SCOPUA Journal of Applied Statistical Research (JASR)

Vol.1, Issue 3, December 2025
DOI: https://doi.org/10.64060/JASR.v1i3.5

Repetitive Sampling Plans for Life Tests Based on Percentiles of the Half-Normal Distribution with Applications to Software Reliability and Device Lifetime Data

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Received: 01 July 2025 / Revised: 30 September 2025 / Accepted: 17 October 2025 / Published online: 24 October 2025

ABSTRACT

Acceptance Sampling Plans (ASPs) are indispensable statistical tools in quality control for making decisions regarding the acceptance or rejection of product lots. While traditional plans often rely on the mean lifetime, percentile-based criteria offer a more robust measure, particularly for capturing tail behavior in lifetime distributions. This paper introduces a novel repetitive sampling plan (RSP) for life tests based on percentiles of the Half-Normal Distribution (HND). The plan is designed to verify that a specified quantile of the product lifetime exceeds a predefined standard. The design parameters, namely the sample size, acceptance number, and rejection number, are obtained through an optimization model that minimizes the Average Sample Number (ASN) while ensuring that both the producer's risk ($\alpha = 0.05$) and the consumer's risk (β) constraints are satisfied. Comprehensive tables are presented for various practical scenarios, examining the effects of the percentile ratio, termination time multiplier, and life percentile on the performance of plan. A comparative analysis demonstrates that the proposed RSP consistently requires a smaller ASN than the comparable single sampling plan, confirming its superior efficiency in reducing inspection effort and cost. The practical utility of the methodology is illustrated through a real-life example using software reliability data, showcasing its straightforward implementation and significant advantages for quality assurance in industrial settings.

Keywords*:* Percentile Lifetime; Truncated Life Test; Repetitive Sampling; Average Sample Number

1. Introduction

In modern industrial environments, the importance of ensuring product quality cannot be overstated. Organizations operate in highly competitive markets, where even marginal deviations from quality standards can have serious economic, reputational, and safety-related consequences. To address these challenges, the field of quality control (QC) has developed a range of methodologies grounded in statistical reasoning. As (Montgomery, 2020)emphasized, statistical quality control techniques may be broadly grouped into three categories: (i) Acceptance Sampling Plans (ASPs), (ii) Statistical Process Control (SPC), and (iii) Design of Experiments (DOE). Each of these methodologies contributes uniquely to the monitoring, maintenance, and improvement of process quality. Among these, Acceptance Sampling Plans (ASPs) occupy a foundational role. Historically one of the earliest developed statistical tools in QC, ASPs provide an organized framework for making informed decisions about the acceptance or rejection of production lots. Manufacturers face a recurring problem when receiving raw materials from suppliers or producing large batches of items: how to determine whether the lot meets quality specifications without incurring prohibitive inspection costs. In such situations, three primary pathways exist: (i) accept the lot without inspection, (ii) perform a 100% inspection of every item in the lot, or (iii) conduct acceptance sampling. While the first option is imprudent and the second is often impractical due to cost, time, or destructive testing requirements, acceptance sampling offers a pragmatic compromise, balancing efficiency with reliability.

In its classical formulation, an Acceptance Sampling Plan (ASP) operates by randomly selecting a sample from a lot and evaluating it against predefined quality criteria. Based on the sample outcome, the lot is either accepted or rejected. The strength of ASPs lies in their flexibility: they are not limited to incoming material inspection but can also be applied at intermediate production stages or as part of final product release decisions. Importantly, ASPs are not merely a substitute for 100% inspection; they are designed to achieve specific statistical risk profiles, thereby protecting both producers and consumers. By reducing costs while maintaining rigorous standards, ASPs remain an essential component of modern quality assurance systems. ASPs are broadly categorized into two main types: attribute-based and variable-based sampling. Attribute sampling evaluates product quality by counting the number of defective items in a sample. If the defect count falls below a specified threshold, the lot is accepted; otherwise, it is rejected. This approach is straightforward and easy to implement, particularly when the quality characteristic is categorical in nature (e.g., defective vs. non-defective). Variable sampling, on the other hand, utilizes quantitative measurements of specific quality characteristics, comparing them against predetermined benchmarks. By incorporating richer information, variable sampling can achieve greater statistical efficiency than attribute sampling, often requiring smaller sample sizes for the same level of protection.

