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A new Bartlett-based homogeneity test for linear regression models

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ABSTRACT

The assumption of homogeneous residuals is crucial in linear regression analysis. In fact, all statistical inferences around regression coefficients are built on the basis of this fundamental assumption. As a result, having a linear model with heterogeneous residuals would destruct all those beneficial reliable inferences, and this specifically means that all extracted predicted values, confidence regions and any other conclusions are then false, misleading and far from reality. Also, heterogeneity underestimates true significance levels, which could lead to considering the importance of an explanatory variable whereas truth is not. These are a few consequences of handling a linear regression model with heterogeneous residuals, and hence, researchers have focused on two things: first, defining a consistent statistical tool to catch deviations from homogeneity, and second, proposing approaches to deal with that violation efficiently. We are not interested here in the last goal, and rather, we are focusing on the first one. Indeed, there were enormous and remarkable efforts seeking to fulfil the first goal in different ways, from plotting residuals against fitted values to applying statistical tests, such as the Breusch-Pagan test, Koenker test, and other related tests. But in effect, analysing plots is highly dependent on self-experience and how one would draw conclusions and thoughts about the plot and, in fact, it is not an easy task in many real-world studies. On the other hand, each of the Breusch-Pagan and Koenker tests has its own deficiencies and misleading conclusions. Other related tests, which we will summarise shortly, are not easily implemented or programmed. So, we aim in the present paper to present a simple statistical method for testing the homogeneity assumption of linear regression residuals, by just employing the well-known Bartlett's test, on the basis of defining two suitable disjoint subsets driven from the original dataset. We evaluate the proposed



approach by a series of simulation studies and analysing a previous historic case study, and it will be shown that the proposed method controls the error rate in a nominal level and has high performance in sense of both homogeneity and heterogeneity detections.

Keywords: Bartlett Test; Breusch-Pagan Test; Homogeneity Test; Koenker Test; Linear Regression

1. Introduction and preliminaries

Recently, statistical linear regression models have become very useful tools in artificial intelligence and machine learning applications. Assessing the homogeneity of regression random error variance is important and a crucial stage in building every linear regression model. Breusch and Pagan (1979) introduced a test for detecting heteroscedasticity, and years later, a studentized version of that test was given by Koenker (Koenker, 1981). But both proposed tests have problems of making a type II error and fail in many scenarios (Zaman, 1995). On the other hand, Dutta and Zaman (1992) introduced a test statistic, denoted by DZ , that performs better than Breusch-Pagan statistics, but they concluded that their test is biased. Until today, plotting the scatter plot of pairs of points (\hat{y}, e) is the most famous procedure to check homogeneity, but this approach also collapses when categorical explanatory variable(s) exist in the model.

The main work of the current paper is to describe and develop an easy-to-apply method of low computational complexity to provide a homogeneity test in the framework of the hypothesis testing problem. To be more precise, we present a Bartlett-based methodology depending on computing the inner products of $\langle e_i, \hat{y}_i \rangle$ conditioned on two cases, namely, positive residuals and the opposite directional ones. This is not the first approach relying on dividing observations. Precisely, in simple linear regression, if the independent variable x is divided into two suitable disjoint parts, then an F variance ratio test or Bartlett test can be deployed. Also, if x is partitioned into more than two disjoint groups, then the Levene test is a good choice. All previously proposed tests, as our new approach, are sensitive to the departures of residuals from normality or the presence of outliers, and in these situations the Fligner-Killeen test is a good nonparametric alternative. But the obvious two problems with previous parametric approaches (F variance ratio, Bartlett and Levene tests) are: first, the problem of splitting x in a proper efficient way is a challenging stage and second, those procedures are unapplicable in multiple linear regression settings.

