SCOPUA Journal of Applied Statistical Research (JASR)

Vol.2, Issue 1, March 2026
DOI: https://doi.org/10.64060/JASR.v2i1.3

A comprehensive review on the advancement of the Xgamma Distribution

Harsh Tripathi [1*], Mahendra Saha [2], Abhimanyu Singh Yadav [3]

[1]Symbiosis Statistical Institute, Symbiosis International (Deemed University) ,Pune, India

[2]Department of Statistics, University of Delhi, India

[3]Department of Statistics, Banaras Hindu University, Varanasi, India

* Corresponding Email: rsearchstat21@gmail.com

Received: 01 September 2025 / Revised: 06 December 2025 / Accepted: 10 December 2025 / Published online: 18 December 2025

ABSTRACT

The present article provides a comprehensive review of the development and applications of the Xgamma distribution (XGD). The Xgamma distribution, conceptually parallel to the Lindley distribution, emerges as a combination of the exponential and gamma distributions with an appropriate weighting coefficient. Owing to its flexibility and analytical tractability, the XGD has gained considerable attention in reliability and lifetime data modeling. In recent years, numerous researchers have proposed various extensions and generalizations of the Xgamma distribution by employing diverse transformation techniques and distributional families. These developments have further enhanced its modeling capability and adaptability to complex data structures. A particularly interesting aspect of this review is the discussion of practical applications, where several real data sets used in previous studies are examined to demonstrate the empirical relevance and versatility of the extended forms of the XGD across different scientific and engineering fields.

Keywords: Xgamma distribution; finite mixture distribution; extended Xgamma families; transformation-based distributions; real data applications.

1. Introduction

Statistics underpins almost every scientific discipline, including medicine, finance, economics, agriculture, engineering, and the social sciences. Its methods guide evidence-based decision-making, support policy formulation, and help in understanding complex real-world phenomena. Within both academic research and applied work, statistics provides the conceptual and computational tools needed to summarize data, quantify uncertainty, and draw reliable conclusions. The discipline of statistics is broadly structured into several major areas such as descriptive statistics, inferential statistics, design of experiments, time series analysis, statistical quality control, econometrics, regression analysis, and multivariate analysis. In each of these areas, researchers continually develop new methodologies and theoretical results to address emerging data structures and practical demands, thereby contributing both to the advancement of statistical science and to societal welfare. These developments have had substantial impact in diverse sectors, including medical research, the corporate and financial industries, manufacturing, agriculture, and public health, where rigorous statistical tools are now integral to planning, monitoring, and evaluation.

Statistical thinking also plays a crucial role in governance and public policy. Almost all modern government policies ranging from health interventions and educational reforms to economic planning and environmental regulation rely heavily on statistical evidence. Large-scale surveys, censuses, clinical trials, and administrative databases provide the empirical foundation upon which policies are designed, implemented, and assessed. Without sound statistical methodology, such large and complex data sources would be difficult to interpret, and policy decisions would risk being inefficient or misguided. Within parametric statistics, probability distributions form one of the central pillars of theory and practice. They provide mathematical models for random phenomena in virtually every field, including medical survival times, insurance claims, financial returns, rainfall amounts, crop yields, and industrial lifetimes. By specifying a probability distribution, one can characterize the behavior of a random variable and derive key statistical properties such as moments, quantiles, reliability measures, and risk functionals, which in turn inform inference, prediction, and decision-making.

Because of their wide utility, an extensive body of work has focused on proposing, studying, and extending families of probability distributions. Over time, numerous univariate, bivariate, and multivariate distributions have been introduced, along with systematic "generating" families that allow new models to be constructed from existing ones. Classical and widely used distributions include the exponential, gamma, normal, Weibull, logistic, Burr, binomial, Poisson, geometric, and hypergeometric distributions, among many others. These models arise naturally in different applied contexts, yet none is universally adequate for all types of data, especially when real data exhibit features such as skewness, heavy tails, multimodality, or complex dependence.

In the last few decades, there has been a marked shift toward the development and expansion of distribution theory through generalizations, extensions, and compound or transformed forms of existing models. Authors have proposed new families by introducing additional shape parameters, applying transformations, or compounding baseline distributions with mixing mechanisms to increase flexibility. This line of research is driven by practitioners' need for models that can accurately capture diverse real-world behaviors, such as varying hazard rate shapes in reliability and survival analysis or extreme events in finance and environmental studies. The ongoing search for flexible, robust, and tractable probability distributions continues to motivate the development of new parametric forms that offer better fit, richer interpretation, and wider applicability across scientific and industrial domains.

XGD is a direct outcome of the ongoing search for flexible probability models capable of providing good fit and meaningful interpretation across a wide variety of real-life situations. Since its introduction, the XGD has been employed by many researchers in diverse applied fields, particularly in reliability and lifetime data analysis, where its structural properties and tractable form make it an attractive alternative to classical distributions. The original work of Sen et al. (2016) demonstrated both the superior performance and the appealing theoretical features of the XGD when compared with several competing models, thereby establishing it as a useful baseline distribution for further generalization and application.

Following this foundational contribution, a rich body of work has emerged that focuses on extending and modifying the XGD to enhance its flexibility and adapt it to different data-analytic contexts. Several notable variants have been introduced, including the weighted Xgamma, quasi Xgamma, wrapped Xgamma, inverse Xgamma, and unit Xgamma distributions. Each of these versions is constructed to address specific modeling needs, such as accommodating different support domains, capturing more complex shapes of the probability density function, or allowing for a wider range of skewness and tail behavior. In most cases, the proposed models are rigorously studied in terms of their structural properties, estimation procedures, and reliability characteristics.

A key aspect of this development is the strong empirical support provided through applications to real data sets. For each extended or generalized form of the XGD, authors typically analyze one or more real-life data sets arising from areas such as engineering reliability, biomedical survival times, environmental measurements, or industrial production. These applications not only illustrate the practical relevance of the models but also allow comparison with classical and competing distributions, often showing that XGD-based models yield better fit or more realistic hazard rate behavior. In particular, the flexibility of the hazard rate function capable of capturing increasing, decreasing, bathtub-shaped, or other complex patterns has been central to justifying the use of these extensions in reliability and survival contexts.

Interestingly, the literature on XGD and its extensions has grown continuously since 2016, reflecting sustained interest in this class of distributions. Recent contributions include, among others, works by Wani et al. (2022), Abulebda et al. (2022), Sen et al. (2024), Alomain et al. (2025), Tripathi et al. (2025), and Boudjerda (2025), each proposing new generalized or transformed versions of the XGD and demonstrating their usefulness through theoretical investigations and real-data applications. These studies collectively expand the XGD family into a broad class of models with varying shapes, supports, and dependence structures, thereby enriching the toolbox available to applied statisticians.

In this context, the main aim of the present review is to provide a systematic and coherent summary of all work related to the generalizations and extensions of the Xgamma distribution. The study brings together the various proposed forms of XGD, outlines their construction mechanisms, details their key statistical and reliability properties, and documents their areas of application. By organizing and synthesizing the scattered literature, this review is intended to serve as a comprehensive reference for researchers and practitioners interested in using XGD-based models, as well as a foundation for future developments in the theory and application of flexible lifetime distributions.

Rest of article is organized as follows: In section 2, we discussed regarding the genesis and evolution of XGD. We placed the new generalization or extension of XGD in section 3. We have reported some of data sets which are used to by different researchers in their articles of variety developed XGD in section 4. Conclusive words regarding the presented study are discussed in section 5.

2. Evolution of Xgamma distribution (XGD)

The exponential and gamma distributions are two of the most elegant and widely used models in probability and statistics, and their beauty lies both in their fundamental properties and in their simple functional forms. For an exponential distribution with rate parameter $\theta > 0$,, the probability density function (PDF) and cumulative distribution function (CDF) are

$$f_{\text{Exp}}(x) = \theta e^{-\theta x}, \quad x > 0, \tag{1}$$

$$F_{\text{Exp}}(x) = 1 - e^{-\theta x}, \quad x > 0. \tag{2}$$

The exponential distribution is celebrated for its mathematical simplicity and its unique memoryless property, which makes it a natural choice for modeling lifetimes with a constant failure rate and inter-arrival times in Poisson processes, especially in reliability and queueing contexts. The gamma distribution generalizes the exponential model by introducing a shape parameter that allows much richer behavior. For a gamma distribution with shape parameter $k = 3$ and rate parameter $\theta > 0$, the PDF and CDF can be written as

$$f_{\Gamma}(x) = \frac{\theta^3 x^2}{2} e^{-\theta x}, \quad x > 0, \tag{3}$$

$$F_{\Gamma}(x) = 1 - e^{-\theta x}\left(1 + \theta x + \frac{\theta^2 x^2}{2}\right), \quad x > 0. \tag{4}$$

By varying the shape parameter in general, the gamma distribution can capture decreasing, increasing, or unimodal density shapes and a wide range of hazard rate patterns, which is particularly useful in reliability, survival analysis, and Bayesian modeling, where conjugacy and tractable moments are highly valued. XGD harnesses the strengths of both exponential and gamma distributions by representing a finite mixture of an exponential distribution with rate θ and a gamma distribution with parameters $(3, \theta)$, with mixing proportions $\theta/(1 + \theta)$ and $1/(1 + \theta)$, respectively. The resulting PDF is

$$f_{\text{XGD}}(x) = \frac{\theta^2}{1+\theta}\left(1 + \frac{\theta x^2}{2}\right) e^{-\theta x}, \quad x > 0, \ \theta > 0, \tag{5}$$

and the CDF is

$$F_{\text{XGD}}(x) = 1 - \frac{1+\theta+\theta x+\frac{\theta^2 x^2}{2}}{1+\theta} e^{-\theta x}, \quad x > 0, \ \theta > 0. \tag{6}$$

Through this mixture construction, the Xgamma distribution retains the tractability and interpretability of its parent distributions while introducing additional flexibility in the shape of the density and hazard rate, making it an appealing model for positive continuous data in reliability, survival, and related fields. The foundational contribution of Sen et al. (2016) formally introduced the XGD, established its key statistical properties, and demonstrated its superiority over several competing models using real data applications in lifetime analysis. Subsequent research has developed a rich family of XGD-based models, including weighted Xgamma, quasi Xgamma, wrapped Xgamma, inverse Xgamma, and unit Xgamma distributions, each designed to address specific modeling needs such as different supports, more complex density shapes, or more flexible hazard rate patterns. More recent extensions and generalizations such as those proposed by Wani et al. (2022), Abulebda et al. (2022), Sen et al. (2024), Alomain et al. (2025), Tripathi et al. (2025), and Boudjerda (2025)—have further expanded the XGD family, providing additional parameters and transformation schemes, and consistently supporting their proposals through detailed theoretical investigations and applications to diverse real-life data sets that highlight the practical power and versatility of Xgamma-based modeling.

3. Different forms of XGD

Motivation is to developed new version of XGD to provide more flexible versions of XGD. In some cases is not a good fit for data or suited well to real situation to overcome this situation, there are various versions of XGD present in literature and make XGD more reliable for the use cases. These new versions allow the researcher to choose one among many good suited versions of XGD distribution. Many authors have taken XGD distribution as a baseline model and generate its different forms by using several transformations and families. All the recent developments of XGD are presented in following table [see, Table 1] along with year and author name.

***Table 1**: List of different forms of XGD*

S.No	Year	Author (Authors)	Model
1	2016	Sen et al.	Xgamma distribution
2	2017	Sen et al.	Weighted Xgamma distribution
3	2017	Sen and Chandra	Quasi Xgamma distribution
4	2018	Sen at al.	Quasi Xgamma Poisson distribution
5	2018	Altun and Hamedani	Log Xgamma distribution
6	2018	Maiti et al.	Discrete Xgamma distribution
7	2019	Hazem and Sen	Wrapped Xgamma distribution
8	2019	Yadav et al.	Inverse Xgamma distribution
9	2019	Sen et al.	Quasi Xgamma-Geometric distribution

10	2020	Cordeiro et al.	Xgamma family
11	2020	Yousof et al.	Xgamma Weibull distribution
12	2020	Bantan et al.	Half-logistic Xgamma distribution
13	2020	Hassan et al.	Lindley-Quasi Xgamma distribution
14	2020	Mazucheli et al.	Discrete Quasi Xgamma distribution
15	2020	Para et al.	Poisson Xgamma distribution
16	2021	Yadav et al.	Exponentiated Xgamma distribution
17	2021	Wani and Shafi	Weighted Lindley-Quasi Xgamma distribution
18	2021	Mohamed et al.	Three parameter Xgamma frechet distribution
19	2022	Shukla et al.	Alpha power transformed Xgamma distribution
20	2022	Tripathi et al.	Marshall-Olkin Xgamma distribution
21	2022	Tripathi and Mishra	Transmuted inverse Xgamma distribution
22	2022	Hashmi et al.	Unit Xgamma distribution
23	2022	Wani et al.	Size Baised Lindley-Quasi Xgamma distribution
24	2022	Abulebda et al.	Bivariate Xgamma Distribution
25	2022	Yadav et al.	Xgamma Exponential Distribution
26	2023	Yadav et al.	Weighted Xgamma Exponential Distribution
27	2024	Sen et al.	Truncated version of Xgamma distribution
28	2025	Alomair et al.	Improved Extension of Xgamma distribution
29	2025	Tripathi et al.	Generalized Inverse Xgamma distribution
30	2025	Boudjerda	New Power Xgamma Distribution

All mentioned models are used in survival studies, reliability theory, censoring scheme, truncation scheme, environmental data etc. It is important to discuss the CDF and hazard rate function (HRF) of each developed model or extension of XGD one by one in detail.

3.1 Weighted Xgamma distribution

Sen et al. (2017) chalked out weighted Xgamma distribution with the concept of weighted distribution. A general form of the weighted PDF is:

$$f(x) = \frac{w(x)f_0(x)}{E[w(x)]}$$

With the baseline PDF of XGD, CDF (G(x)) and HRF (H(x)) of weighted xgamma distribution are:

$$G(x) = \frac{2\eta}{\zeta![2\eta+(1+\zeta)(2+\zeta)]}\left[\gamma(\zeta+1,\eta x)+\frac{1}{2\eta}\gamma(\zeta+3,\eta x)\right]; x>0, \eta>0, \zeta=1,2,3.. \quad (7)$$

and

$$H(x) = \frac{\eta^{\zeta+1}\left(x^{\zeta}+\frac{\eta}{2}x^{\zeta+2}\right)e^{-\eta x}}{\left[\Gamma(\zeta+1,\eta x)+\frac{1}{2\eta}\Gamma(\zeta+3,\eta x)\right]} \quad (8)$$

Authors focused on special case of weighted xgamma distribution, for $\zeta = 1$ and it is formulated as Length biased Xgamma distribution (LBXD). Sen et al. (2017) mentioned that HRF of LBXD is log-concave and having the increasing failure rate nature. Graphical presentation of the HRF of the same placed in Figure 1.