The evolution of Acceptance Sampling Plans (ASPs) reflects a concerted effort to align statistical theory with the practical demands of industrial quality control, prompting researchers to develop increasingly sophisticated methodologies. Recognizing that purely attribute-based or variable-based approaches each have inherent strengths and limitations, hybrid models have been proposed to leverage the advantages of both. An illustrative contribution in this regard is provided by the work of (Aslam, Azam, & Jun, 2013), who demonstrated that such blended strategies significantly enhance decision-making accuracy in complex scenarios. This drive for adaptability is further evident in the progression of sampling schemes themselves, which have transitioned from the basic single sampling plan, a straightforward yet often inefficient method, to more nuanced approaches like double sampling, where an optional second inspection stage is employed to achieve a more optimal balance between cost and risk when initial results are inconclusive.

Building upon these foundational plans, a sophisticated array of advanced sampling schemes has emerged to address more complex industrial needs. These methodologies enhance decision-making by incorporating greater contextual information; for instance, repetitive and multiple dependent state sampling integrate prior inspection outcomes into subsequent evaluations for greater accuracy. Sequential sampling further refines this efficiency by allowing for repeated assessments until a definitive conclusion is reached, often reducing the average sample size required. Other schemes are tailored to specific scenarios: group sampling, which analyzes subsets of items collectively, is designed for high-throughput processes, while resubmitted sampling offers a mechanism for the re-evaluation of rejected lots. Finally, moving beyond mere defect counts, reliability sampling focuses on product lifetime characteristics under stress conditions. This diverse toolkit demonstrates how acceptance sampling has evolved from a simple pass/fail procedure into a sophisticated set of methodologies finely tuned to specific operational and reliability contexts.

Traditionally, acceptance sampling under life testing has relied on mean lifetime as the primary parameter for assessing product quality. Truncated life tests are frequently employed to minimize time and cost, with lot acceptance decisions based on whether the observed mean life meets or exceeds a specified standard. While this approach is convenient, it has significant limitations. The mean does not adequately capture distributional features, particularly for skewed or heavy-tailed lifetime distributions. Consider, for instance, a scenario where the mean life remains acceptable, yet variance increases significantly. In such cases, lower percentiles (e.g., the 10th percentile) may drop sharply, allowing products with a high likelihood of early failure to pass inspection. This issue highlights the inadequacy of relying solely on mean life as a quality metric. In practical applications, engineers and decision-makers often prioritize percentile-based criteria, which more accurately capture distribution tails and ensure consumer safety. For example, in structural applications, the 5th or 10th percentile of material strength may serve as the relevant benchmark, as failure below such thresholds can lead to catastrophic consequences.