The novelty of our new approach comes from three things. First, it fills the gap of testing the homogeneity assumption depending on splitting the inner products of $\langle e_i, \hat{y}_i \rangle$, instead of splitting the original observations, which was the case before. Second, whereas F variance ratio, Bartlett and Levene tests are suitable just for simple linear regression models, the new method can handle simple and multiple linear regression models as well. Third, in the presence of a categorical

explanatory variable (or more), both of Breusch-Pagan and Koenker tests fail as completely linear relationships do not straightly exist anymore, and this is where our proposed method fills the gap, again.

2. The linear regression model

We start with the simplest form of linear regression models, the simple linear regression model. For a sample of pairs $(x_i, y_i)_{i=1,2,\dots,n}$, the simple linear regression model of y on x is given by

$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i; \quad i = 1, 2, \dots, n$$

Where it is assumed that the random errors $\varepsilon_1, \varepsilon_2, \dots, \varepsilon_n$ are independent and identically distributed (iid) with $\mathcal{N}(0, \sigma^2)$. To verify the error terms assumptions, the residuals $e_i = y_i - \hat{y}_i$ are computed and investigated.

The usual procedure to check the homogeneity of e_i 's, is to plot them against the fitted values \hat{y}_i 's. If the scatter plot does not reflect any pattern, then we conclude that the homogeneity assumption is met. But building statistical judgments by relying only on graphs may be misleading. It depends significantly on how people draw conclusions. So, there is an urgent statistical mission to propose a technique that leads to computing a p-value, and hence, there would be no conflicting decisions.

Analysing model residuals is a crucial step in building any prediction model. Altindal and Askan (2022) analysed 1053 earthquakes in Turkey recorded at 178 different stations. They found that they are dealing with non-normal and heterogeneous residuals, so they proceeded with applying Box-Cox transformations to handle these issues. But other researchers deviate from dealing with or treating heterogeneous residuals by adopting non-parametric procedures. For example, Al-Nasrawi et al. (2021) deployed Sen's slope and Mann Kendall test in analysing the annual maps of water occurrence. Further, Demel et al. (2025) studied the social empathy during two recent refugee crises, namely, the Syrian and the Ukrainian refugee crisis. They managed to introduce a linear regression model to predict social closeness on the basis of nationality, gender and age of the refugee, along with other factors. The assumption of homogeneity was merely verified visually.

3. The Bartlett-based homogeneity test

It is well known that heterogeneity means that the error variance is not constant over all observations. As a result, overall observations, the variance of positive residuals is significantly



different from the variance of negative residuals. From the last statement, a proposed approach is proposed in the light of the following theorem.

Theorem 3.1. In a linear regression model $y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$. For all $i = 1, 2, \dots, n$, the argument that the error terms ε_i are homogeneous across all observations is equivalent to the argument that the terms $\varepsilon_i \hat{y}_i$ are homogeneous across the negative and positive residuals. In other words, $\forall i = 1, 2, \dots, n$,

$$\mathbb{V}(\varepsilon_i) = \sigma^2 \Leftrightarrow \mathbb{V}(\varepsilon_i \hat{y}_i | \varepsilon_i \leq 0) = \mathbb{V}(\varepsilon_i \hat{y}_i | \varepsilon_i > 0) \quad (3.1)$$

As a result, to test the null hypothesis $H_0: \mathbb{V}(\varepsilon_i) = \sigma^2; \forall i \in \{1, \dots, n\}$ against $H_1: \mathbb{V}(\varepsilon_i) \neq \sigma^2; \exists i \in \{1, \dots, n\}$, it is sufficient to test

$$H_0: \sigma_1^2 = \sigma_2^2 \quad \text{against} \quad H_1: \sigma_1^2 \neq \sigma_2^2 \quad (3.2)$$

where $\sigma_1^2 = \mathbb{V}(\varepsilon_i \hat{y}_i | e_i \leq 0)$ and $\sigma_2^2 = \mathbb{V}(\varepsilon_i \hat{y}_i | e_i > 0)$.

To perform the addressed test in (3.2), the well-known Bartlett's test can be employed, which is equivalent to performing the newly defined test in (3.1), and this is the new approach we propose in this paper.