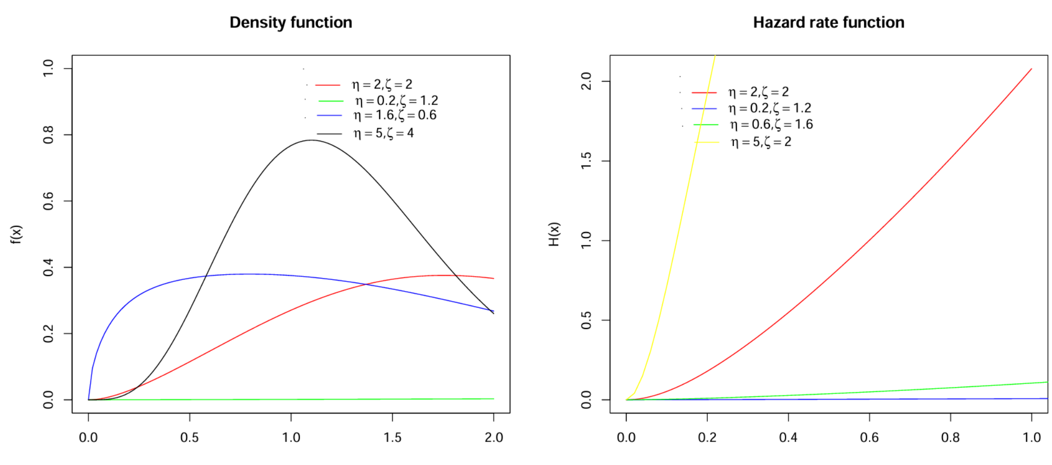

***Figure 1**: Weighted XGD*

3.2 Quasi Xgamma distribution

Sen and Chandra (2017) developed the extension of XGD and called this extension as Quasi Xgamma distribution. Authors have added one more parameter to XGD to make it more flexible. CDF and HRF of Quasi Xgamma distribution are given below:

$$G(x) = 1 - \frac{\left(1+\zeta+\eta x+\frac{\eta^2 x^2}{2}\right)}{(1+\zeta)} e^{-\eta x}, \quad x > 0,\ \eta > 0,\ \zeta > 0 \tag{9}$$

and

$$H(x) = \frac{\eta\left(\zeta+\frac{\eta^2}{2}x^2\right)}{1+\zeta+\eta x+\frac{\eta^2}{2}x^2}, \quad x > 0,\ \eta > 0,\ \zeta > 0 \tag{10}$$

,respectively. They explained the special cases of it along with the statistical properties. Also authors evaluated the mode of Quasi Xgamma distribution and the same displayed by using graphs. Graphical presentation of the HRF of the same placed in Figure 2.

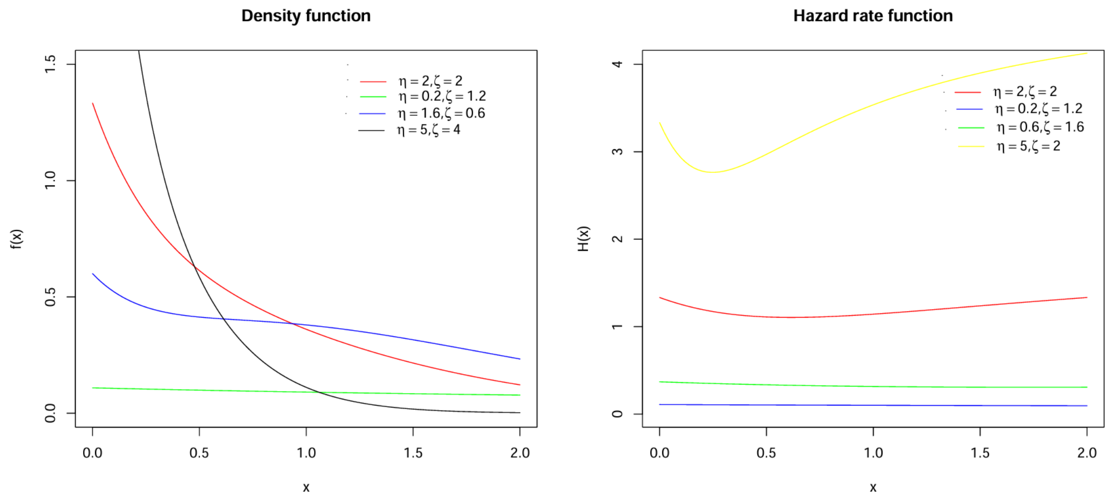

Figure 2: *Quasi XGD*

3.3 Quasi Xgamma Poisson distribution

Modeling of lifetime data can be done by Quasi Xgamma Poisson distribution. Need of Quai Xgamma Poisson distribution is established by Sen et al. (2018) and showed through real life data. Quasi Xgamma Poisson distribution coincide with different distributions like: Xgamma Poisson, Quasi Xgamma and Xgamma distribution. CDF and HRF are:

$$G(x) = \frac{e^{\lambda} - \exp\left(\frac{\lambda e^{-\eta x}}{(1+\zeta)}\left(\frac{\eta^2}{2}x^2+\eta x+\zeta+1\right)\right)}{e^{\lambda}-1}, \quad x > 0,\ \eta > 0,\ \zeta > 0,\ \lambda > 0 \tag{11}$$

and

$$H(x) = \frac{\frac{\lambda\eta}{(1+\zeta)}\left(\zeta+\frac{\eta^2}{2}x^2\right)\exp\left(\frac{\lambda e^{-\eta x}}{(1+\zeta)}\left(\frac{\eta^2}{2}x^2+\eta x+\zeta+1\right)-\eta x\right)}{\exp\left(\frac{\lambda e^{-\eta x}}{(1+\zeta)}\left(\frac{\eta^2}{2}x^2+\eta x+\zeta+1\right)-1\right)}, \quad x > 0,\ \eta > 0,\ \zeta > 0,\ \lambda > 0 \tag{12}$$

HRF of Quasi Xgamma Poisson distribution can take different shapes which makes it applicable for real life situation and the same illustrated through the graph for various parameters. Graphical presentation of the HRF of the same placed in Figure 3.

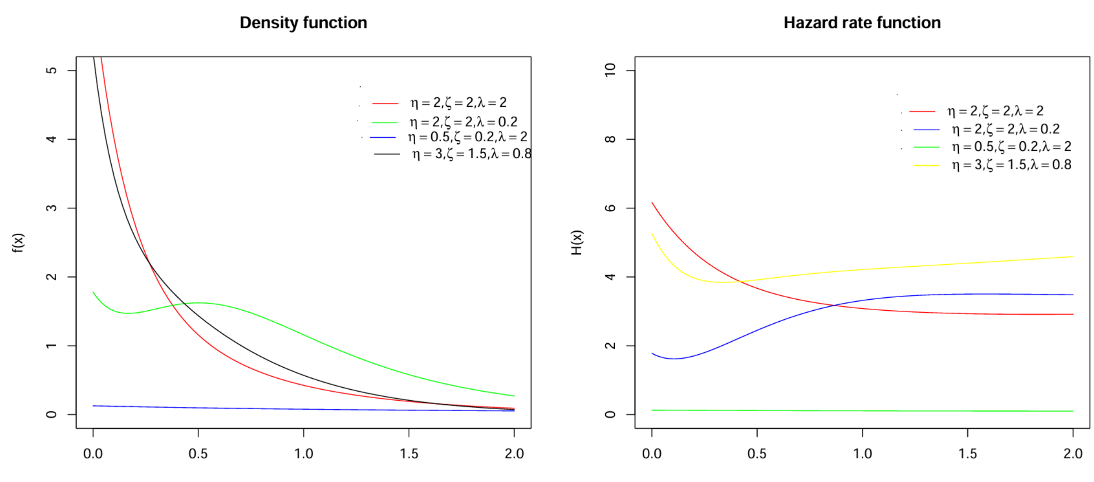

Figure 3: *Quasi XGD Poisson*

3.4 Log Xgamma distribution

Altun & Hamedani outline the Log Xgamma in 2018. Researchers are very much inclined to develop the probability distributions of unbounded support. Log Xgamma distribution is used in the situation where values are in bounded range such as percentage and proportions. CDF and HRF of the Log Xgamma are given below:

$$G(x) = x^{\eta}(\eta+1)^{-1}\left[1+\eta-\eta log(x)+\frac{\eta^2 log(x)^2}{2}\right]; 0<x<1, \eta>0 \quad (13)$$

and

$$H(x) = \frac{\frac{\eta^2}{1+\eta}\left(1+\frac{\eta}{2}log(x)^2\right)x^{\eta-1}}{1-x^{\eta}(\eta+1)^{-1}\left[1+\eta-\eta log(x)+\frac{\eta^2 log(x)^2}{2}\right]} \quad (14)$$

HRF of Log Xgamma distribution having increasing and bathtub shape property and to validate the same Altun & Hamedani (2018) used a data set those having the range of observation between 0 to 1. Graphical presentation of the HRF of the same placed in Figure 4.

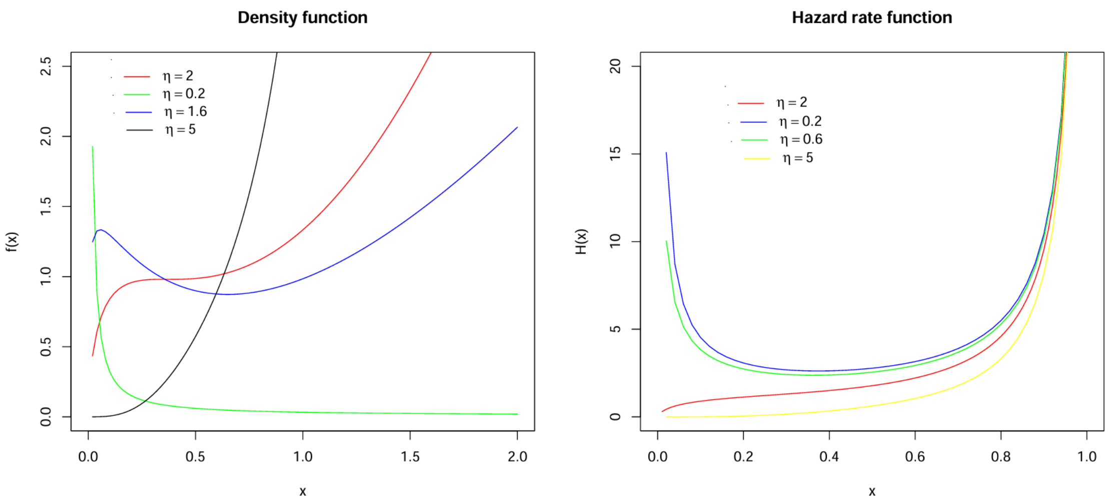

***Figure 4**: Log Xgamma distribution*

3.5 Discrete Xgamma distribution

Maiti et al. (2018) developed the Discrete Xgamma distribution, called as dxgamma-I to model the discrete real life situations and authors continued to strive to derive the statistical properties, estimation and real life application for the Discrete Xgamma distribution. CDF and HRF are:

$$G(x) = 1-\frac{1-\log p-\log p(x+1)+(\log(p))^2/2(x+1)^2}{1-\log(p)}p^{x+1}; x=0,1,2... \quad (15)$$

and

$$H(x) = \frac{[1+p-e^{p}(1+2p+p^2/2)+p(1-e^{-p}(1+p))x+(p^2)/2(1-e^{-p})x^2]\frac{e^{-px}}{1+p}}{\frac{1-\log(p)-\log(p)(x+1)+(\log(p))^2/2(x+1)^2}{1-\log(p)}p^{x+1}} \quad (16)$$

Graphical presentation of the probability mass function of the same placed in Figure 5.

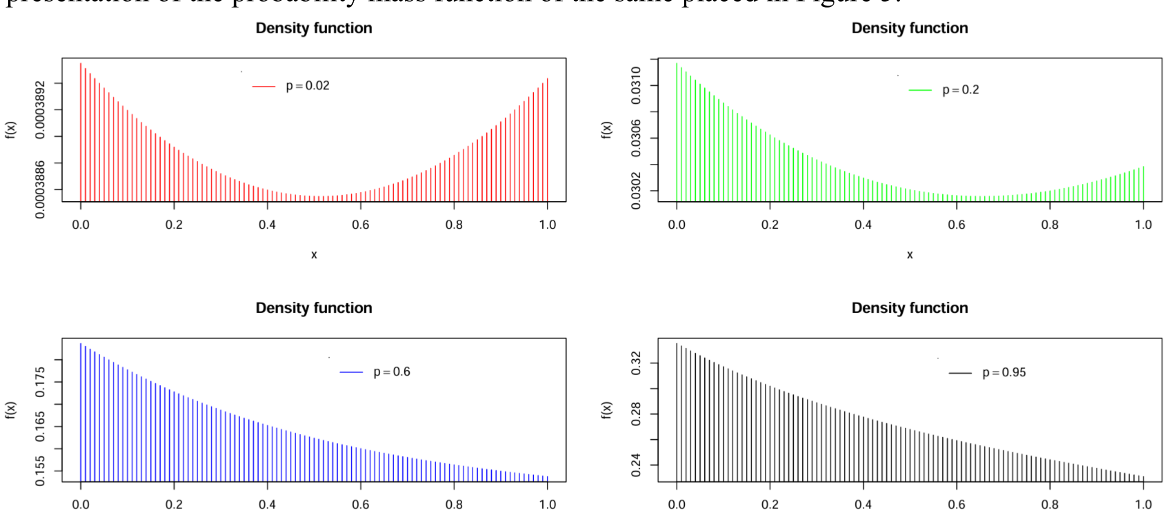

***Figure 5**: Discrete Xgamma distribution*

3.6 *Wrapped Xgamma distribution*

Hazem and Sen (2019) described the circular version of XGD and denoted as Wrapped Xgamma distribution. Wrapped xgamma distribution is very useful to model the circular data. CDF and HRF are:

$$G(x)=\left(1-\frac{1+\lambda(1+\eta)+\frac{\eta^2\lambda^2}{2}}{1+\lambda}e^{-\eta\lambda}\right)\frac{1}{1-e^{-2\pi\lambda}}+\frac{2\pi\lambda}{\lambda+1}(1-(1+\eta\lambda))e^{-\eta\lambda}\frac{e^{-2\pi\lambda}}{(1-e^{-2\pi\lambda})^2}$$
$$+\frac{2\pi^2\lambda^2}{\lambda+1}(1-e^{-\eta\lambda})\frac{e^{-2\pi\lambda}(1+e^{-2\pi\lambda})}{(1-e^{-2\pi\lambda})^3};0\le x<2\pi,\lambda>0$$

and

$$H(x)=\frac{\frac{\lambda^2e^{-\eta\lambda}}{(\lambda+1)(1-e^{-2\pi\lambda})}\left(1+\frac{\lambda\eta^2}{2}+2\pi\lambda[(\pi-\eta)e^{-2\pi\lambda}+(\eta+\pi)]\frac{e^{-2\pi\lambda}}{(1-e^{-2\pi\lambda})^2}\right)}{1-\left(1-\frac{1+\lambda(1+\eta)+\frac{\eta^2\lambda^2}{2}}{1+\lambda}e^{-\eta\lambda}\right)\frac{1}{1-e^{-2\pi\lambda}}+U_1+\frac{2\pi^2\lambda^2}{\lambda+1}(1-e^{-\eta\lambda})\frac{e^{-2\pi\lambda}(1+e^{-2\pi\lambda})}{(1-e^{-2\pi\lambda})^3};0\le x<2\pi,\lambda>0} \quad (17)$$

where, $U_1=\frac{2\pi\lambda}{\lambda+1}(1-(1+\eta\lambda))e^{-\eta\lambda}\frac{e^{-2\pi\lambda}}{(1-e^{-2\pi\lambda})^2}$

Measurements in direction is common in science and real life data observations. The popular encounter of such data sets might be related to direction of flight of a bird, orientation of certain animals, direction of magnetic field in a place, etc., could be two dimensional or three dimensional depending on the nature of measurements. To support the behavior of HRF, authors have used glaciologist data which mathched to HRF of Wrapped Xgamma distribution.

3.7 *Inverse Xgamma distribution*

Inverse transformation is used on XGD to develop the Inverse Xgamma distribution and it is developed by Yadav et al. (2019). Following are the CDF and HRF are:

$$G(x)=\left[1+\frac{\eta}{\eta+1}\frac{1}{x}+\frac{\eta^2}{2(1+\eta)}\frac{1}{x^2}\right]e^{-\eta/x};x>0,\eta>0 \quad (18)$$

and

$$H(x)=\frac{\frac{\eta^2}{1+\eta}\frac{1}{x^2}\left(1+\frac{\eta}{2x^2}\right)e^{-\eta/x}}{1-\left[1+\frac{\eta}{\eta+1}\frac{1}{x}+\frac{\eta^2}{2(1+\eta)}\frac{1}{x^2}\right]e^{-\eta/x}} \quad (19)$$

Yadav et al. (2018) plotted the HRF for various η and found that found that it is suitable for non monotone failure pattern which often occurs in clinical trials and reliability studies. Graphical presentation of the HRF of the same placed in Figure 6.