The significance of percentile-based acceptance sampling was initially highlighted by (Lio, Tsai, & Wu, 2009), who introduced percentile-based ASPs for the Birnbaum–Saunders distribution under truncated life testing. Their methodology established the minimum sample sizes necessary to ensure specified percentiles while managing consumer risk. They also provided operating characteristic values and analyses of producer's risk, validated with real-world data sets. Building on this foundational work, (G. S. Rao & Kantam, 2010) expanded the framework to the log-logistic distribution, while (G. S. Rao, 2013) suggested similar methods for the Marshall–Olkin extended exponential distribution. (B. S. Rao, Kumar, & Rosaiah, 2013)) further contributed by concentrating on percentiles of the Half-Normal Distribution (HND) under truncated life tests. Additional distributions, such as the Linear Failure Rate (B. Srinivasa Rao, Priya, & Kantam, 2014) and the Extended Generalized Exponential (Al-Omari & Alomani, 2024), have also been examined, with applications validated through industrial datasets. More recent contributions like (S Rao & Naidu, 2014), (Azam, Aslam, Balamurali, & Javaid, 2015), (Kaviyarasu & Fawaz, 2017), (G. S. Rao, Rosaiah, & Prasad, 2019), (G. S. Rao, Rosaiah, Sivakumar, & Kalyani, 2019), (Kaviyarasu & Sivasankari, 2020),(Gupta, Rao, Nagasailaja, & Rao, 2020), (Jayalakshmi & Vijilamery, 2022), (Jayalakshmi & Aleesha, 2024) illustrate the continuing significance of percentile-based ASPs in enhancing quality control methodology.

The concept of repetitive sampling was introduced by (Sherman, 1965), who demonstrated that repetitive sampling plans often outperform single sampling plans by offering better protection at lower inspection costs. Repetitive sampling strategies depend on repeated lot evaluations based on previous outcomes, thereby enhancing efficiency while managing risks. This framework was significantly expanded by (Aslam, Niaki, Rasool, & Fallahnezhad, 2012) , who proposed RSP plans for Weibull and generalized exponential distributions using median life as the quality parameter. Their work incorporated Type I and Type II error considerations into the design of sampling tables, providing decision-makers with a practical framework for real-world applications. (Aslam, Lio, & Jun, 2013) further advanced the methodology by developing RSP under truncated life tests for the Burr Type XII distribution, while (Kalyani, Srinivasa Rao, Rosaiah, & Sivakumar, 2021) proposed RSP for the odds exponential log-logistic distribution, optimizing the average sample number (ASN) while simultaneously meeting producer's and consumer's risk levels. These contributions collectively underscore the importance of repetitive sampling in modern acceptance sampling research. By reducing average inspection effort while maintaining statistical rigor, repetitive sampling offers a promising framework for integrating more complex life test criteria, such as percentile lifetimes. More work using RSP can be seen in (Tripathi, Kiapour, & Qomi, 2024) (Jeyadurga & Balamurali, 2025), (Kiapour, Tripathi, & Masoumi, 2025), (Naveed et al., 2025), (Qomi & Aslam, 2025), (Mahdizadeh, 2025), (Qomi, Piadeh, Pérez-González, & Fernández, 2025)

The Half-Normal Distribution (HND) is of particular relevance to the present study due to its applicability in lifetime data analysis. Formally defined as a truncated normal distribution constrained to non-negative values, with a mean of zero and a scale parameter θ, the HND possesses an increasing failure rate, making it suitable for reliability modeling where the symmetric normal distribution is inappropriate. Its utility has been demonstrated across various disciplines, including quality control, fatigue analysis, and strength–stress reliability. Foundational work by (Chou & Liu, 1998) established key properties of the HND, while (Castro, Gómez, & Valenzuela, 2012) illustrated its effectiveness in modeling fatigue-related data. Within acceptance sampling, (Rao, Kumar, & Rosaiah, 2013) and (Rao, Kumar, & Rosaiah, 2014)developed single and group sampling plans for truncated life tests assuming HND lifetimes, with subsequent extensions to truncated single and double sampling designs by (Lu, Gui, & Yan, 2013) and(Al-Omari, Al-Nasser, & Gogah, 2016)respectively. More recently, (Geetha, Jayabharathi, & Uddin, 2024) proposed two-stage group acceptance sampling plans that achieve efficiency gains through reduced average sample sizes.

Synthesis and Identification of the Research Gap

Two major research streams, percentile-based acceptance sampling plans and repetitive sampling procedures, have evolved largely independently. Studies on percentile-based plans have emphasized robustness to distributional asymmetry but have not utilized the efficiency benefits inherent in repetitive sampling. Conversely, the literature on repetitive sampling plans (RSPs) for life tests has primarily focused on mean or median lifetime parameters, without incorporating the percentile-based criteria that enhance reliability assessment under skewed lifetime models. This divergence defines a distinct research gap, summarized in Table 1.