To prove Theorem 3.1, we need to remember some basic results from the linear regression model.

Lemma 3.2. (Rencher and Schaalje (2007) p.159). For an iid random sample $y_i \sim \mathcal{N}(\beta_0 + \beta_1 x_i, \sigma^2)$, with $i = 1, 2, \dots, n$. Let $\hat{\beta}_0, \hat{\beta}_1$ and $\hat{\sigma}_{MLE}^2$ be the maximum likelihood estimators of β_0, β_1 and σ^2 , respectively. Then, each of $\hat{\beta}_0$ and $\hat{\beta}_1$ is randomly independent of $\hat{\sigma}_{MLE}^2$.

It follows from this lemma that the random variable $\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x$ is also randomly independent of $\hat{\sigma}_{MLE}^2$. And as a result, \hat{y} is also independent of SSE (error sum of squares), and hence it is independent of e as well (Rencher and Schaalje (2007) p.229).

Furthermore, we need to recall the following famous lemma.

Lemma 3.3. If the conditional two random variables $M|S$ and $N|S$ are independent, then

$$\mathbb{V}(MN|S) = \mathbb{V}(M|S)\mathbb{V}(N|S) + \mathbb{V}(M|S)\mathbb{E}[N|S]^2 + \mathbb{V}(N|S)\mathbb{E}[M|S]^2$$

Proof. We have that

$$\begin{aligned} \mathbb{V}(MN|S) &= \mathbb{E}[M^2|S]\mathbb{E}[N^2|S] - \mathbb{E}[M|S]^2\mathbb{E}[N|S]^2 \\ &= (\mathbb{V}(M|S) + \mathbb{E}[M|S]^2)(\mathbb{V}(N|S) + \mathbb{E}[N|S]^2) - \mathbb{E}[M|S]^2\mathbb{E}[N|S]^2 \end{aligned}$$

and the result follows.

Now, we can proceed to prove Theorem 3.1.

Proof. Since $\hat{y}_i | e_i$ is independent of $e_i | e_i$, which is e_i , for $i = 1, 2, \dots, n$, we may write

$$\begin{aligned} \mathbb{V}(\hat{y}_i e_i | e_i) &= \mathbb{V}(e_i | e_i)\mathbb{V}(\hat{y}_i | e_i) + \mathbb{V}(\hat{y}_i | e_i)\mathbb{E}[e_i | e_i]^2 + \mathbb{V}(e_i | e_i)\mathbb{E}[\hat{y}_i | e_i]^2 \\ &= 0\mathbb{V}(\hat{y}_i) + \mathbb{V}(\hat{y}_i)e_i^2 + 0\mathbb{E}[\hat{y}_i]^2 \\ &= \mathbb{V}(\hat{y}_i)e_i^2 \end{aligned}$$



Hence, assuming $\mathbb{V}(e_i) = \sigma^2$ is actually equivalent to assuming $\mathbb{V}(\hat{y}_i e_i | e_i \leq 0) = \sigma^2 e_i^2$ and also $\mathbb{V}(\hat{y}_i e_i | e_i > 0) = \sigma^2 e_i^2$.

In the next two sections, we check the performance of Theorem 3.1 by a series of simulation studies and a case study.

4. Simulation study

In the following two examples, we denote by y to the dependent variable and by x to the independent one. The results are obtained as follows. In each repetition of the 200 repetitions, we generate at random a sample of we denote by $n = 100$ paired observations (x_i, y_i) , and then computing the p -values of the Breusch-Pagan, Koenker and the Bartlett-based (3.2) tests.

After that, for each test, we compute the rejection rates and the p -values average over repetitions. A reliable test should reflect two facts. Fact number one: if the homogeneity assumption is true, then we would expect that the empirical power, which is the rejection rates, is close to the significance level of $\alpha = .05$ and the p -values average is near 0.5. Fact number two: if the homogeneity assumption is false, then we would expect that the empirical power is close to 1 and the p -values average is near zero. The R language codes are available by contacting the first author.