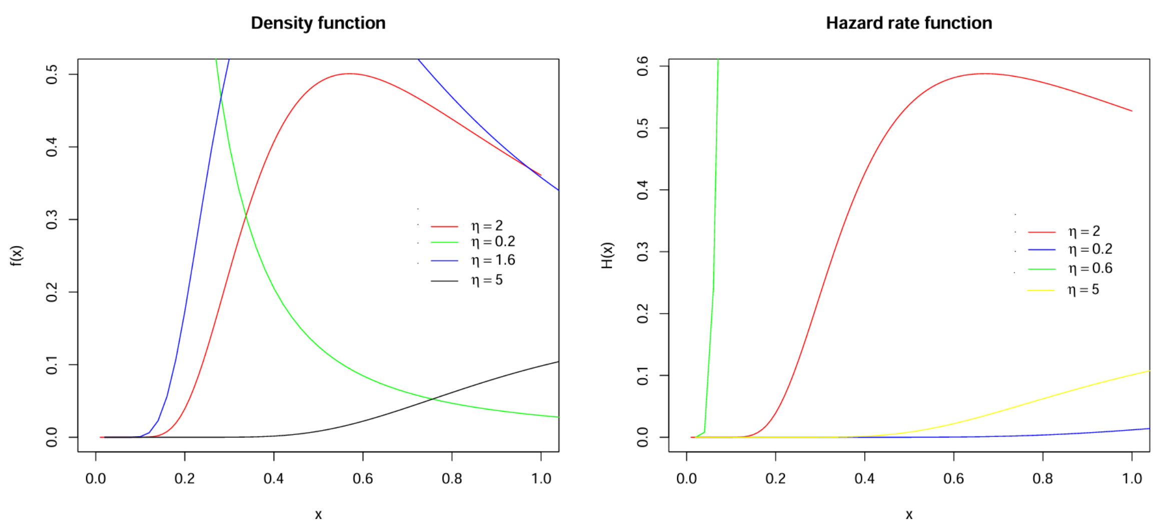

***Figure 6**: Inverse Xgamma distribution*

3.8 *Quasi Xgamma-Geometric distribution*

It is introduced by Sen et al. (2019) and Quasi Xgamma-Geometric distribution is the result of applying compound technique geometric distribution. CDF and HRF of Quasi Xgamma-Geometric distribution are:

$$G(x) = \left[1 - \frac{e^{-\theta x}\left(1+\alpha+\theta x+\frac{\theta^2}{2}x^2\right)}{(1+\alpha)}\right]\left[1 - \frac{pe^{-\theta x}\left(1+\alpha+\theta x+\frac{\theta^2}{2}x^2\right)}{(1+\alpha)}\right]^{-1},$$
$$x > 0, \theta > 0, \alpha > 0. \tag{20}$$

and

$$H(x) = \frac{\theta\left(\alpha+\frac{\theta^2}{2}x^2\right)}{\left(1+\alpha+\theta x+\frac{\theta^2}{2}x^2\right)}\left[1 - \frac{pe^{-\theta x}\left(1+\alpha+\theta x+\frac{\theta^2}{2}x^2\right)^{-1}}{(1+\alpha)}\right]^{-1} \tag{21}$$

The HRF of the Quasi Xgamma-Geometric distribution can be constant, decreasing, increasing, decreasing-increasing, upside down bathtub or bathtub failure rate shapes. Graphical presentation of the HRF of the same placed in Figure 7.

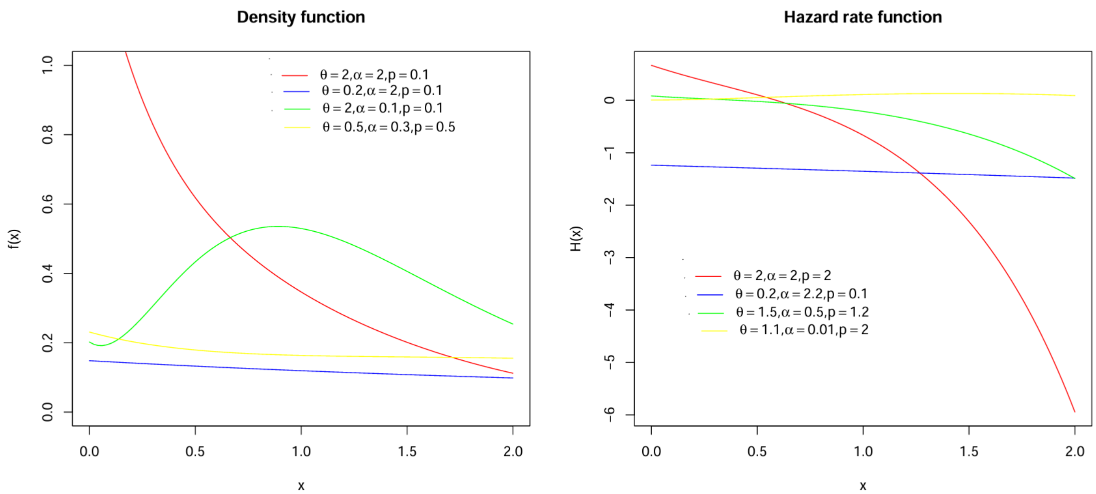

***Figure** 7: Quasi Xgamma-Geometric distribution*

3.9 Xgamma family

Cordeiro et al. (2020) described Xgamma family and developed the censored regression modelling. They introduced the special submodels like Xgamma lindley distribution, Xgamma weibull distribution and Xgamma Burr XII distribution. Glass fiber data, Diabetic retinopathy study and Leukemia data are used to prove the applicability of special submodels of Xgamma family and made the point that HRF of several submodels of Xgamma family are coincide with the real life situations.

3.10 Xgamma Weibull distribution

A new extension of weibull distribution is derived and named as Xgamma Weibull distributions [see, Yousof et al. (2020)] and it can be obtained by just doing the integration of Xgamma distribution with range 0 to $-\log[1 - G_W(x; b)]$. CDF and HRF are:

$$G(x) = 1 - \left[1 + \eta + \eta x^{\lambda} + \frac{\eta^2 x^{2\lambda}}{2}\right]\frac{e^{-\eta x^{\lambda}}}{1+\eta}; x > 0, \lambda > 0, \eta > 0 \tag{22}$$

and

$$H(x) = \frac{\frac{\lambda\eta^2}{1+\eta}x^{\lambda-1}e^{-\eta x^{\lambda}}\left(1+\frac{\eta x^{2\lambda}}{2}\right)}{\left[1+\eta+\eta x^{\lambda}+\frac{\eta^2 x^{2\lambda}}{2}\right]\frac{e^{-\eta x^{\lambda}}}{1+\eta}} \tag{23}$$

Increasing, decreasing and bathtub shape HRF can be accommodate with implementation of Xgamma Weibull distribution and the same trends of HRF are seen into the used data of manufacturing field. Graphical presentation of the HRF of the same placed in Figure 8.

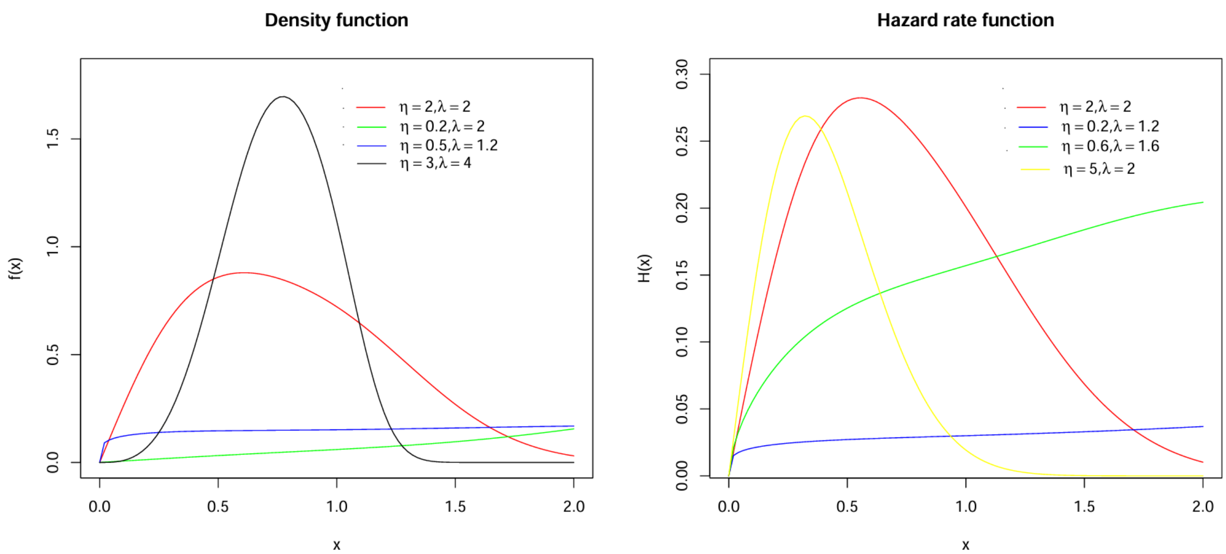

***Figure 8**: Xgamma Weibull distribution*

3.11 Half-logistic Xgamma distribution

Bantan et al. (2020) provided the flexible version of XGD by introducing one more parameter and called as Half-Logistic Xgamma distribution. To model the right skewed, uni-modal and J shaped data and CDF, HRF of its are:

$$G(x) = 2[1-\mathcal{A}(x,\eta)]^{\lambda}(1+[1-\mathcal{A}(x,\eta)]^{\lambda})^{-1}; x>0, \eta>0, \lambda>0 \tag{24}$$

and

$$H(x) = \frac{2\lambda\eta^2(1+\eta x^2/2)e^{-\eta x}[1-\mathcal{A}(x,\eta)]^{\lambda-1}/(1+\eta)(1+[1-\mathcal{A}(x,\eta)]^{\lambda})^2}{1-2[1-\mathcal{A}(x,\eta)]^{\lambda}(1+[1-\mathcal{A}(x,\eta)]^{\lambda})^{-1}} \tag{25}$$

where, $\mathcal{A}(x,\eta) = (1+\eta+\eta x+\eta^2x^2/2/(1+\eta))e^{-\eta x}$.

HRF of Half-Logistic Xgamma distribution is able to capture the increasing, decreasing and semibathtub hazard rate scenarios, authors have shown these trends of HRF with the help of three different data sets of the manufacturing, medical field. Graphical presentation of the HRF of the same placed in Figure 9.

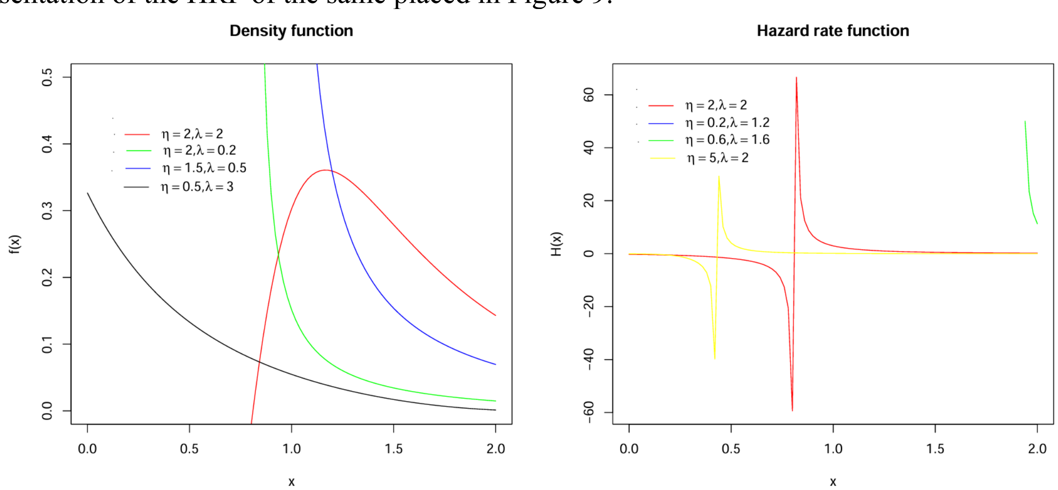

***Figure 9**: Half-logistic Xgamma distribution*

3.12 Lindley-Quasi Xgamma distribution

Hassan et al. (2020) published a paper introduced a new distribution, Lindley-Quasi Xgamma distribution. Authors dealt with properties, estimation and application of the Lindley-Quasi Xgamma distribution. CDF, HRF are:

$$G(x) = \frac{1}{2(\eta+\lambda)^2}[U_2], \quad x>0,\ \eta>0,\ \lambda>0 \tag{26}$$

and

$$H(x) = \frac{2\eta e^{-\eta x}\{(\lambda+\eta)\left(\lambda+\frac{x^2\eta^2}{2}\right)+\eta(\eta-1)(1+\lambda x)\}}{1-\frac{1}{2(\eta+\lambda)^2}[U_2]} \tag{27}$$

Where, $U_2 = (\lambda+\eta)\{2\lambda+2-(2\lambda+x^2\eta^2+2\eta x+2)e^{-\eta x}\}+2(\eta-1)\{\eta+\lambda-(\eta+\lambda\eta x+\lambda)e^{-\eta x}\}$ To justify the HRF, authors have used two data sets from the reccuring time of kidney infection and flood peaks of wheaten river. Graphical presentation of the HRF of the same placed in Figure 10.

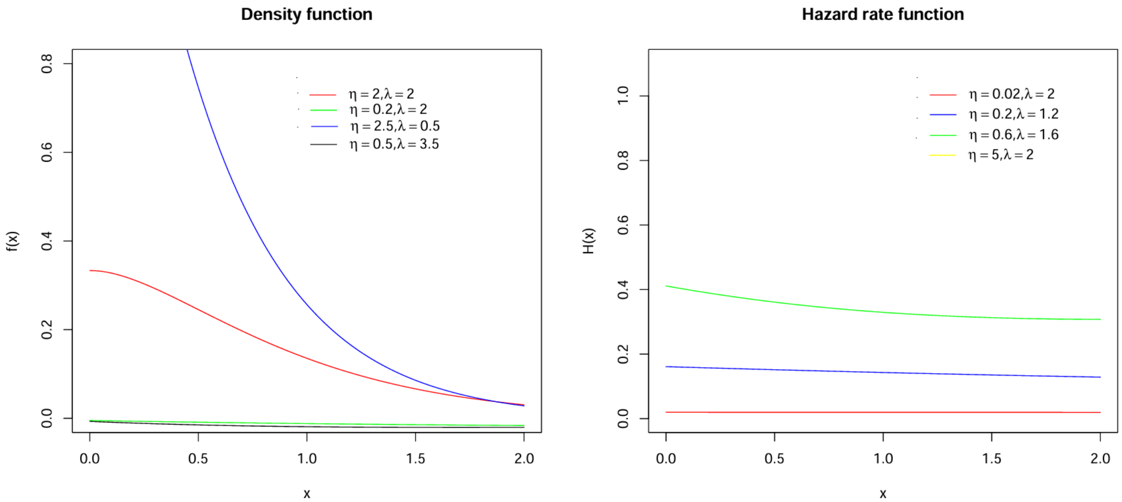

Figure 10: *Lindley-Quasi Xgamma distribution*

3.*13 Discrete Quasi Xgamma distribution*

It is discretization of XGD and developed by Mazucheli et al. (2019). Authors proposed alternatives to model under and over dispersed datasets. Authors provided the probability mass function (PMF) when x is continuous random variable, following is the PMF for the support (−inf, +inf) and (0,inf) of x random variable.

$$PMF1 = \frac{f_X(y,\eta)}{\sum_{j=-\infty}^{\infty} f_X(j,\eta)}; y \in Z \tag{28}$$

and

$$PMF2 = \frac{f_X(y,\eta)}{\sum_{j=0}^{\infty} f_X(j,\eta)}; y \in Z+ \tag{29}$$

where y is continuous random variable. Authors derived both type of PMF for different range and computed several statistical properties with applications of both. HRF suitability for real life scenario shown through two real life examples: Corn Borers data and outbreaks of strikes.