Table 1. *Positioning of the Proposed Research within the Existing Literature*

Sampling Scheme	Quality Parameter	Lifetime Distribution	Exemplary Works
Single Sampling	Percentile Lifetime	Various (Birnbaum–Saunders, Log-logistic, etc.)	(Lio, Tsai, & Wu, 2009); (G. S. Rao & Kantam, 2010)
Single Sampling	Percentile Lifetime	Half-Normal (HND)	Rao et al. (2013)
Repetitive Sampling	Mean/Median Lifetime	Various (Weibull, Burr Type XII)	Aslam et al. (2012); Aslam et al. (2013)
Proposed Plan	Percentile Lifetime	Half-Normal (HND)	This Study

As shown in Table 1, the specific combination of a repetitive sampling plan for life tests based on percentiles of the Half-Normal Distribution (HND) has not been previously addressed. The HND is selected for its analytical simplicity and suitability for modeling failure data with an increasing failure rate, while the percentile-based criterion enhances robustness against skewed lifetime behavior. The repetitive sampling structure further contributes by improving efficiency through reduced average sample numbers.

Accordingly, this study introduces a novel integration of three components—(i) percentile-based life testing, (ii) the HND as the underlying lifetime model, and (iii) a repetitive sampling framework, into a unified methodology. The proposed plan effectively captures key distributional characteristics through percentile estimation while maintaining sampling efficiency, thereby providing a more robust and economical approach for practitioners engaged in reliability and quality assurance analysis. The rest of the paper is organized as follows: Section 2 details the formulation and design of the proposed sampling plan. Section 3 provides a comparative analysis with existing methodologies. Section 4 illustrates the practical utility of the proposed plan through real life applications. Finally, Section 5 concludes with a summary of findings and suggestions for future research.

2. Design of the Proposed Repetitive Acceptance Sampling Plan

This section outlines the methodology for designing a RSP for life tests where the lifetime of product follows a half-normal distribution (HND). The plan is based on truncating the life test at a pre-specified time related to a percentile of the lifetime distribution.

2.1. Half-Normal Distribution for Lifetime Data

Acceptance Sampling Plans (ASPs) for life tests require a statistical model that accurately represents the lifetime distribution of the product under study. In this paper, we employ the Half-Normal Distribution (HND), a flexible and positively skewed distribution particularly suitable for modeling lifetime data where the failure rate increases over time, a common characteristic of wear-out failure modes in mechanical and electronic components.

The HND can be derived as a truncated form of the normal distribution, constrained to non-negative values. Let T be a continuous random variable representing the lifetime of product, which follows HND. The probability density function (PDF) of T is given by:

$$f(t) = \frac{2}{\lambda\sqrt{\pi}} exp\left(-\frac{t^2}{\lambda^2\pi}\right),\ t \geq 0, \lambda > 0 \tag{1}$$

Here, λ is the scale parameter of the distribution. A larger value of λ indicates a greater spread of lifetimes, corresponding to a generally longer-lived product.

The corresponding cumulative distribution function (CDF), which provides the probability that a product fails before time t, is expressed using the error function (erf):

$$F(t) = erf\left(\frac{t}{\lambda\sqrt{\pi}}\right) \tag{2}$$

The error function, $erf(\cdot)$, is a standard mathematical function defined as:

$$erf(z) = \frac{2}{\sqrt{\pi}}\int_0^z \exp(-x^2)\, dx \tag{3}$$

The mean lifetime μ and the variance σ^2 of the HND are derived from the scale parameter λ as follows:

$$\mu = \frac{\lambda\sqrt{\pi}}{2} \tag{4}$$

$$\sigma^2 = \lambda^2\left(\frac{\pi}{2} - 1\right) \tag{5}$$

Its increasing failure rate and simple closed-form cumulative distribution function (CDF) make it a suitable and efficient choice for developing tractable acceptance sampling plans in life testing applications.