4.1. First example

Assume that x is an $\mathcal{N}(2,3)$ variable, and assume that y is generated by the formula of $y = 1 + x$, which is a completely linear relationship. As Zaman (1995) noted, the Breusch-Pagan test performs well since it can detect linear relationships properly, but fails otherwise. As a result, we expect that both Breusch-Pagan and Bartlett-based tests perform well in this scenario.

Figure 1 shows the scatter plot of e_i against \hat{y}_i , for $i = 1, 2, \dots, 100$ in some random repetition. By looking at this figure, it is not a simple task for many to decide whether these residuals are homogeneous or not.

The results listed in Table 1 show that the homogeneity assumption is not verified, since the rejection rates tend to 1 and the p -values averages are close to zero. This is true for both Breusch-Pagan and Bartlett-based tests. But as for the Koenker test, the results reflect that $H_0: \mathbb{V}(\varepsilon) = \sigma^2$ is true mostly, and this is wrong.



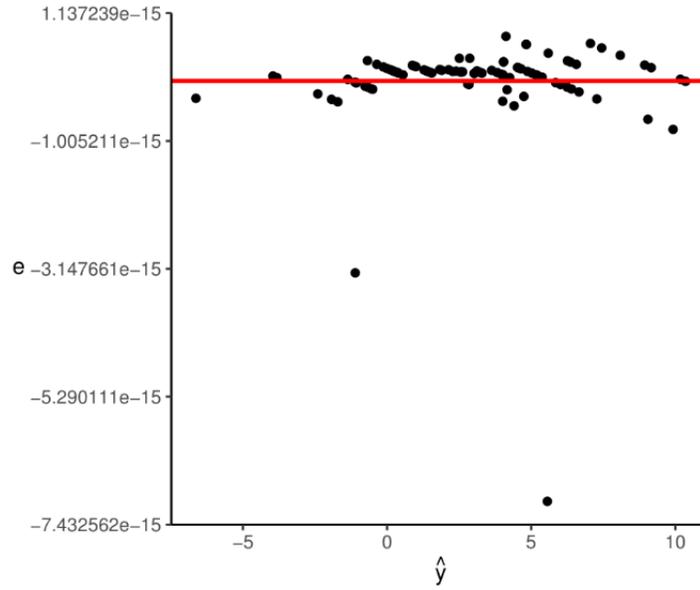


Figure 1: Scatter plot of (e, \hat{y}) , for example, 4.1

Table 1:

Rejection rates and p-value averages of testing $H_0: \mathbb{V}(\varepsilon_i) = \sigma^2$ in Figure 1, over 200 repetitions

	Breusch-Pagan	Koenker	Bartlett-based
rejection rate	0.705	0.065	0.895
ave(<i>p-value</i>)	0.119	0.489	0.048

4.2. Second example

Again, in this scenario, we generate x from $\mathcal{N}(2,3)$ distribution, but we now assume that y is generated as $y = 1 + x^2$, which is a quadratic smooth function, not a linear one. Hence, we anticipate that Breusch-Pagan performs better than the Koenker test, but it is poor in the sense of test power. On the contrary, we will see that the Bartlett-based test outperforms the two previous tests in this new scenario.

Obviously, by looking at [Figure 2](#), we conclude that the residuals are not homogeneous.

Now, considering the previously addressed tests, [Table 2](#) shows that the Breusch-Pagan test is statistically more powerful than the Koenker test. But the Bartlett-based test (3.2) can completely distinguish between $H_0: \mathbb{V}(\varepsilon_i) = \sigma^2$ and the alternatives, where the rejection rates is one (without rounding) and the *p-value* average is totally zero.