3.14 Poisson Xgamma distribution

Para et al. (2020) showed the importance of Poisson Xgamma distribution and it is a discrete model for count data. Authors showed that it beats so many known old and new discrete distributions. CDF and HRF are:

$$G(x) = \frac{\eta^2+(\eta x)^2/2+5\eta x^2/2+5\eta^2+x\eta+4\eta+1}{(1+\eta)^{x+4}}; x = 0,1,2,3\ldots, \eta > 0 \tag{30}$$

and

$$H(x) = \frac{\frac{\eta^2}{2(1+\eta)^{x+4}}[2(1+\eta)^2+\eta(x+1)(x+2)]}{1-\frac{\eta^2+(\eta x)^2/2+5\eta x^2/2+5\eta^2+x\eta+4\eta+1}{(1+\eta)^{x+4}}} \tag{31}$$

Two data sets: epileptic seizure counts and accidents to 647 women working on high explosive shells in 5 weeks are the representation of the matching the HRF of Poisson Xgamma distribution to the real life situation. Graphical presentation of the probability mass function of the same placed in Figure 11.

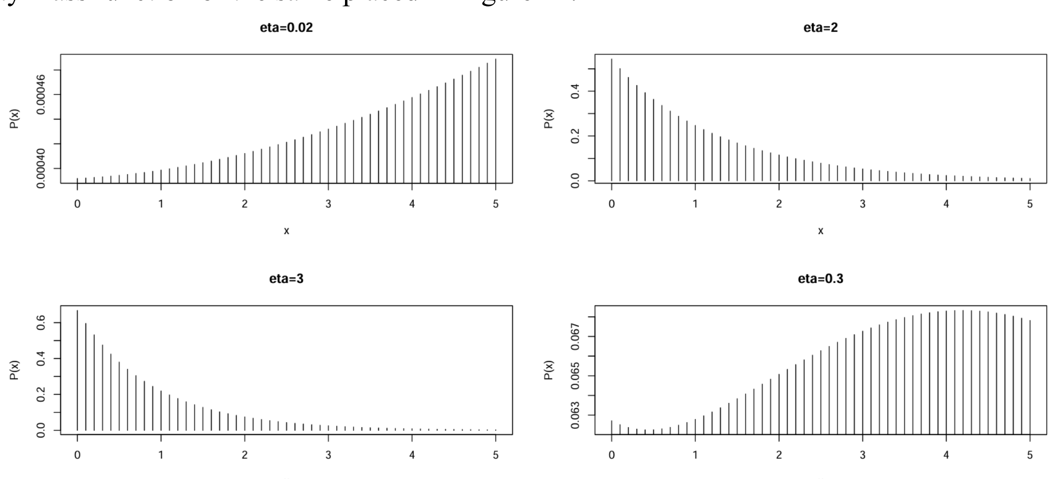

Figure 11: *Poisson Xgamma distribution*

3.15 Exponentiated Xgamma distribution

Exponentiated version of Xgamma distribution is introduced by Yadav et al. (2021) and proved that this new version of XGD is very useful in some real situations. CDF and HRF are:

$$G(x) = \left[1 - \frac{\left(1+\eta+\eta x+\frac{\eta^2x^2}{2}\right)}{(1+\eta)}e^{-\eta x}\right]^{\lambda} \; ; x > 0, \eta > 0, \lambda > 0 \tag{32}$$

and

$$H(x) = \frac{\frac{\lambda\eta^2}{1+\eta}\left[1-\left(1+\eta+\eta x+\frac{\eta^2x^2}{2}\right)\frac{e^{-\eta x}}{1+\eta}\right]^{(\lambda-1)}\left(1+\frac{\eta x^2}{2}\right)e^{-\eta x}}{1-\left[1-\frac{\left(1+\eta+\eta x+\frac{\eta^2x^2}{2}\right)}{(1+\eta)}e^{-\eta x}\right]^{\lambda}} \tag{33}$$

For $\lambda \geq 1, \eta \geq 1$, it follows the pattern of increasing failure rate, decreasing failure rate when $\lambda < 1, \eta < 1$ and the pattern of bathtub shaped hazard rate may be traced for $\lambda < 1, \eta < 1$. Graphical presentation of the HRF of the same placed in Figure 12.

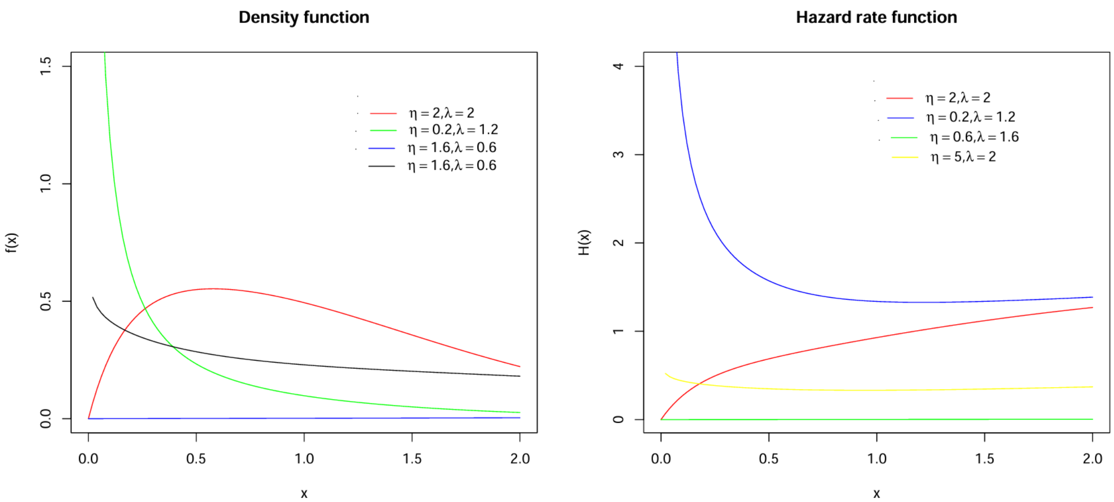

***Figure 12**: Exponentiated Xgamma distribution*

3.16 Weighted Lindley-Quasi Xgamma distribution

Wani and Shafi (2021) developed the Weighted Lindley-Quasi Xgamma distribution and applied the model to on two data sets. CDF and HRF are:

$$G(x) = \frac{(\eta+\lambda)2\eta(\gamma(c+1,\eta x)+\gamma(c+3,\eta x))+2(\eta-1)(\eta\gamma(c+1,\eta x)+\lambda\gamma(c+2,\eta x))}{c!(\eta+\lambda)(2\lambda+(c+1)(c+2))+2(\eta-1)((\eta+\lambda(c+1)))}; \quad x > 0, \eta > 0, \lambda > 0 \tag{34}$$

and

$$H(x) = \left(\frac{\eta^{c+1}x^c((\lambda+\eta)(2\lambda+x^2\eta^2)+2\eta(\eta-1)(1+\lambda x))e^{-\lambda x}}{U_6-U_7}\right)$$

Where, U_6 is $c!\ ((\eta+\lambda)(2\lambda+(c+1)(c+2))+2(\eta-1)(\eta+\lambda(c+1)))$ and U_7 is $((\lambda+\eta)(2\lambda\gamma(c+1,\eta x)+\gamma(c+3,\eta x))+2(\eta-1)(\eta\gamma(c+1,\eta x)+\lambda\gamma(c+2,\eta x)))$.

Hazard rate of it, revealed that model possesses non-decreasing hazard rate and it can be also seen that hazard rate becomes constant as value of x increases. Graphical presentation of the HRF of the same placed in Figure 13.

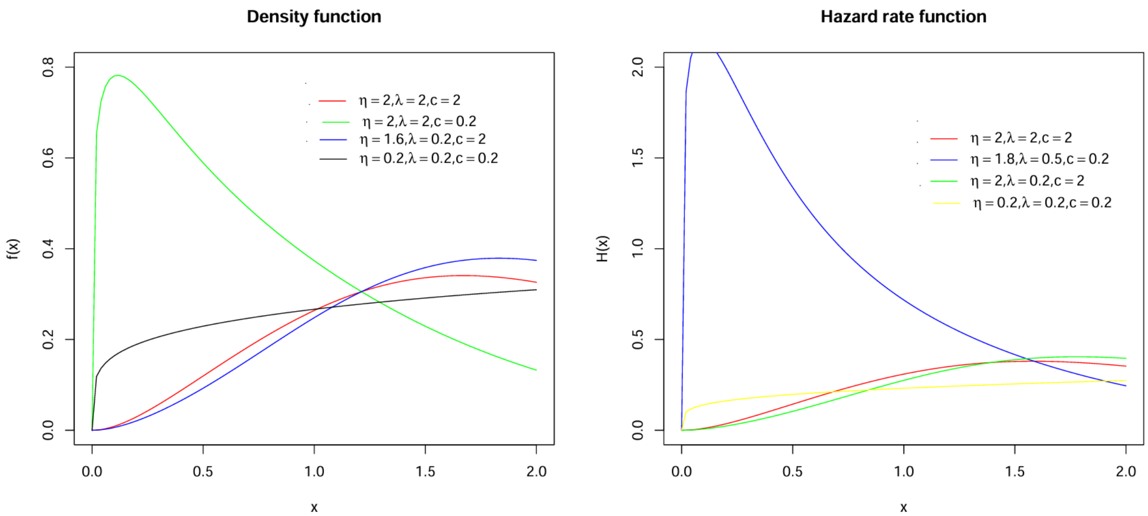

***Figure 13**: Weighted Lindley-Quasi Xgamma distribution*

3.17 Three Parameter Xgamma frechet distribution

Ibrahim et al. (2021) derived Three Parameter Xgamma frechet distribution and computed statistical properties of it. CDF and HRF are:

$$G(x)=\left[1-\frac{1}{1+\eta}\left(1-e^{-\left(\frac{a}{x}\right)^{b}}\right)^{\eta}\left(1+\eta-\eta\log\left[1-e^{-\left(\frac{a}{x}\right)^{b}}\right]+\frac{1}{2}\eta^{2}\left\{\log\left[1-e^{-\left(\frac{a}{x}\right)^{b}}\right]\right\}^{2}\right)\right],$$
$$x>0,\eta>0,a>0,b>0. \tag{35}$$

and

$$H(x)=\frac{\frac{\eta}{1+\eta}ba^{b}x^{-(b+1)}e^{-\left(\frac{a}{x}\right)^{b}}\left[1-e^{-\left(\frac{a}{x}\right)^{b}}\right]^{\eta-1}\left(\eta+\frac{1}{2}\eta^{2}\left\{\log\left[1-e^{-\left(\frac{a}{x}\right)^{b}}\right]\right\}^{2}\right)}{1-\left[1-\frac{1}{1+\eta}\left(1-e^{-\left(\frac{a}{x}\right)^{b}}\right)^{\eta}\left(1+\eta-\eta\log\left[1-e^{-\left(\frac{a}{x}\right)^{b}}\right]+\frac{1}{2}\eta^{2}\left\{\log\left[1-e^{-\left(\frac{a}{x}\right)^{b}}\right]\right\}^{2}\right)\right]} \tag{36}$$

Authors used real data sets to prove the validity of developed model as literature full with new models. Considered data set for this model are from the manufacturing and medical field and they showed that HRF is monotonically increasing. Graphical presentation of the HRF of the same placed in Figure 14.

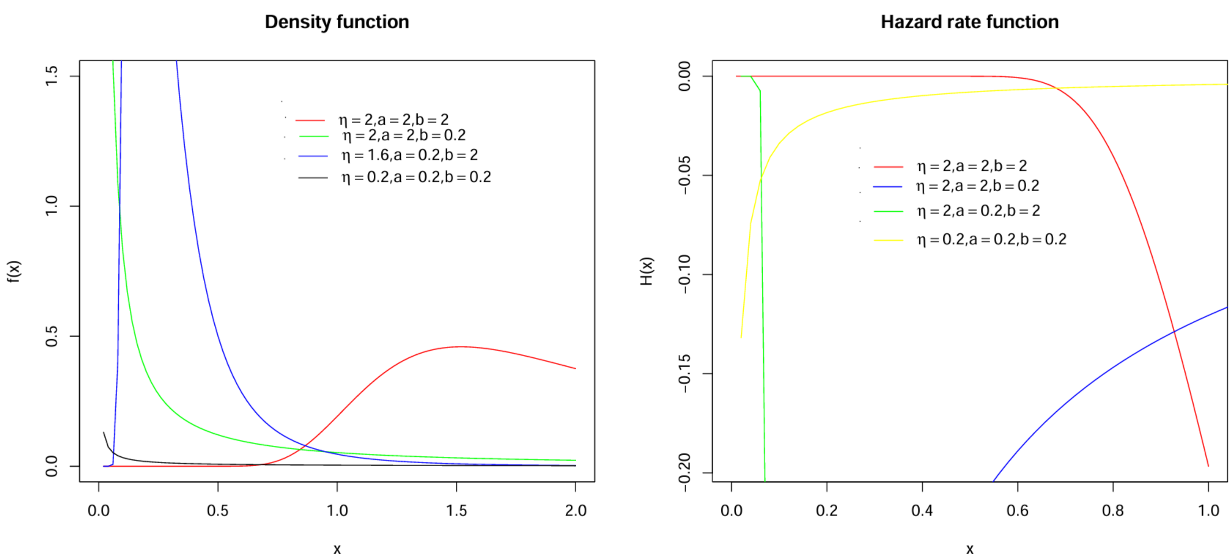

***Figure 14**: Three Parameter Xgamma Frechet distribution*

3.18 Alpha Power Transformed Xgamma distribution

Shukla et al. (2022) introduced two parameter XGD and made tis flexible version of XGD and called it as Alpha Power Transformed Xgamma distribution. CDF and HRF are:

$$G(x)=\begin{cases}\frac{\lambda^{1-u_0}-1}{\lambda-1};\ \lambda>0,\lambda\neq 1\\ 1-u_0;\ \alpha=1\end{cases} \tag{37}$$

Where, $u_0 = \frac{\left(1+\eta+\eta x+\frac{\eta^2 x^2}{2}\right)}{(1+\eta)} e^{-\eta x}$

$$H_{APTXGD}(x) = \begin{cases} \dfrac{\frac{\log\lambda}{\lambda-1}\frac{\eta^2}{(1+\eta)} u_{01}\lambda^{1-u_0}}{1-\left[\frac{\lambda^{1-u_0}-1}{\lambda-1}\right]} \\ ;\lambda > 0, \lambda \neq 1 \\ \dfrac{\frac{\eta^2}{(1+\eta)}\left(1+\frac{\eta}{2}x^2\right)e^{-\eta x}}{1-[1-u_0]} ;\alpha = 1 \end{cases} \tag{38}$$

where, $u_{01} = \left(1+\frac{\eta}{2}x^2\right)e^{-\eta x}$. Authors found that HRF takes increasing and decreasing shape by using Glaser (1980) description about it. Authors justified the application of this by model four data sets of different field. Graphical presentation of the HRF of the same placed in Figure 15.