2.2. Percentile Lifetime and Failure Probability in Truncated Life Tests

While the mean lifetime is a common metric, quality standards often mandate reliability at a specific percentile. For instance, a specification might require that 95% of units survive a certain duration, which corresponds to the $5th$ percentile. This section derives the formulas necessary to construct a sampling plan based on such a percentile lifetime criterion.

Let t_q denote the $100q - th$ percentile of the lifetime distribution. By definition, this is the time by which a proportion q of the product population fails. Therefore, it satisfies the condition $\left(F(t_q) = q\right)$. Substituting into the CDF from Equation (2):

$$erf\left(\frac{t}{\lambda\sqrt{\pi}}\right) = q \tag{6}$$

To isolate t_q, we apply the inverse error function, erf^{-1}, to both sides:

$\frac{t}{\lambda\sqrt{\pi}} = erf^{-1}(q)$

Solving for t_q yields the fundamental equation for the $100q - th$ percentile of the Half-Normal Distribution:

$$t_q = \lambda\sqrt{\pi}erf^{-1}(q) \qquad 0 < q < 1 \tag{7}$$

Equation (7) establishes a direct, linear relationship between the scale parameter λ and the percentile lifetime t_q. For a fixed quantile q, any improvement in the lifetime of product (a larger t_q is equivalent to an increase in the scale parameter λ of the underlying distribution. For the purpose of designing the sampling plan, it is useful to express the scale parameter in terms of the percentile. Solving Equation (7) for λ gives:

$$\lambda = \frac{t_q}{\sqrt{\pi}erf^{-1}(q)} \tag{8}$$

Equation (8) establishes a critical relationship between the distribution parameter λ and the quality characteristic t_q to be controlled. This expression enables the complete characterization of the lifetime distribution based on a specified percentile value.

We now derive the probability that a unit will fail during the truncated life test. To make the test practical and time-bound, it is terminated at a pre-specified time t_0. To connect this time to our percentile-based quality standard, we define it as a multiple of the specified percentile standard, ${t_q}^0$ as

$t_0 = \tau_q t_q^0$

Here, τ_q is the termination time multiplier. For example, if the specified $10th$ percentile lifetime ${t_q}^0$ is 1000 hours and $\tau_q = 0.5$, the life test is stopped at 500 hours. The key variable for our attribute-based sampling plan is p, the probability that a single unit fails before the test time t_0. This is given directly by the CDF evaluated at t_0:

$$p = F(t_0) = erf\left(\frac{t_0}{\lambda\sqrt{\pi}}\right) \tag{9}$$

Equation (9) depends on the scale parameter λ. Our goal is to express the failure probability p in terms of the ratio $d = \frac{t_q}{{t_q}^0}$, which measures how much the true, unknown percentile t_q exceeds the required standard ${t_q}^0$. A lot is considered acceptable if $d \geq 1$.

We achieve this by substituting the expression for λ from Equation (8) into Equation (9). We also substitute the definition of the test time $t_0 = \tau_q t_q^0$

$p = erf\left(\frac{\tau_q t_q^0}{\left(\frac{t_q}{\sqrt{\pi}erf^{-1}(q)}\right)\sqrt{\pi}}\right)$

Simplifying this expression, the $\sqrt{\pi}$ terms cancel out:

$p = erf\left(\frac{\tau_q t_q^0 erf^{-1}(q)}{t_q}\right)$

Finally, we introduce the ratio $d = \frac{t_q}{{t_q}^0}$

$$\mathrm{p} = \mathrm{erf}\left(\frac{\tau_q \mathrm{erf}^{-1}(\mathrm{q})}{\mathrm{d}}\right) \tag{10}$$

Practical Interpretation and Use: Equation (10) is the cornerstone for designing our sampling plan.