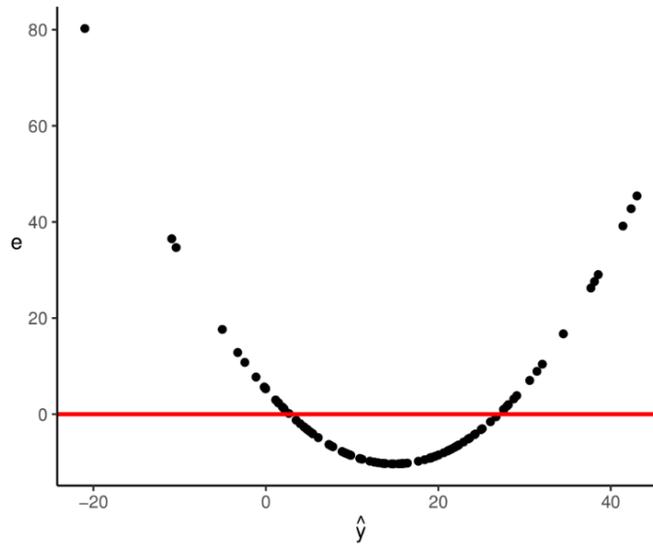


Figure 2: Scatter plot of (e, \hat{y}) for example 4.2

Table 2:

Rejection rates and p-value averages of $H_0: \mathbb{V}(\varepsilon_i) = \sigma^2$ in testing Figure 2, over 200 repetitions

	Breusch-Pagan	Koenker	Bartlett-based
rejection rate	0.495	0.215	1
ave(p-value)	0.221	0.362	0

4.3. Third example

In the current scenario, we handle a linear regression model with a categorical explanatory variable. In this case, the visual interpretation of the residuals plot against fitted values is meaningless. A nutrition specialist expects that total muscle mass in the human body depends on one's gender. So, he collected a sample size of sixteen fully grown males (coded by 1) and females (coded by 2), and then he measured their muscle mass in pounds. The observations in [Table 3](#) represent the results of this experiment.

Table 3:

Muscle mass against gender

x	2	2	2	1	1	2	2	1
y	82	91	100	68	87	73	78	80
x	1	2	2	1	1	2	1	2
y	65	84	116	76	97	100	105	77

As we see in [Figure 3](#), it is impossible to draw any conclusions about the homogeneity assumption, and in this setting, the visual interpretation fails. After regressing y on x and computing the residuals, we can split those residuals into two disjoint subsets, according to males and females. [Tables 4 & 5](#) show these results.



Table 4:
Residuals for subset x=males

x	1	1	1	1	1	1	1	1
e ₁	-14.571	4.429	-2.571	-17.571	-6.571	14.429	22.429	

Table 5:
Residuals for subset x=females

x	2	2	2	2	2	2	2	2
e ₂	-7	2	11	-16	-11	-5	27	-12

For x=1 (males), the Anderson-Darling test shows that the e₁ sample is normally distributed, where sig = 0.9947314. Likewise, for x=2 (females), the e₂ sample is also normally distributed, where sig = 0.8953762. As a result, we may proceed to perform the F variance ratio test, Bartlett test, and our Bartlett-based test, as well as Breusch-Pagan and its correction.

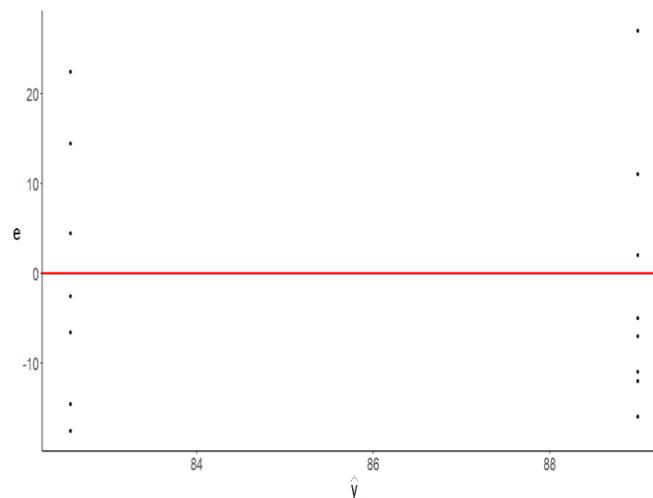


Figure 3: Scatter plot in the presence of a categorical explanatory variable

The results listed in Table 6 reflect the fact that all five addressed tests accept the hypothesis of homogeneous residuals.

