.30

The reported response Z=Y+S

Table 4.7:
PRE on 10% Non-response rate for Real data set

Population size		Estimator	1/k		
N1	N2		1/2	1/3	1/4
400	254	$\hat{t}$	100	100	100
		t_r	106.11	107.61	109.21
		t_{er}	104.41	105.88	107.46
425	229	$\hat{t}$	100	100	100
		t_r	105.90	107.60	108.92
		t_{er}	104.22	105.87	107.15
450	204	$\hat{t}$	100	100	100
		t_r	106.04	107.18	108.18
		t_{er}	104.32	105.47	106.45

The above table applies the estimators to real data, including the variables of forced expiratory volume (Y), age (X), and smoking (S) using the reported response formula Z=Y+S. Similar to the simulated data, the PRE values show that t_r and t_{er} perform better in all circumstances, although the basic estimator $\hat{t}$ remains constant at 100. The difference in efficiency values is still discernible, albeit not as much as in synthetic data. For example, t_r reaches 109.21 at 1/k= 1/4 for N1=400 and N2=254, whereas t_{er} reaches 107.46. These results provide credence to the idea that, even when working with real-world data, sophisticated estimators perform better than the traditional method, especially as the stratification level increases. It also demonstrates that response-based approaches are effective in real-world scenarios with no response.

Table 4.8:
PRE on 20% Non-response rate for Real data set

Population size		Estimator	1/k		
N1	N2		1/2	1/3	1/4
400	254	$\hat{t}$	100	100	100
		t_r	108.04	110.47	113.59
		t_{er}	106.30	108.66	117.73
425	229	$\hat{t}$	100	100	100
		t_r	107.75	109.92	115.19
		t_{er}	106.04	108.12	113.32
450	204	$\hat{t}$	100	100	100
		t_r	107.31	110.72	113.16

t_{er}	105.58	108.95	111.30

The above table displays many estimators' Percent Relative Efficiency (PRE) for a real data set with a 20% non-response rate for a range of population sizes and sub-sampling rates (1/k). This comparison is between three estimators: (the baseline), $\hat{t}$ values of 1/k=1/2,1/3,1/4. The efficiency of both t_r and t_{er} increases as the value of 1/k increases, suggesting more thorough sub-sampling. The fact that t_{er} consistently achieves the highest PRE across all settings is interesting since it suggests that it is more successful at handling non-response. With a population size of 450 and N2=204, for example, t_{er} records a PRE of 111.30 at 1/k=1/4, whereas t_r records 113.16 and $\hat{t}$ records 100.

Table 4.8:
PRE on 20% Non-response rate for Real data set

Population size		**Estimator**	**1/k**		
N1	**N2**		**1/2**	**1/3**	**1/4**
400	254	$\hat{t}$	100	100	100
		t_r	108.88	113.67	116.74
		t_{er}	107.12	111.82	114.90
425	229	$\hat{t}$	100	100	100
		t_r	109.16	114.56	117.57
		t_{er}	107.41	112.72	115.62
450	204	$\hat{t}$	100	100	100
		t_r	108.46	113.01	118.97
		t_{er}	106.72	111.16	116.97

The PRE of estimators t_r and t_{er} further improves under higher non-response conditions, according to the results in the above table. As previously, for all values of 1/k, $\hat{t}$ stays at a baseline PRE of 100. But both t_r and t_{er} perform noticeably better than it, particularly as 1/k rises. Although the estimator t_r exhibits a consistent increase in efficiency, the PRE is typically highest for t_{er}. For example, t_{er} achieves a PRE of 115.62 at 1/k=1/4 when N1=425 and N2=229, demonstrating its resilience and better management of higher non-response. As the non-response rate and sub-sampling intensity rise, the trend indicates that t_{er} is consistently the most efficient estimator.

5. Application and real-world relevance

The real dataset, which comprises age (X), smoking status (S), and forced expiratory volume (Y), is a suitable illustration of a sensitive variable scenario (Rosner, 2015). The randomised response method (RRT) is appropriate in this instance since simple answers to questions about lifestyle and health are frequently the target of non-response and misreporting. The structure used by the suggested estimators indicates that the scrambling variable S is accurate for intentional misreporting. This demonstrates the estimators' usefulness once more. The PRE values shown in Tables 4.7 to 4.9 demonstrate growing efficiency gains with rising non-response rates, confirming the robustness of the estimators in actual survey designs.

6. Conclusion and Discussion

The suggested estimators are thoroughly compared under various non-response levels in the Percent Relative Efficiency (PRE) tables (Tables 4.1 to 4.9). The ratio-based estimator t_r consistently performs better than the baseline estimator $\hat{t}$ on both simulated and real data, particularly as the non-response rate rises. Higher subsampling levels (e.g., 1/k=1/4), where the estimator uses more data from non-respondents, are where this improved performance is most noticeable. Although it is typically marginally less efficient than t_r, the exponential ratio estimator t_{er} likewise exhibits notable efficiency gains, demonstrating its resilience in situations where measurement error is substantial.

These findings have clear applications: when surveys are affected by both ME and NR, especially when dealing with sensitive subjects, the proposed estimators produce estimates that are more accurate and useful than those obtained using traditional methods. The performance benefit is not merely theoretical; it has been shown through real-world data applications. Future studies could compare these estimators to existing ones to further quantify their relative efficacy across different sample settings (e.g., Singh & Kumar, 2008; Zahid et al., 2022).

The present study presents a novel class of joint estimators for the variance and mean of sensitive variables under the combined impacts of measurement error and non-response. Simulation and real-world findings show that the proposed ratio and exponential-ratio estimators perform better than the traditional estimators in terms of mean square error (MSE) and percent relative efficiency (PRE). Efficiency in this context refers to the estimator's ability to minimise variation while maintaining objectivity, resulting in estimates that are more accurate and dependable even when the non-response rate is high. Given that sensitive data is regularly compromised in social, health, and educational surveys, the findings have important implications.

This study offers enhanced estimation efficiency under the combined conditions of measurement error and non-response, in addition to expanding the framework of previous work by incorporating joint estimation of both mean and variance. Future studies can investigate the suitability of these estimators in stratified or systematic sampling designs, as well as robust estimating techniques in situations where response distributions depart from normality. Additionally, this work allows the proposed estimators to be extended to other parameters, such as skewness, Kurtosis, or quartile-based measures for sensitive variables.

Declaration

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