- It defines the failure probability (p) as a function of three known or chosen values: the termination time multiplier (τ_q), the life percentile (q), and the critical ratio (d).
- The equation has an intuitive behavior: if a lot has high quality (d) is large, meaning the true percentile (t_q) is much larger than the required $({t_q}^0)$, then the argument of the error function is small, leading to a small failure probability (p). Conversely, a bad lot (d) close to 1) will have a higher failure probability (p).
- In the optimization model (Section 2.5), we use this formula to calculate the specific failure probabilities for the Acceptable Quality Level (AQL) scenario, where $(d > 1)$ $(e.g\ (d = 2,4,6 ...)$, and the Limiting Quality Level (LQL) scenario, where $(d = 1)$. These probabilities, $(p_1 \text{and } p_2)$, are then used to find the plan parameters (n, c_a, c_r) that satisfy the producer's and consumer's risk constraints.

2.3. The Repetitive Acceptance Sampling Plan

The operational procedure for the proposed RSP under a truncated life test is as follows:

1. Draw a Sample: Select a random sample of n units from the lot and subject them to a life test until the predetermined time $t_0 = \tau_q t_q^0$.
2. Make a Decision:

- Accept the lot if the number of failures D observed by time t_0 is less than or equal to the acceptance number c_a.
- Reject the lot if D exceeds the rejection number c_r $(where\ c_r > c_a)$.
- Repeat the process (return to Step 1) if $c_a < D \le c_r$.

The plan is characterized by the parameters (n, c_a, c_r). Note that if$c_a = c_r$, the RSP reduces to a standard single sampling plan.

2.4. Operating Characteristic (OC) and Average Sample Number (ASN)

The performance of a sampling plan is evaluated by its Operating Characteristic (OC) function, which gives the probability of accepting a lot as a function of the failure probability p. For a RSP, the OC function is given by (Sherman, 1965):

$$OC(p) = \frac{P_a}{P_a + P_r} \tag{11}$$

where $P_a and\ P_r$ denote the probabilities of acceptance and rejection for a single sample, respectively. These probabilities are computed from the binomial distribution, with P_a given by the cumulative probability as

$$P_a = P(d \le c_a) = P_a = \sum_{d=0}^{c_a} \binom{n}{d} p^d (1-p)^{n-d}$$

$$P_a = \sum_{d=0}^{c_a} \binom{n}{d} \left\{ erf\left(\frac{\tau_q\,(erf^{-1}(q))}{\left(\frac{t_q}{t_q^0}\right)} \right) \right\}^d \left\{ 1 - erf\left(\frac{\tau_q\,(erf^{-1}(q))}{\left(\frac{t_q}{t_q^0}\right)} \right) \right\}^{n-d} \tag{12}$$

and P_r defined as

$$P_r = P(d > c_r) = 1 - P(d \le c_r) = P_r = 1 - \sum_{d=0}^{c_r} \binom{n}{d} P^d (1-P)^{n-d}$$

$$P_r = 1 - \sum_{d=0}^{c_r} \binom{n}{d} \left\{ erf\left(\frac{\tau_q\,(erf^{-1}(q))}{\left(\frac{t_q}{t_q^0}\right)} \right) \right\}^d \left\{ 1 - erf\left(\frac{\tau_q\,(erf^{-1}(q))}{\left(\frac{t_q}{t_q^0}\right)} \right) \right\}^{n-d} \tag{13}$$

The finally OC function for proposed plan is given as

$$OC(p) = \frac{\sum_{d=0}^{c_a} \binom{n}{d} \{p\}^d \{1-p\}^{n-d}}{\sum_{d=0}^{c_a} \binom{n}{d} \{p\}^d \{1-p\}^{n-d} + 1 - \sum_{d=0}^{c_r} \binom{n}{d} \{p\}^d \{1-p\}^{n-d}} \tag{14}$$