Table 6:
Significance levels of testing residuals homogeneity in the third example

	Breusch-Pagan	Koenker	F variance ratio	Bartlett	Bartlett-based
p-value	0.928	0.905	0.464	0.307	0.153

4.4. Fourth example

Let us turn to another example that handles building a multiple linear regression model of regressing y on two explanatory variables. x1 is a continuous variable that refers to the quantity of an oil-based paint for houses. x2 is a categorical variable that refers to the season in which the house was painted (1=summer, 2=fall, 3=winter). The dependent variable y refers to customer satisfaction and Table 7 shows all assumed dataset.



Table 7:

Data of fourth example

i	y	x1	x2	i	y	x1	x2	i	y	x1	x2
1	1.0	4.05	1	11	-.1	4.45	2	21	.8	4.30	3
2	.1	3.81	1	12	.5	4.45	2	22	.5	4.55	3
3	.5	4.20	1	13	.5	4.25	2	23	.4	4.15	3
4	.7	4.35	1	14	.6	4.25	2	24	.6	4.15	3
5	-.1	4.35	1	15	.0	4.05	2	25	-.7	4.25	3
6	.4	4.05	1	16	-.2	4.22	2	26	-.2	3.95	3
7	.2	4.20	1	17	.0	4.10	2	27	.3	4.35	3
8	.3	4.32	1	18	.2	4.28	2	28	.1	4.15	3
9	.7	4.21	1	19	-.1	4.32	2	29	.4	4.16	3
10	.5	4.17	1	20	.6	4.12	2	30	-.6	3.85	3

By considering $x_2=2$ as a reference group, we need to define two dummy variables that represent x_2 in the equation. In fact, the prediction model for this example is

$$\hat{y} = -2.462 + .626(x_1) + .279(\text{dummy1}) - .001(\text{dummy2})$$

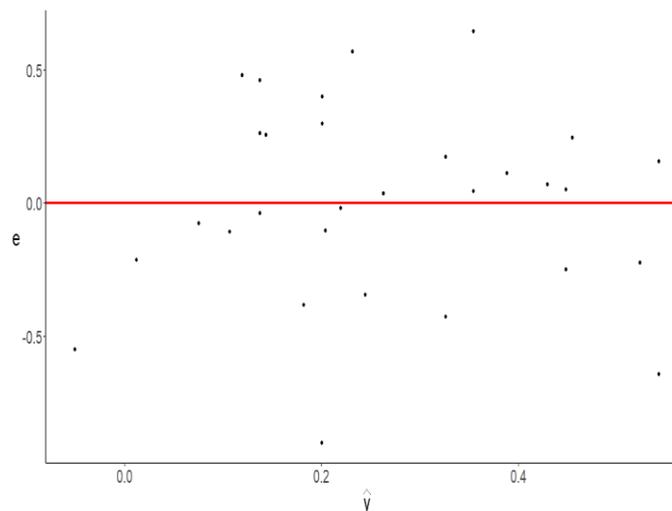
By looking at [Figure 4](#), one should not adopt or consider any decisive decision, knowing and keeping in mind that our model involves a categorical explanatory variable. On the contrary, and at $\alpha = .05$, [Table 8](#) tells us another story. Both of Breusch-Pagan and Koenker tests demonstrate that residuals are homogeneous, that is, accepting the null hypothesis of test (3.2). But, as for the proposed Bartlett-based approach, the decision is to reject the null.