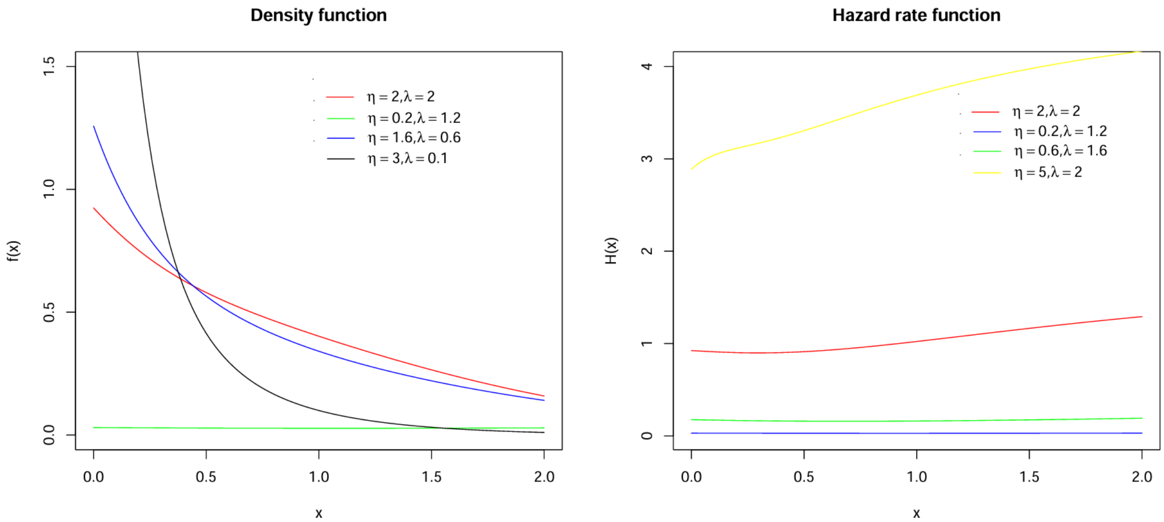

***Figure 15**: Alpha Power Transformed Xgamma distribution*

3.19 Flexible extension of Xgamma distribution

New extension of XGD based on Marshall-Olkin family of distribution is developed by Tripathi et al. (2022). They mentioned the different statistical properties of it. CDF and HRF are:

$$G(x) = 1 - \frac{\lambda^{\frac{\left(1+\eta+\eta x+\frac{\eta^2x^2}{2}\right)}{(1+\eta)}e^{-\eta x}}}{\left[1-\bar{\lambda}^{\frac{\left(1+\eta+\eta x+\frac{\eta^2x^2}{2}\right)}{(1+\eta)}e^{-\eta x}}\right]} \tag{39}$$

and

$$H(x) = \frac{\frac{\lambda\eta^2}{(1+\eta)}\frac{\left(1+\frac{\eta}{2}x^2\right)e^{-\eta x}}{\left[1-\bar{\lambda}^{\frac{\left(1+\eta+\eta x+\frac{\eta^2x^2}{2}\right)}{(1+\eta)}e^{-\eta x}}\right]^2}}{1-\frac{\lambda^{\frac{\left(1+\eta+\eta x+\frac{\eta^2x^2}{2}\right)}{(1+\eta)}e^{-\eta x}}}{\left[1-\bar{\lambda}^{\frac{\left(1+\eta+\eta x+\frac{\eta^2x^2}{2}\right)}{(1+\eta)}e^{-\eta x}}\right]}} \tag{40}$$

HRF of it can take every possible shape, i.e, increasing, decreasing and bathtub shape for the chosen value of the parameters. Authors used four real data sets to explore the areas of applications of it. Graphical presentation of the HRF of the same placed in Figure 16.

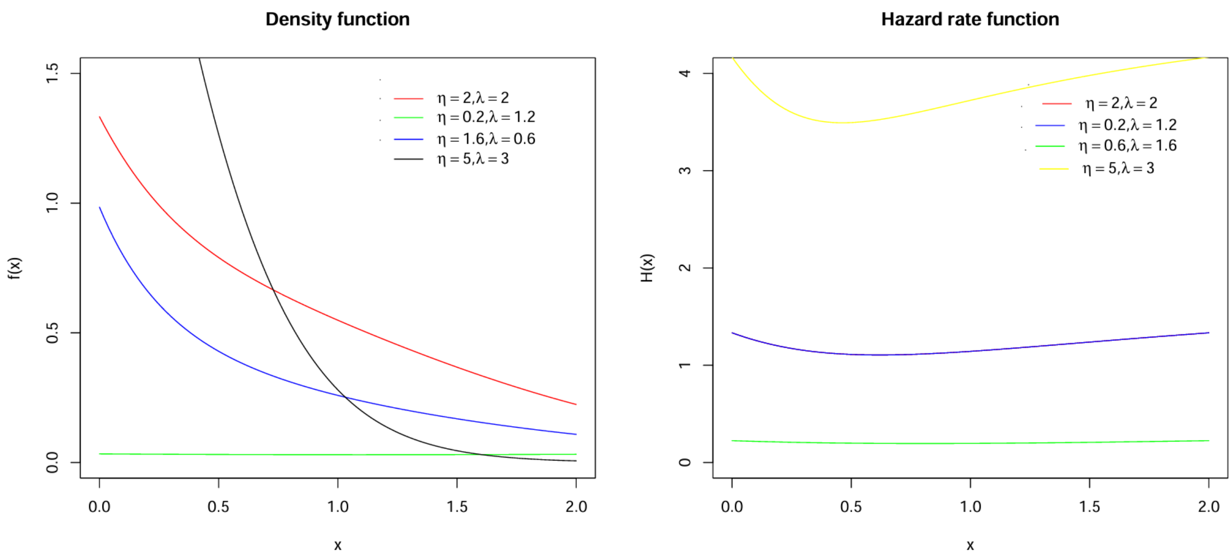

***Figure 16**: Flexible extension of Xgamma distribution*

3.20 Transmuted Inverse Xgamma distribution

Tripathi and Mishra (2022) developed the transmuted version of inverse xgamma distribution and they derived some of the properties for Transmuted Inverse Xgamma distribution. CDf and HRF are:

$$G(x) = (1+\lambda)\left[1+\frac{\eta}{\eta+1}\frac{1}{x}+\frac{\eta^2}{2(1+\eta)}\frac{1}{x^2}\right]e^{-\eta/x} - \lambda\left(\left[1+\frac{\eta}{\eta+1}\frac{1}{x}+\frac{\eta^2}{2(1+\eta)}\frac{1}{x^2}\right]e^{-\eta/x}\right)^2;\quad x>0, \eta>0, |\lambda|\le 1 \tag{41}$$

and

$$H(x) = \frac{\left(1+\lambda-2\lambda\left[1+\frac{\eta}{\eta+1}\frac{1}{x}+\frac{\eta^2}{2(1+\eta)}\frac{1}{x^2}\right]e^{-\eta/x}\right)\frac{\eta^2}{1+\eta}\frac{1}{x^2}\left(1+\frac{\eta}{2x^2}\right)e^{-\eta x}}{1-(1+\lambda)\left[1+\frac{\eta}{\eta+1}\frac{1}{x}+\frac{\eta^2}{2(1+\eta)}\frac{1}{x^2}\right]e^{-\eta/x}+\lambda\left(\left[1+\frac{\eta}{\eta+1}\frac{1}{x}+\frac{\eta^2}{2(1+\eta)}\frac{1}{x^2}\right]e^{-\eta/x}\right)^2} \tag{42}$$

This model is applicable to the situation where researchers having keen interest to apply the proposed model for modeling of the data with increasing and deacreasing trend of HRF. Graphical presentation of the HRF of the same placed in Figure 17.

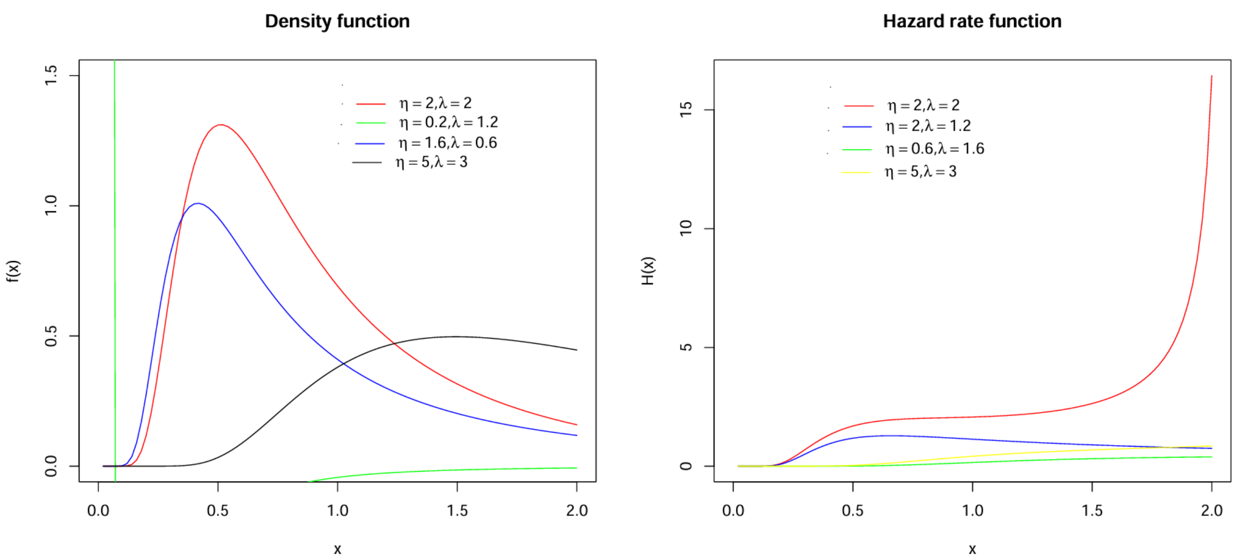

***Figure 17**: Transmuted Inverse Xgamma distribution*

3.21 Unit Xgamma distribution

Hashmi et al. (2022) presented a evolved version of XGD, called as unit Xgamma distribution. Unit Xgamma distribution is based on unit interval and it is good for modeling of such data sets. CDF and HRF are:

$$G(x) = 1-\frac{1+\eta+\eta\left(\frac{x}{1-x}\right)+\frac{\eta^2}{2}\left(\frac{x}{1-x}\right)^2}{1+\eta}\,e^{-\eta\left(\frac{x}{1-x}\right)},\quad 0<x<1, \eta>0 \tag{43}$$

and

$$H(x) = \frac{\eta^2\left\{1+\frac{x^2\eta}{2(1-x)^2}\right\}}{(1-x)^2\left[1+\eta+\frac{x\eta}{1-x}+\frac{x^2\eta^2}{2(1-x)^2}\right]} \tag{44}$$

It is suitable for negatively skewed data and analyze the data having increasing hazard rate function. Also to support the real life scenario which can be model by using Unit Xgamma distribution, authors used real data set is used for the validation of the same. Graphical presentation of the HRF of the same placed in Figure 18.

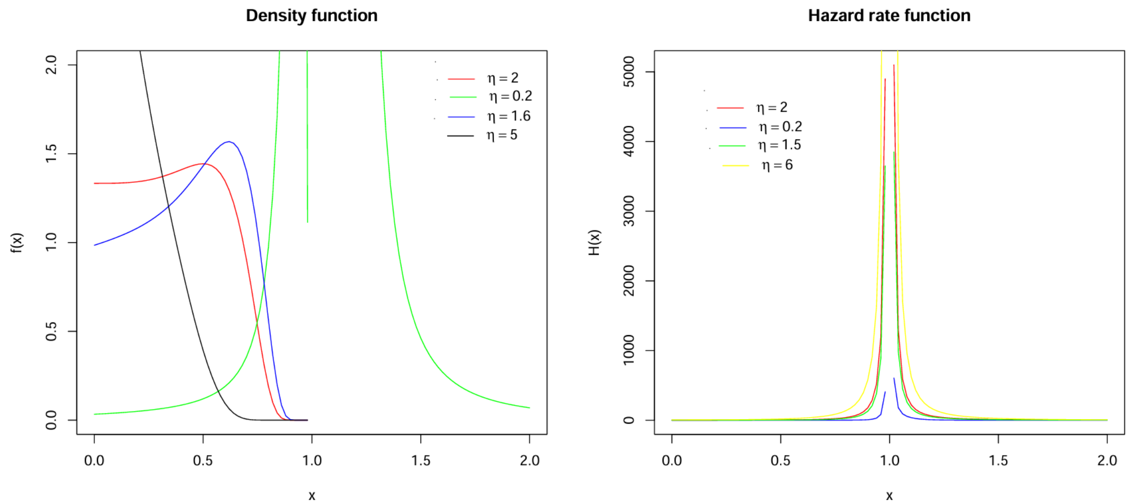

***Figure 18**: Unit Xgamma distribution*

3.22 Size Biased Lindley-Quasi Xgamma distribution

Wani et al. (2022) introduced Size Biased Lindley-Quasi Xgamma distribution and applied it to survival times data. CDF and HRF are:

$$G(x) = 1 - \left[1 + \frac{(2(\eta-1)(\eta+2\lambda+x\eta\lambda)+(\eta+\lambda)(2\lambda+6+3x\eta+x^2\eta^2))\eta x}{2(\eta^2+\lambda^2+3\eta\lambda+2\eta+\lambda)}\right] e^{-\eta x};$$
$$x > 0, \eta > 0, \lambda > 0 \tag{45}$$

and

$$H(x) = \frac{2\eta^2(\eta(\eta-1)(x+\lambda x^2)+(\eta+\lambda)(\lambda x+\frac{\eta^2 x^3}{2}))e^{-\eta x}}{[U_4+(2(\eta-1)(\eta+2\lambda+x\eta\lambda)+(\eta+\lambda)(2\lambda+6+3x\eta+x^2\eta^2))\eta x]e^{-\eta x}} \tag{46}$$

where, U_4 is $2(\eta^2+\lambda^2+3\lambda\eta+2\eta+\lambda)$.

Developed model have non -decreasing hazard rate function and authors proved the necessity of this model over other models when HRF is decreasing in real life phenomena. Graphical presentation of the HRF of the same placed in Figure 19.

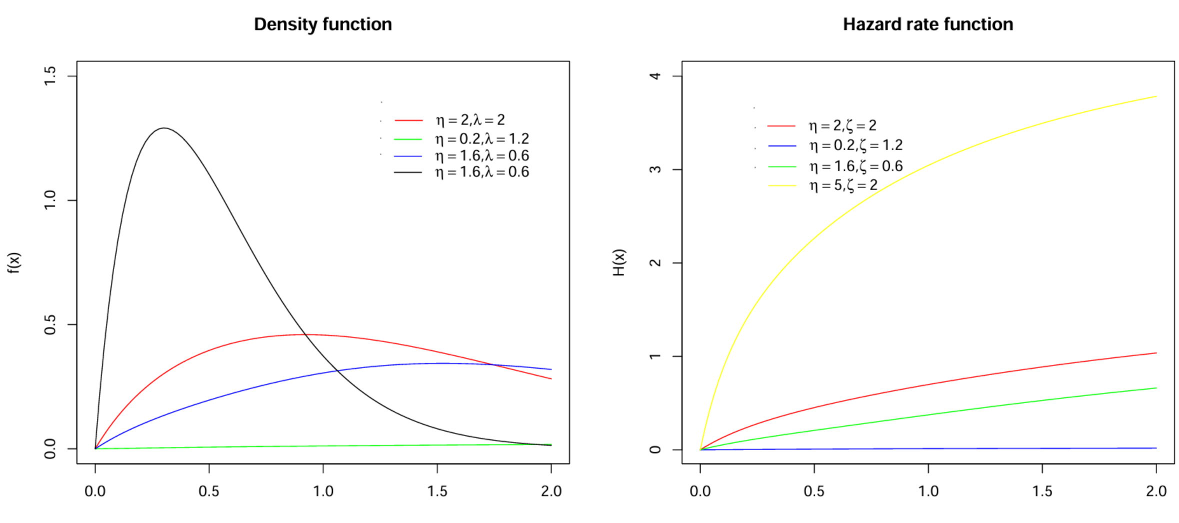

***Figure 19**: Size Biased Lindley Quasi Xgamma distribution*

3.23 Bivariate Xgamma distribution

Abulebda et al. (2022) developed Bivariate Xgamma distribution. Authors used FGM coupla to develop bivariate version of XGD. CDF and HRF are:

$$G(x_1,x_2) = U_5\left[1+\delta\left(\frac{\left(1+\eta_1+\eta_1 x_1\frac{\eta_1^2 x_1^2}{2}\right)}{(1+\eta_1)}e^{-\eta_1 x_1}\right)\left(\frac{\left(1+\eta_2+\eta_2 x_2\frac{\eta_2^2 x_2^2}{2}\right)}{(1+\eta_2)}e^{-\eta_2 x_2}\right)\right] \quad (47)$$

where, U_5 is $\left[1-\frac{\left(1+\eta_1+\eta_1 x_1\frac{\eta_1^2 x_1^2}{2}\right)}{(1+\eta_1)}e^{-\eta_1 x_1}\right]\left[1-\frac{\left(1+\eta_2+\eta_2 x_2\frac{\eta_2^2 x_2^2}{2}\right)}{(1+\eta_2)}e^{-\eta_2 x_2}\right]$.

HRFs of the Bivariate Xgamma distribution, η_1 and η_2 can be calculated by following equations:

$$\eta_1 = \frac{-\partial}{\partial x_1}\log S(x_1,x_2)$$

and

$$\eta_2 = \frac{-\partial}{\partial x_2}\log S(x_1,x_2)$$

where $S(x_1,x_2)$ is the survival function of Bivariate Xgamma distribution. HRF of Bivariate Xgamma distribution helps to researcher for modeling the increasing, decreasing and bathtub scenario. To demonstrate the use of the Bivariate Xgamma distribution, authors considered UEFA Champion's League data set and represents the time(in minutes) of the first kick goal scored by any team(X1) and time of first goal of any types scored by the home team(X2).

3.24 Xgamma exponential distribution

To Xgamma exponential distribution is introduced by Yadav et al. (2022) as an alternative to the one-parameter exponential distribution. The beauty of this distribution has been fully justified based on its monotone and bathtub-shaped hazard rate. CDF and HRF are given below:

$$G(x) = 1-\frac{1}{2}e^{-\lambda x}\left[2+\lambda x+\frac{1}{2}(\lambda x)^2\right], \quad x>0, \lambda>0 \quad (48)$$

and,

$$H(x) = \frac{\lambda\left[1+\frac{1}{2}(\lambda x)^2\right]}{2+\lambda x+\frac{1}{2}(\lambda x)^2}. \quad (49)$$

Xgamma exponential distribution have the nature of increasing, decreasing and bathtub shape of HRF. Graphical presentation of the HRF of the same placed in Figure 20.

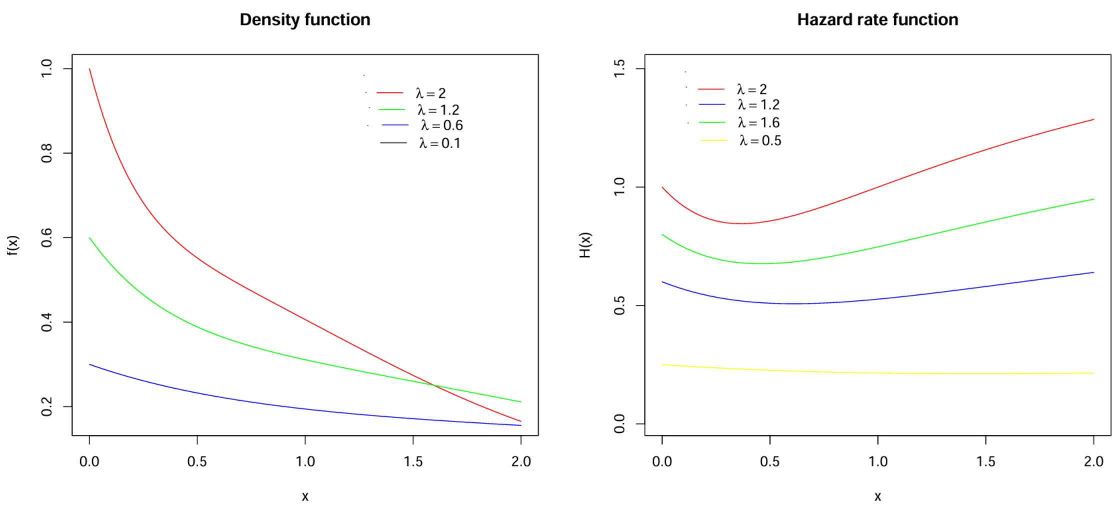

***Figure 20**: Xgamma Exponential distribution*

3.25 Weighted Xgamma exponential distribution

Yadav et al. (2023) introduced a weighted version of the Xgamma-exponential distribution following a similar methodological framework of weighted distributions. Their comprehensive study presented a thorough characterization of this new model along with diverse practical applications. The PDF and CDF of Weighted Xgamma exponential distribution are derived by employing a weight function $w(x) = x^{\eta}$ with the Xgamma exponential distribution as the baseline distribution. CDF and HRF are:

$$G(x) = \frac{2I_L(\eta+1,\lambda x)+I_L(\eta+3,\lambda x)}{\eta!(\eta^2+3\eta+4)}, \quad x>0, \quad (50)$$

and

$$H(x)=\frac{\frac{2\lambda^{\eta+1}x^{\eta}e^{-\lambda x}\left[1+\frac{(\lambda x)^2}{2}\right]}{\eta!(\eta^2+3\eta+4)}}{1-\frac{2I_L(\eta+1,\lambda x)+I_L(\eta+3,\lambda x)}{\eta!(\eta^2+3\eta+4)}} \tag{51}$$

where I_L is the lower incomplete gamma function. This model is particularly noteworthy due to its capability to accommodate various hazard rate shapes including monotone as well as bathtub-shaped forms, making it highly flexible and applicable to a broad spectrum of real-world scenarios. Additionally, when η is set to 1 or 2, the model reduces to the length-biased and area-biased versions of Weighted Xgamma exponential distribution, respectively. Graphical presentation of the HRF of the same placed in Figure 21.

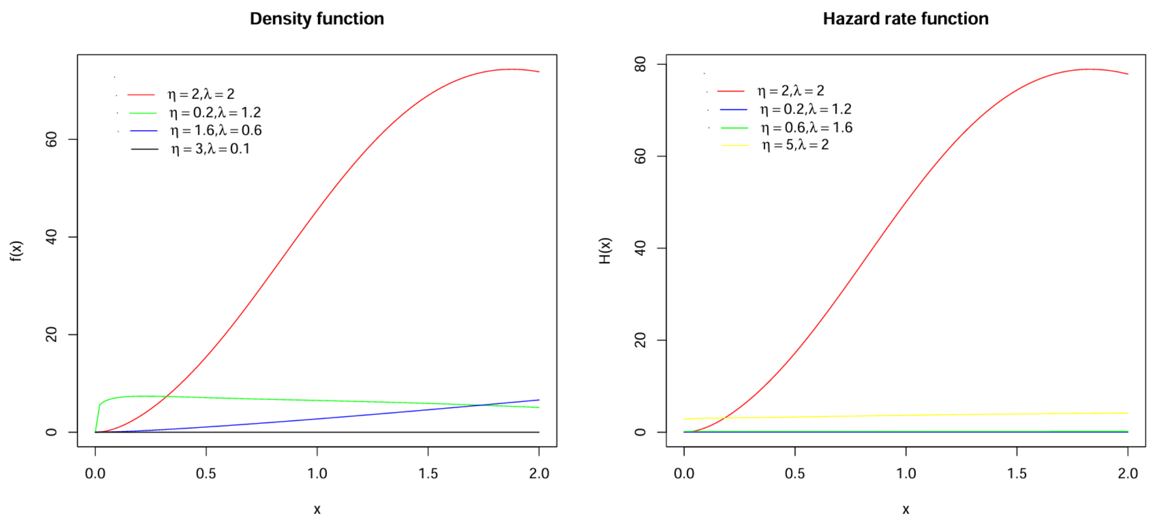

***Figure 21**: Weighted Xgamma Exponential distribution*

3.26 Truncated version of Xgamma distribution

Sen et al. (2024) proposed truncated version of Xgamma distribution and various estimation methods are discussed for the developed distribution. CDF is:

$$G(x)=\frac{F(x)-F(\alpha)}{F(\beta)-F(\alpha)};\alpha\le x\le\beta \tag{52}$$

where F(.) denote the CDF of baseline distribution and HRF is:

$$H(x)=\frac{f(x)/(F(\beta)-F(\alpha))}{1-\frac{F(x)-F(\alpha)}{F(\beta)-F(\alpha)}} \tag{53}$$

Application of truncated version of Xgamma distribution is shown by strength data of glass of aircraft window and that is the proof of HRF of truncated version of Xgamma distribution matched with real life situation.

3.27 Improved Extension of Xgamma distribution

Improved extension of Xgamma distribution is introduced by Alomair et al. (2024) and it is called as Power Quasi Xgamma distribution. CDF and HRF are:

$$G(x)=1-\left[\frac{1+\eta+\lambda x^{\alpha}+(\lambda^2x^{2\alpha}/2)}{1+\eta}\right]e^{-\lambda x^{\alpha}};x>0,\eta>0,\lambda>0,\alpha>0 \tag{54}$$

and

$$H(x)=\frac{\frac{\lambda\alpha x^{\alpha-1}}{1+\eta}(\eta+(\lambda^2x^{2\alpha}/2))e^{-\lambda x^{\alpha}}}{\frac{1+\eta+\lambda x^{\alpha}+(\lambda^2x^{2\alpha}/2)}{1+\eta}e^{-\lambda x^{\alpha}}} \tag{55}$$

HRF of this can take every possible shape such as increasing, decreasing and bathtub which is essential for the real life application and authors used two data sets to show the application of the proposed distribution. Graphical presentation of the HRF of the same placed in Figure 22.

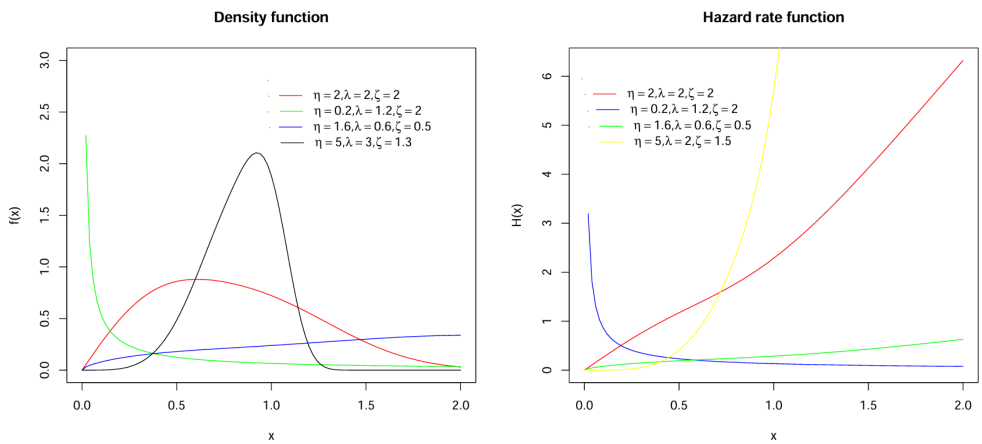

Figure 22*: Power Quasi Xgamma distribution*

3.28 Generalized Inverse Xgamma distribution

Tripathi et al. (2025) derived the generalized form of inverse xgamma distribution which is having two parameter and more flexible than XGD. CDF and HRF are:

$$G(x) = \left(1 + \frac{\eta^2}{2(\eta+1)}\frac{1}{x^{(2\alpha)}} + \frac{\eta}{\eta+1}\frac{1}{x^{\alpha}}\right) e^{-\eta/x^{\alpha}}; \quad x > 0, \alpha > 0, \theta > 0, \tag{56}$$

and

$$H(x) = \frac{\frac{\alpha\eta^2}{1+\eta}\frac{1}{x^{(\alpha+1)}}\left(1+\frac{\eta}{2x^{2\alpha}}\right)e^{-\theta/x^{\alpha}}}{1-\left(1+\frac{\eta^2}{2(\eta+1)}\frac{1}{x^{(2\alpha)}}+\frac{\eta}{\eta+1}\frac{1}{x^{\alpha}}\right)e^{-\eta/x^{\alpha}}} \tag{57}$$

Generalized Inverse Xgamma distribution is positively skewed and uni-modal distribution, also HRF is increasing and reaches to a peak after that declined slowly, which indicates that the model possesses the hump or upside-down bathtub property of hazard rate. Graphical presentation of the HRF of the same placed in Figure 23.

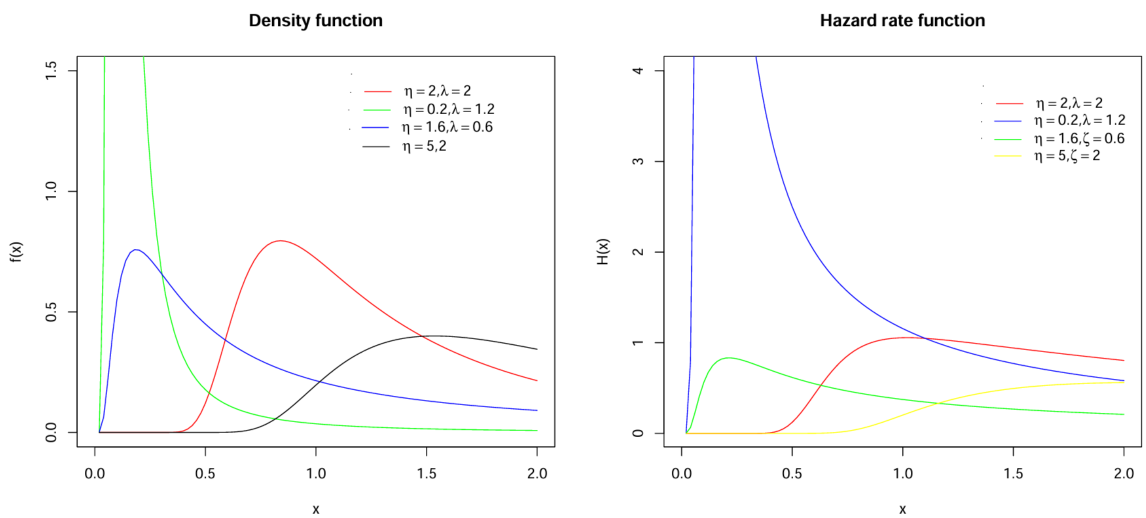

Figure 23*: Generalized Inverse Xgamma distribution*

3.29 Power Xgamma distribution

If X follows the new power xgamma distribution [see, Boudjerda (2025)] which is generated by taking the transformation by X=(1/Y) $^{\beta}$ where Y follows one parameter XGD. CDF and HRF of Power Xgamma distribution this is given below.

$$G(x) = \frac{1+\eta+\eta x^{\lambda}+\frac{\eta^2}{2}x^{2\lambda}}{1+\eta} e^{-\eta x^{\lambda}}, \quad x > 0, \ \eta > 0, \ \lambda > 0 \tag{58}$$

and

$$H(x) = \frac{\frac{\eta^2\lambda}{1+\eta}x^{\lambda-1}\left(\frac{\eta^2}{2}x^{2\lambda}+1\right)e^{-\eta x^{\lambda}}}{\frac{1+\eta+\eta x^{\lambda}+\frac{\eta^2}{2}x^{2\lambda}}{1+\eta}e^{-\eta x^{\lambda}}} \tag{59}$$

This probability distribution modeled the situation of both monotonic and non-monotonic HRF which is common behavior in real life. Moreover Boudjerda (2025) claimed that in some situation it gives better result than XGD, gamma, Weibull, gamma-Lindley, Lomax and Lindley distributions. Graphical presentation of the HRF of the same placed in Figure 24.

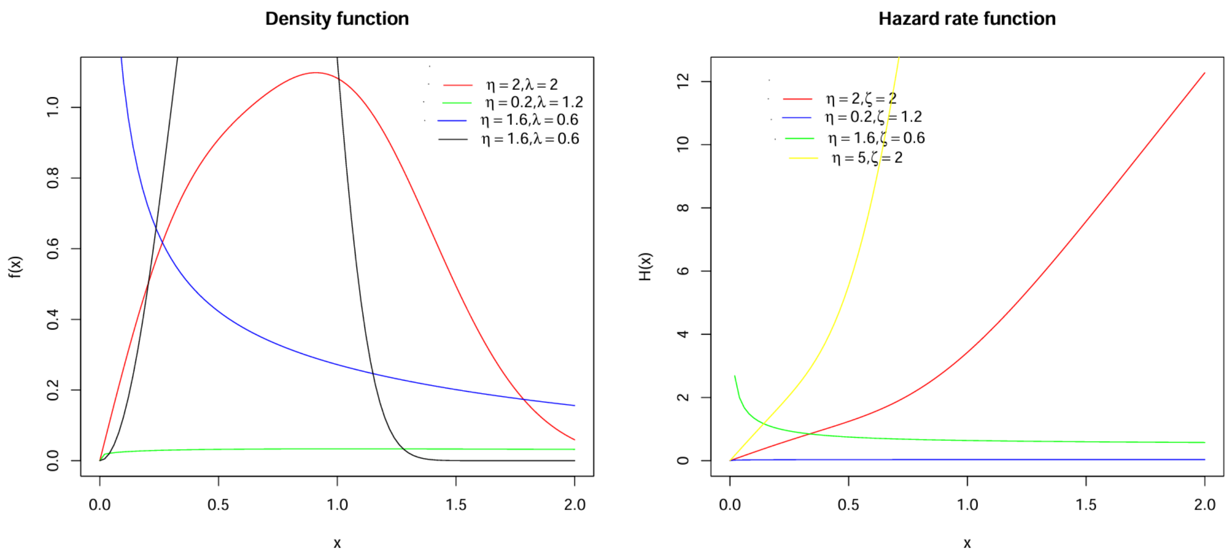

***Figure 24**: Power Xgamma distribution*

4. Application of Various Xgamma family of distributions

Real-life applications form a crucial component of any statistical study because they provide empirical validation for the proposed models and methodologies. In the context of the XGD and its various generalizations, real data analyses demonstrate not only the goodness of fit but also the practical interpretability of the underlying distributional assumptions in diverse applied settings. These applications help to bridge the gap between theoretical development and practical utility, thereby strengthening the credibility and relevance of the proposed models. In this study, several data sets that have been used in the literature to validate the XGD and its extensions are compiled and presented. For each data set, the observations together with the corresponding bibliographic reference are reported in this section, enabling readers to trace the original source and the specific modeling context in which XGD-type distributions have been employed. The selected data sets originate from a wide range of application areas, including medical studies (such as survival times or remission durations), engineering and reliability experiments (such as component or system lifetimes), and other industrial or environmental contexts where modeling of positive continuous data is essential.