Table 8:

P-values of testing the homogeneity assumption in fourth example

	Breusch-Pagan	Koenker	Bartlett-based
p-value	0.701	0.675	0.028

As another supportive descriptive procedure to our final decision that residuals are not homogeneous, let us take a look at the box plots of the residuals for the three different seasons.

**Figure 4:** Scatter dots of residuals against fitted values in the fourth example

A quick glimpse at Figure 5 leaves no room for doubt that we are studying a linear regression model with heterogeneous residuals.

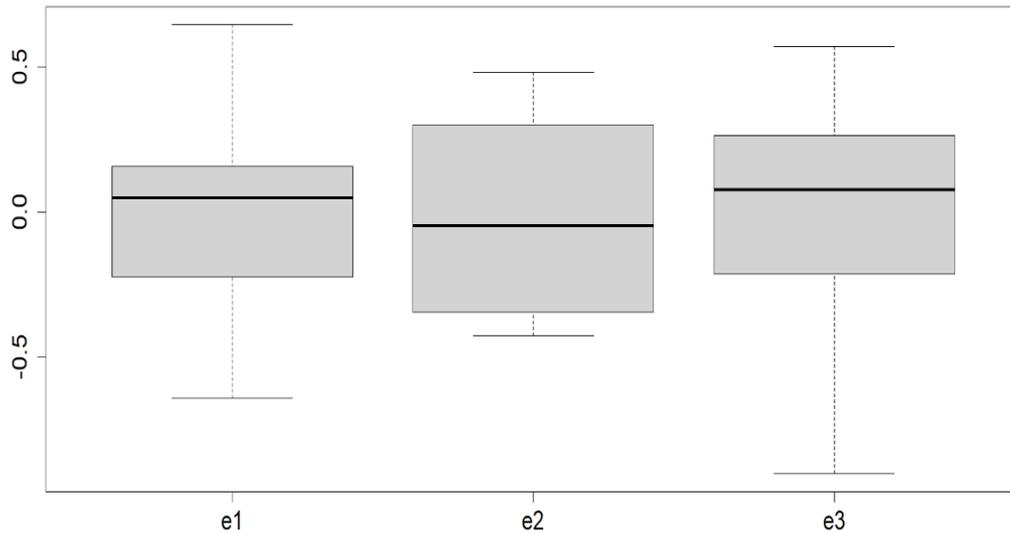


Figure 5: Box plots of the residuals for three different seasons

5. Case study

Pearson and Lee (1903) collected the data of 1078 fathers' heights and their fully grown sons in inches, in order to predict the son's height relying on knowing the father's. But, since the computer was not there, the original data was rounded to the nearest integer. So, Freedman et al. (2007) added random noise as a continuity correction to the data. In this section, we use the corrected data, which is available from the authors in this link <https://myweb.uiowa.edu/pbreheny/data/pearson.html>. Figure 6 demonstrates the fitted regression line, by considering fathers' heights as the observations of x , and their fully grown sons' heights as the observations of y . The linear regression model is $\hat{y} = 33.8866 + 0.514093x$.

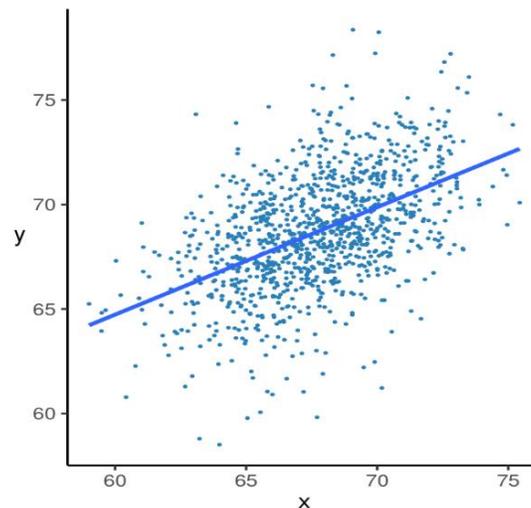


Figure 6: Scatter plot and fitted line of regressing y on x



Figure 7 shows, descriptively, that it is suitable to assume e_i 's as homogeneous residuals, and we will investigate the performance of the three named tests. But, since Bartlett's test is sensitive to the Gaussianity assumption, we need to check the Gaussianity of the two samples proposed in the (3.2) test.