An important feature of the considered data sets is their ability to exhibit different types of hazard rate behavior, which is one of the main motivations for employing flexible lifetime distributions like XGD and its generalizations. Collectively, these data sets display increasing, decreasing, bathtub-shaped, inverted-bathtub, and other non-monotone hazard rate patterns, thereby providing a rich testbed for assessing the performance of competing models. By showing that XGD-based distributions can successfully accommodate such diverse hazard structures, the applications underscore the flexibility and robustness of this family of models for practical reliability and survival analysis.

Data 1: Following data used by XGD, Sen et al. (2016) and it shows the application of XGD version in virulent tubercle bacilli. 72 guinea pigs infected with virulent tubercle bacilli and data is:

10, 33, 44, 56, 59, 72, 74, 77, 92, 93, 96, 100, 100, 102, 105, 107, 107, 108, 108, 108, 109, 112, 113, 115, 116, 120, 121, 122, 122, 124, 130, 134, 136, 139, 144, 146, 153, 159, 160, 163, 163, 168, 171, 172, 176, 183, 195, 196, 197, 202, 213, 215, 216, 222, 230, 231, 240, 245, 251, 253, 254, 254, 278, 293, 327, 342, 347, 361, 402, 432, 458, 555.

Data 2: Data values represents the Fatigue lifetime of 23 deep-groove ball bearings For detail of data, see, Weighted Xgamma distribution, Sen et al. (2017).

17.88, 28.92, 33.00, 41.52, 42.12, 45.60, 48.48, 51.84, 51.96, 54.12, 55.56, 67.80, 68.64, 68.64, 68.88, 84.12, 93.12, 98.64, 105.12, 105.84, 127.92, 128.04, 173.40

Data 3: Monitoring and modelling the cancer decease data is big task in medical industry. Sen and Chandra (2017) used random sample of 128 bladder cancer patients to show the modeling of cancer decease datawith the help of Quasi Xgamma distribution.

0.08, 2.09, 3.48, 4.87, 6.94, 8.66, 13.11, 23.63, 0.20, 2.23, 0.26, 0.31, 0.73,0.52,4.98,6.97,9.02, 13.29, 0.40, 2.26, 3.57, 5.06, 7.09, 11.98, 4.51, 2.07,0.22,13.8,25.74,0.50,2.46,3.64,5.09,7.26,9.47, 14.24, 19.13, 6.54, 3.36,0.82,0.51,2.54,3.70,5.17,7.28,9.74,14.76,26.31,0.81,1.76,8.53,6.93, 0.62, 3.82, 5.32, 7.32, 10.06, 14.77, 32.15, 2.64, 3.88, 5.32, 3.25, 12.03,8.65,0.39,10.34,14.83,34.26, 0.90,2.69, 4.18, 5.34, 7.59, 10.66, 4.50,20.28,12.63,0.96,36.66,1.05,2.69,4.23,5.41,7.62,10.75, 16.62,43.01, 6.25, 2.02, 22.69, 0.19, 2.75, 4.26, 5.41, 7.63, 17.12, 46.12, 1.26, 2.83, 4.33, 8.37, 3.36, 5.49, 0.66, 11.25, 17.14, 79.05, 1.35, 2.87, 5.62, 7.87, 11.64,17.36,12.02,6.76,0.40, 3.02,4.34,5.71, 7.93, 11.79, 18.1, 1.46, 4.40, 5.85, 2.02, 12.07

Data 4: Stress-rupture life of kevlar 49/epoxy strands are recorded in terms of given observations and it is used by Sen et al. (2018) to discussed the real life application of Quasi Xgamma Poisson distribution.

0.01, 0.01, 0.02, 0.02, 0.02, 0.03, 0.03, 0.04, 0.05, 0.06, 0.07, 0.07, 0.08, 0.09, 0.09, 0.10, 0.10, 0.11, 0.11, 0.12, 0.13, 0.18, 0.19, 0.20, 0.23, 0.24, 0.24, 0.29, 0.34, 0.35, 0.36, 0.38, 0.40, 0.42, 0.43, 0.52, 0.54, 0.56, 0.60, 0.60, 0.63, 0.65, 0.67, 0.68, 0.72, 0.72, 0.72, 0.73, 0.79, 0.79, 0.80, 0.80, 0.83, 0.85, 0.90, 0.92, 0.95, 0.99, 1.00, 1.01, 1.02, 1.03, 1.05, 1.10, 1.10, 1.11, 1.15, 1.18, 1.20, 1.29, 1.31, 1.33, 1.34, 1.40, 1.43, 1.45, 1.50, 1.51, 1.52, 1.53, 1.54, 1.54, 1.55, 1.58, 1.60, 1.63, 1.64, 1.80, 1.80, 1.81, 2.02, 2.05, 2.14, 2.17, 2.33, 3.03, 3.03, 3.34, 4.20, 4.69, 7.89

Data 5: Altunand and Hamedani (2018) dealt with the comparison of two different algorithms called SC16 and P3 for estimating unit capacity factors to implement the application of Log Xgamma distribution.

0.853,0.759,0.866,0.809,0.717,0.544,0.492,0.403,0.344,0.213,0.116,0.116, 0.092,0.070,0.059, 0.048,0.036,0.029,0.021,0.014,0.011,0.008, 0.006

Data 6: Sen et al. (2019) proved the importance of Quasi Xgamma-Geometric distribution by using remission times(in months) of bladder cancer patients and the observations of remission times are given below:

0.08, 2.09, 3.48, 4.87, 6.94, 8.66, 13.11, 23.63, 0.20, 2.23, 0.26, 0.31, 0.73,0.52,4.98,6.97,9.02, 13.29, 0.40, 2.26, 3.57, 5.06, 7.09, 11.98, 4.51, 2.07,0.22,13.8,25.74,0.50,2.46,3.64,5.09,7.26,9.47, 14.24, 19.13, 6.54, 3.36,0.82,0.51,2.54,3.70,5.17,7.28,9.74,14.76,26.31,0.81,1.76,8.53,6.93, 0.62, 3.82, 5.32, 7.32, 10.06, 14.77, 32.15, 2.64, 3.88, 5.32, 3.25, 12.03,8.65,0.39,10.34,14.83,34.26, 0.90,2.69, 4.18, 5.34, 7.59, 10.66, 4.50,20.28,12.63,0.96,36.66,1.05,2.69,4.23,5.41,7.62,10.75, 16.62,43.01, 6.25, 2.02, 22.69, 0.19, 2.75, 4.26, 5.41, 7.63, 17.12, 46.12, 1.26, 2.83, 4.33, 8.37, 3.36, 5.49, 0.66, 11.25, 17.14, 79.05, 1.35, 2.87, 5.62, 7.87, 11.64,17.36,12.02,6.76,0.40, 3.02,4.34,5.71, 7.93, 11.79, 18.1, 1.46, 4.40, 5.85, 2.02, 12.07, 21.73, 2.07, 3.36,6.93,8.65,12.63,22.69

Data 7: Given observations are the survival time (in weeks) of patient suffering from acute Myelogeneous and this positive skewed data handeled by Sen et al. (2019) to illustrate the importance of Quasi Xgamma-Geometric distribution.

65, 156, 100, 134, 16, 108, 121, 4, 39, 143, 56, 26, 22, 1, 1, 5, 65, 56, 65, 17, 7, 16, 22, 3, 4, 2, 3, 8, 4, 3, 30, 4, 43

Data 8: To show the superiority of Xgamma Weibull distribution over other exiting version of XGD, Yousof et al. (2020) used a data set and data represents the strength of 1.5 cm glass fibres, measured at National physical laboratory, England.

0.55, 0.93, 1.25, 1.36, 1.49, 1.52, 1.58, 1.61, 1.64, 1.68, 1.73, 1.81, 2.00, 0.74, 1.04, 1.27, 1.39, 1.49, 1.53, 1.59, 1.61, 1.66, 1.68, 1.76, 1.82, 2.01, 0.77, 1.11, 1.28, 1.42, 1.50, 1.54, 1.60, 1.62, 1.66, 1.69, 1.76, 1.84, 2.24, 0.81, 1.13, 1.29, 1.48, 1.50, 1.55, 1.61, 1.62, 1.66, 1.70, 1.77, 1.84, 0.84, 1.24, 1.30, 1.48, 1.51, 1.55, 1.61, 1.63, 1.67, 1.70, 1.78, 1.89.

Data 9: Bantan et al. (2020) showed the application of Half-logistic Xgamma distribution to the manufacturing industry and data represents 100 observations of breaking stress of carbon fibers (in Gba).

3.7, 3.11, 4.42, 3.28, 3.75, 2.96, 3.39, 3.31, 3.15, 2.81, 1.41, 2.76, 3.19, 1.59, 2.17, 3.51, 1.84, 1.61, 1.57, 1.89, 2.74, 3.27, 2.41, 3.09, 2.43, 2.53, 2.81, 3.31, 2.35, 2.77, 2.68, 4.91, 1.57, .65, 2.12, 1.8, 0.85, 4.38, 2, 1.18, 1.71, 1.17, 2.17, 0.39, 2.79, 1.08, 2.73, 0.98, 1.73, 1.59, 1.92, 2.38, 3.56, 2.55, 3.22, 3.39, 4.9, 1.69, 3.11, 3.6, 2.05, 1.61, 2.03, 2.48, 1.25, 2.48, 1.12, 2.88, 2.87, 3.19, 1.87, 2.95, 2.67, 4.2, 2.85, 2.55, 2.17, 2.97, 3.68, 0.81, 1.22, 5.08, 1.69, 3.68, 4.70, 2.03, 2.82, 2.50, 3.22, 3.15, 1.47, 5.56, 1.84, 1.36, 2.59, 2.83, 2.56, 3.33, 2.93, and 2.97.

Data 10: Following data set having 63 observation and represents gauge length of 10mm and this data sets is used by Bantan et al. (2020) for application purpose.

1.901, 2.132, 2.203, 2.228, 2.257, 2.350, 2.361, 2.396, 2.397, 2.445, 2.454, 2.474, 2.518, 2.522, 2.525, 2.532, 2.575, 2.614, 2.616, 2.618, 2.624, 2.659, 2.675, 2.738, 2.740, 2.856, 2.917, 2.928, 2.937, 2.937, 2.977, 2.996, 3.030, 3.125, 3.139, 3.145, 3.220, 3.223, 3.235, 3.243, 3.264, 3.272, 3.294, 3.332, 3.346, 3.377, 3.408, 3.435, 3.493, 3.501, 3.537, 3.554, 3.562, 3.628, 3.852, 3.871, 3.886, 3.971, 4.024, 4.027, 4.225, 4.395, and 5.020.

Data 11: Death time (in weeks) of patient cancer of the tongue with an aneuploid DNA profile modeled by using Half-logistic Xgamma distribution [see, Bantan et al. (2020)].

1, 3, 3, 4, 10, 13, 13, 16, 16, 24, 26, 27, 28, 30, 30, 32, 41, 51, 61, 65, 67, 70, 72, 73, 74, 77, 79, 80, 81, 87, 87, 88, 89, 91, 93, 93, 96, 97, 100, 101, 104, 104, 108, 109, 120, 131, 150, 157, 167, 231, 240, and 400.

Data 12: Following data is having 58 recurrence times to infection, at the point of insertion of the catheter and this data is used by Hassan et al. (2020) [see, Lindley-Quasi Xgamma distribution].

8, 16, 23, 22, 28, 447, 318, 30, 12, 24, 245, 7, 9, 511, 30, 53, 196, 15, 154, 7, 333, 141, 96, 38, 536, 17, 185, 177, 292, 114, 15, 152, 562, 402, 13, 66, 39, 12, 40, 201, 132, 156, 34, 30, 2, 25, 130, 26, 27, 58, 43, 152, 30, 190, 119, 8, 78, 63

Data 13: 72 exceedances of flood peaks (in m3/s) of the Wheaton River for the year 1958-1984 are reported in following data set and it is used by Hassan et al. (2020) to justify the development of Lindley-Quasi Xgamma distribution.

1.7, 2.2, 14.4, 1.1, 0.4, 20.6, 5.3, 0.7, 13.0, 12.0, 9.3, 1.4, 18.7, 8.5, 25.5, 11.6, 14.1, 22.1, 1.1, 2.5, 14.4, 1.7, 37.6, 0.6, 2.2, 39.0, 0.3, 15.0, 11.0, 7.3, 22.9, 1.7, 0.1, 1.1, 0.6, 9.0, 1.7, 7.0, 20.1, 0.4, 14.1, 9.9, 10.4, 10.7, 30.0, 3.6, 5.6, 30.8, 13.3, 4.2, 25.5, 3.4, 11.9, 21.5, 27.6, 36.4, 2.7, 64.0, 1.5, 2.5, 27.4, 1.0, 27.1, 20.2, 16.8, 5.3, 9.7, 27.5, 2.5, 27.0, 1.9, 2.8

Data 14: Exponentiated Xgamma distribution developed by Yadav et al. (2021) and demonstrated the application by using thirty successive values of March precipitation in Minneapolis/St Paul.

0.77, 1.74, 0.81, 1.2, 1.95, 1.2, 0.47, 1.43, 3.37, 2.2, 3, 3.09, 1.51, 2.1, 0.52, 1.62, 1.31, 0.32, 0.59, 0.81, 2.81, 1.87, 1.18, 1.35, 4.75, 2.48, 0.96, 1.89, 0.9, 2.05.

Data 15: Yadav et al. (2021) introduced new version of XGD, Exponentiated Xgamma distribution and used this model for modelling of new cases of Covid-19 in Italy during 31st May 2020 to 30th June 2020.

334, 200, 319, 322, 177, 519, 270, 197, 280, 283, 202, 380, 163, 347, 337, 301, 210, 329, 332, 251, 264, 224, 221, 113, 190, 296, 255, 175, 174, 126, 142.

Data 16: For application purpose of Weighted Lindley-Quassi Xgamma distribution, Wani and Shafi (2021) utilized the data set of tensile strength measures in GPa of 69 carbon fibres tested under tension at gauge lengths of 20mm.

1.312,1.314,1.479,1.552,1.700,1.803,1.861,1.865,1.944,1.958,1.966,1.997,2.006,2.021,2.027,
2.055,2.063,2.098,2.140,2.179,2.224,2.240,2.253,2.270,2.272,2.274,2.301,2.301,2.359,2.382,
2.382,2.426,2.434,2.435,2.478,2.490,2.511,2.514,2.535,2.554,2.566,2.570,2.586,2.629,2.633,
2.642,2.648,2.684,2.697,2.726,2.770,2.773,2.800,2.809,2.818,2.821,2.848,2.880,2.954,3.012,
3.067,3.084,3.090,3.096,3.128, 3.233, 3.433, 3.585, 3.585

Data 17: Alpha Power transformed Xgamma distribution [see, Shukla et al. (2022)] is very useful to deal with the data which is observed under accelerated life test, following data is the sample failure time (in minutes) of 15 electronic components.

1.4,5.1,6.3,10.8,12.1,18.5,19.7,22.2,23,30.6, 37.3,46.3,53.9,59.8,66.2"

Data 18: Following data is generated from clean up monitoring wells by using vinyl chloride which is a volatile organic compound. Shukla et al. (2022) applied developed Alpha power transformed Xgamma distribution to model the following data.

5.1,1.2,1.3,0.6,0.5,2.4,0.5,1.1,8.0,0.8,0.4,0.6,0.9,0.4,2.0,0.5,5.3,3.2,2.7,2.9,2.5,2.3,1.0,
0.2,0.1,0.1,1.8,0.9,2.0,4.0,6.8,1.2,0.4,0.2

Data 19: In recent time modelling of corona-virus cases distribution is the key interest of researchers and in same fashion, Alpha Power transformed Xgamma distribution [see, Shukla et al. (2022)] used to model the corona-virus cases distribution among the fifteen countries viz.,France, Italy, Spain, US, Germony, UK, Turkey, Iran, Russia, China, Brazil, Canada, Belgium, Netherlands and Switzerland.

5.37,6.56,7.61,32.83,5.24,5.06,3.65,3.03 2.89,2.74,2.10,1.57,1.55,1.27,0.97

Data 20: Following observation represents the water capacity month-wise from the Shasta reservoir in California in the month of February from 1991 to 2010. Sharqa et al. (2022) modeled the water capicity with the help of Unit Xgamma distribution.

0.338936, 0.431915, 0.759932, 0.724626, 0.757583, 0.811556, 0.785339, 0.783660, 0.815627, 0.847413, 0.768007, 0.843485, 0.787408, 0.849868, 0.695970, 0.842316, 0.828689, 0.580194, 0.430681 and 0.742563.

Data 21: Survival times (in months) of 46 patients of melanoma (non-censored data) is reported in following data and Wani et al. (2022) implemented Size Biased Lindley-Quasi Xgamma distribution to mentioned data.

3.25, 3.50, 4.75, 4.75, 5.00, 5.25, 5.75, 5.75, 6.25, 6.50, 6.50, 6.75, 6.75, 7.78 ,8.00, 8.50, 8.50, 9.25, 9.50, 9.50, 10.00, 11.50, 12.5, 13.25, 13.5, 14.25, 14.50, 14.75, 15.00, 16.25, 16.25, 16.50, 17.5, 21.75, 22.50, 24.50, 25.50, 25.75, 27.50, 29.50, 31.00, 32.50, 34.00, 34.50, 35.25, 58.50

Data 22: Truncated data of strength of glass of aircraft window is given below and to tackle this truncated data, Truncated version of Xgamma distribution is developed by Sen et al. (2024).

18.83, 20.80, 21.657, 23.03, 23.23, 24.05, 24.321, 25.5, 25.52, 25.80, 26.69, 26.770, 26.78, 27.05, 27.67, 29.90, 31.11, 33.20, 33.73, 33.76, 33.890, 34.76, 35.75, 35.91, 36.98, 37.08, 37.09, 39.58, 44.045, 45.29, 45.381

Data 23: Generalized Nnverse Xgamma distribution introduced by Tripathi et al. (2025) and placed the real life application section by using the data of survival times (in days)guinea pigs with different doses of tubercle bacilli, and this data is reported below:

12, 15, 22, 24, 24, 32, 32, 33, 34, 38, 38, 43, 44, 48, 52, 53, 54, 54, 55, 56, 57, 58, 58, 59, 60, 60, 60, 60, 61, 62, 63, 65, 65, 67, 68, 70, 70, 72, 73, 75, 76, 76, 81, 83, 84, 85, 87, 91, 95, 96, 98, 99, 109, 110, 121, 127, 129, 131, 143, 146, 146, 175, 175, 211, 233, 258, 258, 263, 297, 341, 341, 37

Data 24: To prove the applicability of Generalized Inverse Xgamma distribution [see, Tripathi et al. (2025)] utilized the data of 44 patients suffering from head and neck cancer disease and were treated using combined radio therapy and chemotherapy, and this data is placed below:

12.20, 23.56, 23.74, 25.87, 31.98, 37, 41.35, 47.38, 55.46, 58.36, 63.47, 68.46, 78.26, 74.47, 81.43, 84, 92, 94, 110, 112, 119, 127, 130, 133, 140, 146, 155, 159, 173, 179, 194, 195, 209, 249, 281, 319, 339, 432, 469, 519, 633, 725, 817, 1776

Data 25: Newly version of Power Xgamma distribution derived by Boudjerda (2025) and it is used to model the repair times for an airborne communication transceiver.

0.50, 0.60, 0.60, 0.70, 0.70, 0.70, 0.80, 0.80,1.00, 1.00, 1.00, 1.00, 1.10, 1.30, 1.50, 1.50, 1.50, 1.50, 2.00,2.00, 2.20, 2.50, 2.70, 3.00, 3.00, 3.30, 4.00, 4.00, 4.50, 4.70,5.00, 5.40, 5.40, 7.00, 7.50, 8.80, 9.00, 10.20, 22.00, 24.50

Data 26: Boudjerda (2025) formulated the new Power Xgamma distribution and incorporated this newly developed distribution to develop the times of failure of fatigue fracture of Kevlar 373/epoxy.

1.2985, 1.3211, 1.3503, 1.3551, 1.4595, 1.4880, 1.5728, 1.5733, 1.7083, 1.7263, 1.7460, 1.7630, 1.7746, 1.8275, 0.0251, 0.0886, 0.0891, 0.2501, 0.3113, 0.3451, 0.4763, 0.5650, 0.5671, 0.6566, 0.6748, 0.6751, 0.6753, 0.7696, 0.8375, 0.8391, 0.8425, 0.8645, 0.8851, 0.9113, 0.9120, 0.9836, 1.0483, 1.0596, 1.0773, 1.1733, 1.2570, 2.2100, 3.7455, 3.9143, 4.8073, 5.4005, 5.4435, 5.5295, 6.5541, 9.0960, 2.2460, 2.2878, 2.3203, 2.3470, 2.3513, 2.4951, 2.5260, 2.9911, 3.0256, 3.2678, 3.4045, 3.4846, 3.7433, 1.2766, 1.8375, 1.8503, 1.8808, 1.8878, 1.8881, 1.9316, 1.9558, 2.0048, 2.0408, 2.0903, 2.1093, 2.1330.

Fitting of all data sets for XGD are reported in Table 2 and results of this table indicate that authors developed better versions of XGD where they claimed, new versions of XGD are better than the XGD. Comparison between the new versions of XGD and XGD are done based on different measures such as AIC(Akaike Information Criterion), BIC(Bayesian Information Criterion), KS value and p-value.

***Table 2**: Model fitting summary of data sets for XGD*

Data Set	Estimate	LL	AIC	BIC	KS value	p-value
1	0.01678	-425.5916	853.1832	855.4599	0.10096	0.45530
2	0.04071	-113.9656	229.9312	231.0667	0.13232	0.81550
3	0.28606	-425.1691	852.3382	855.1902	0.18487	0.00031
4	1.69785	-104.1007	210.2015	212.8166	0.08271	0.49420
5	4.75336	-6.371815	10.74363	9.608136	0.21181	0.25350
6	0.28259	-449.0842	900.1684	903.0736	0.17829	0.00037
7	0.06475	-176.3484	354.6967	356.1932	0.35452	0.00049
8	1.33761	-85.80016	173.6003	175.7435	0.41961	4.634e-10
9	0.86076	-183.4777	368.9554	371.5605	0.28607	1.559e-07
10	0.76304	-122.2032	246.4064	248.5495	0.45868	6.143e-12
11	0.03533	-286.5651	575.1302	577.0814	0.18236	0.06294
12	0.02397	-377.3694	756.7389	758.7993	0.40089	1.602e-08
13	0.20448	-266.4647	534.9293	537.206	0.25620	0.00015
14	1.18998	-44.57353	91.14705	92.54825	0.22263	0.10220
15	0.01167	-187.9499	377.8997	379.3337	0.20905	0.11480
16	0.91375	-121.5597	245.1194	247.3535	0.43273	1.198e-11
17	0.10031	-64.91847	131.8369	132.545	0.15700	0.80000
18	1.03129	-56.48505	114.9701	116.4965	0.13838	0.53300
19	0.42634	-43.55587	89.11175	89.8198	0.25012	0.25870
20	2.32157	-14.23687	30.47375	31.46948	0.42648	0.00081
21	0.17419	-168.2985	338.5971	340.4257	0.11814	0.54210
22	0.09375	-122.2735	246.5469	247.9809	0.32455	0.00208
23	0.03095	-391.7607	785.5215	787.7981	0.16467	0.04030
24	0.01319	-303.383	608.7659	610.5501	0.28070	0.00147
25	0.54521	-101.9971	205.9941	207.683	0.22053	0.04087
26	1.03319	-126.326	254.6521	256.9828	0.14741	0.06621

5. Conclusions

In this paper, a rigorous review work is done for XGD. Evolution of XGD is discussed in detail. Different forms of XGD are systematically summarized in a tabular format tabular form with 4 columns: serial number, year, authors and model. CDF and HRF are useful to characterize any probability distribution. Hence, we reported the CDF and HRF of all developed model for baseline XGD are described with their mathematical formula to summarize all development or extension of XGD. Also all the data sets of various fields are reported which are used by authors for validation of several extended or modified XGD distribution. This study serves as a valuable resource for researchers, providing a comprehensive understanding of the wide applicability and acceptance of XGD. It also offers a foundation for developing new versions of XGD by highlighting the existing advancements. Furthermore, the findings provide industrial practitioners with multiple options to select from XGD and its variants, as the review establishes that XGD and its extensions are suitable for a broad range of data types and application domains. In future, researchers can use these different models in industry and also find gap in the presented literature, and may develop new version XGD which is not attempted by authors in past.

References

Abulebda, M., Pathak, A. K., Pandey, A., & Tyagi, S. (2022). On a Bivariate XGamma Distribution Derived from Copula. Statistica, 82(1), 15-40.

Al-Mofleh, H., & Sen, S. (2019). The wrapped xgamma distribution for modeling circular data appearing in geological context. arXiv preprint arXiv:1903.00177.

Alomair, A. M., Babar, A., Ahsan-ul-Haq, M., & Tariq, S. (2025). An improved extension of Xgamma distribution: Its properties, estimation and application on failure time data. Heliyon, 11(2).

Altun, E., & Hamedani, G. G. (2018). The log-xgamma distribution with inference and application. Journal de la Société Française de Statistique, 159(3), 40-55.

Bantan, R., Hassan, A. S., Elsehetry, M., & Kibria, B. M. (2020). Half-logistic xgamma distribution: Properties and estimation under censored samples. Discrete Dynamics in Nature and Society, 2020.

Boudjerda, K. (2025). A new power xgamma distribution: statistical properties, estimation methods and application. Statistics, Optimization & Information Computing, 13(2), 716-740.

Cordeiro, G. M., Altun, E., Korkmaz, M. C., Pescim, R. R., Afify, A. Z., & Yousof, H. M. (2020). The xgamma family: Censored regression modelling and applications. Revstat-Statistical Journal, 18(5), 593-612.

Hashmi, S., Ahsan-ul-Haq, M., Zafar, J., & Khaleel, M. A. (2022). Unit Xgamma distribution: its properties, estimation and application: Unit-Xgamma distribution. Proceedings of the Pakistan Academy of Sciences: A. Physical and Computational Sciences, 59(1), 15-28.

Hassan, A., Wani, S. A., & Shafi, S. (2020). Pak. j. statist. 2020 vol. 36 (1), 73-89 lindley-quasi xgamma distribution: Properties and applications. Pak. J. Statist, 36(1), 73-89.

Maiti, S. S., Dey, M., & Sarkar, S. (2018). Discrete xgamma distributions: properties, estimation and an application to the collective risk model. Journal of Reliability and Statistical Studies, 117-132.

Mazucheli, J., Bertoli, W., Oliveira, R. P., & Menezes, A. F. (2020). On the discrete quasi Xgamma distribution. Methodology and Computing in Applied Probability, 22(2), 747-775.

Mohamed, M. I., Handique, L., Chakraborty, S., Butt, N. S., & Yousof, H. M. (2021). A new three-parameter Xgamma Fréchet distribution with different methods of estimation and applications. Pakistan Journal of Statistics and Operation Research, 291-308.

Para, B. A., Jan, T. R., & Bakouch, H. S. (2020). Poisson Xgamma distribution: A discrete model for count data analysis. Model Assisted Statistics and Applications, 15(2), 139-151.

Sen, S., & Chandra, N. (2017). The quasi xgamma distribution with application in bladder cancer data. Journal of data science, 15(1), 61-76.

Sen, S., Afify, A. Z., Al-Mofleh, H., & Ahsanullah, M. (2019). The quasi xgamma-geometric distribution with application in medicine. Filomat, 33(16), 5291-5330.

Sen, S., Alizadeh, M., Aboraya, M., Ali, M. M., Yousof, H. M., & Ibrahim, M. (2024). On truncated versions of xgamma distribution: Various estimation methods and statistical modelling. Statistics, Optimization & Information Computing, 12(4), 943-961.

Sen, S., Chandra, N., & Maiti, S. S. (2017). The weighted xgamma distribution: Properties and application. Journal of Reliability and Statistical Studies, 10(1), 43-58.

Sen, S., Korkmaz, M. C., & Yousof, H. M. (2018). The quasi xgamma-Poisson distribution. Theory and Applications of Statistics and Information, 18(3), 65-76.

Sen, S., Maiti, S. S., & Chandra, N. (2016). The xgamma distribution: statistical properties and application. Journal of Modern Applied Statistical Methods, 15(1), 38.

Shukla, S., Yadav, A. S., Rao, G. S., Saha, M., & Tripathi, H. (2022). Alpha Power Transformed Xgamma Distribution and Applications to Reliability, Survival and Environmental Data. Journal of Scientific Research, 66(3).

Tripathi, H., & Mishra, S. (2022). The transmuted inverse xgamma distribution and its statistical properties. International Journal of Mechanical Engineering, Kalahari Journals , 7.

Tripathi, H., Yadav, A. S., Saha, M., & Kumar, S. (2025). Generalized Inverse Xgamma Distribution: Properties, Estimation and Its Applications To Survival Data. Thailand Statistician, 23(3), 573-597.

Tripathi, H., Yadav, A. S., Saha, M., & Shukla, S. (2022). A New Flexible Extension of Xgamma Distribution and its Application to COVID-19 Data. Nepal Journal of Mathematical Sciences, 3(1), 11-30.

Wani, S. A., & Shafi, S. (2021). Generalized Lindley-Quasi Xgamma distribution. Journal of Applied Mathematics, Statistics and Informatics, 17(1), 5-30.

Wani, S. A., Hassan, A., Sheikh, B. A., & Dar, S. A. (2022). Size Biased Lindley-Quasi Xgamma Distribution Applicable to Survival Times. Thailand Statistician, 20(2), 452-474.

Yadav, A. S, Shukla, S., Jaiswal, N., Singh, S. K., & Koley, D. (2023). Development and Estimation of Weighted Xgamma Exponential Distribution with Applications to Lifetime Data. Statistica, 83(4), 247–269. https://doi.org/10.6092/issn.1973-2201/16940

Yadav, A. S., Maiti, S. S., & Saha, M. (2021). The inverse xgamma distribution: statistical properties and different methods of estimation. Annals of Data Science, 8(2), 275-293.

Yadav, A. S., Saha, M., Tripathi, H., & Kumar, S. (2021). The Exponentiated XGamma Distribution: a New Monotone Failure Rate Model and Its Applications to Lifetime Data. Statistica, 81(3), 303-334.

Yadav, A. S., Shukla, S., Goual, H., Saha, M., & Yousof, H. M. (2022). Validation of Xgamma exponential model via Nikulin–Rao–Robson goodness-of-fit test under complete and censored samples with different methods of estimation. Statistics, Optimization & Information Computing, 10(2), 457–483.

Yousof, H. M., Korkmaz, M. Ç., & Sen, S. (2021). A new two-parameter lifetime model. Annals of Data Science, 8(1), 91-106.

Declaration

Conflict of Study: The author declares that he has no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Funding Statement: This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

Availability of Data and Material: Data will be made available by the corresponding author on reasonable request.

Ethical Approval: Not Applicable

Consent to Participate: Not Applicable

Acknowledgement: Not Applicable