Concerning the sample of $\{\varepsilon\hat{y}|e > 0\}$, the Kolmogorov-Smirnov test gives $p\text{-value}=0.830$, so the Gaussianity assumption is verified. Similarly, the Kolmogorov-Smirnov $p\text{-value}$ is 0.851 for the sample $\{\varepsilon\hat{y}|e \leq 0\}$.

Now, we may proceed to perform the homogeneity test according to Breusch-Pagan, Koenker and the Bartlett-based approaches. The results listed in Table 9 show that all three methods are successfully distinguishing between the null and the alternative hypothesis, namely, each $p\text{-value}$ is greater than $\alpha = 0.05$.

Table 9:

P-values of testing $H_0: \mathbb{V}(\varepsilon_i) = \sigma^2$

	Breusch-Pagan	Koenker	Bartlett-based
<i>p-value</i>	0.952	0.960	0.810

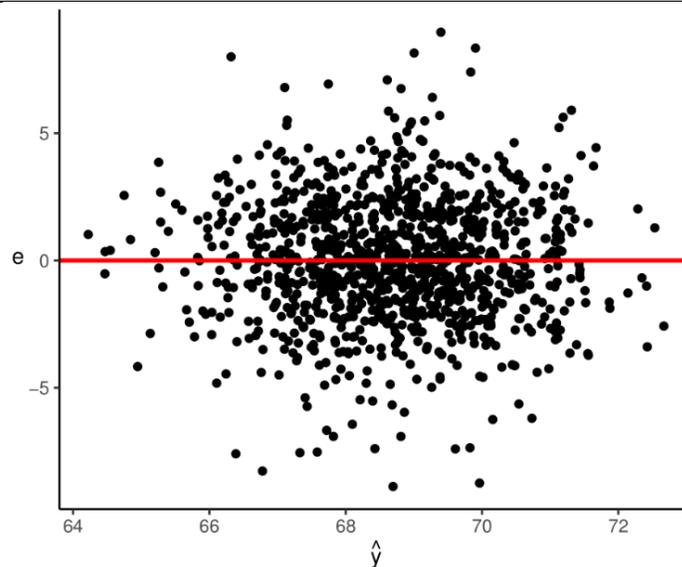


Figure 7: Scatter plot of (\hat{y}, e) and the fitted line of father and son's heights

8. Conclusion

This paper has proposed an easy-to-apply method for testing the homogeneity assumption of residuals in linear regression models. It can distinguish between the null and the alternative hypotheses with a completely satisfactory performance, in a way that we are not forced to visualise or analyse the different shapes of the scatter plots of (\hat{y}, e) . In short, although previous tests do exist for the purpose of testing the constancy of regression errors variance, every test has its own



problems or difficulties, and we are confident that our presented approach is much easier and more applicable. Unfortunately, the proposed approach is sensitive to departures from normality since the Bartlett test is itself sensitive to this issue. As a future study, we seek to address this limitation and propose more robust tests in sense of deviations from normality. The Bartlett-based test presented here outperforms Breusch-Pagan and Koenker tests, especially in preventing type II error, and the capability of dealing not just with completely linear relationships. Also, the defined Bartlett-based test outperforms F variance ratio, Bartlett, and Levene tests, especially in easing the challenging issue of dividing the sample into disjoint subsamples. The notable remark here to mention is that the F variance ratio, Bartlett and Levene tests are familiar in simple linear regression, not the multiple framework, and here our proposed approach confirms its superiority and flexibility.

Declaration

Conflict of Study: The author declares that he has no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper

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