The Average Sample Number (ASN), representing the average number of units inspected before a final decision (accept or reject) is reached, is:

$$ASN = \frac{n}{P_a + P_r} \tag{15}$$

$$ASN = \frac{n}{\sum_{d=0}^{c_a} \binom{n}{d} \{p\}^d \{1-p\}^{n-d} + 1 - \sum_{d=0}^{c_r} \binom{n}{d} \{p\}^d \{1-p\}^{n-d}} \tag{16}$$

2.5. Optimization Model for Plan Parameters and Justification of AQL/LQL Selection

The design parameters of the proposed Repetitive Sampling Plan (RSP) namely the sample size (n), acceptance number (c_a), and rejection number (c_r), are determined by solving a constrained optimization problem. The objective is to minimize the inspection effort while rigorously protecting both the producer and the consumer by controlling their respective risks.

2.5.1. Definition of Quality Levels and Associated Risks

In acceptance sampling, two critical quality levels are defined:

- Acceptable Quality Level (AQL): This represents the worst permissible quality level that is still considered satisfactory for the process average. A lot operating at the AQL should have a high probability of acceptance. The producer's risk (α) is the probability of incorrectly rejecting a lot that is at the AQL. In this study, we fix $\alpha = 0.05$, meaning a good lot has a 95% chance of acceptance.
- Limiting Quality Level (LQL): Also known as the Lot Tolerance Percent Defective (LTPD), this represents the worst quality level that the consumer is willing to accept on an individual lot basis. The consumer's risk (β) is the probability of incorrectly accepting a lot that is at the LQL. We consider various consumer risk levels $(\beta = 0.25, 0.10, 0.05, 0.01)$ to cater to different stringency requirements.

In the context of our percentile-based life test, quality is defined by the ratio $d = \frac{t_q}{{t_q}^0}$ where t_q is the true percentile of the lot and ${t_q}^0$ is the specified standard.

- LQL Definition: The most conservative and universally accepted definition for the LQL is the point where the product merely meets the specified standard, i.e., its true percentile equals the required percentile. Therefore, we set the LQL at d = 1 $(t_q = {t_q}^0)$. A lot with $d = 1$ is on the boundary of being unacceptable, and the sampling plan is designed to have a low probability (β) of accepting it.
- AQL Definition: The AQL represents a lot with good quality, where the true percentile is substantially larger than the required standard. To provide flexibility for different quality regimes and to comprehensively study the performance of proposed plan, we consider AQL values corresponding to $d = 2, 4, 6, and\ 8$. This range is chosen for the following practical reasons:
    - ➢ $d = 2$ signifies that the true lifetime percentile is twice the required standard, representing a good safety margin.
    - ➢ $d = 4, 6, 8$ represent increasingly higher quality levels, from very good to excellent. A producer consistently operating at $d = 8$ is delivering a product whose lifetime percentile is eight times the minimum requirement.
    - ➢ Presenting plans for this range allows a quality manager to select a plan that matches their specific quality capability and cost-of-quality balance. A more stringent AQL (higher d) will result in a plan with a smaller ASN, rewarding high-quality producers with reduced inspection effort.

The failure probabilities (p_1 and p_2) associated with the AQL and LQL are derived from Equation (10). For a given τ_q *and* q :

- $p_1 = \mathrm{erf}\left(\frac{\tau_q \mathrm{erf}^{-1}(q)}{d_1}\right)$, where d_1 is the AQL ratio $(2, 4, 6, 8)$.
- $p_2 = \mathrm{erf}\left(\frac{\tau_q \mathrm{erf}^{-1}(q)}{d_2}\right)$, since $d_2 = 1$ for the LQL.

2.5.2. The Optimization Model

The optimal parameters (n, c_a, c_r) are found by solving the following model:

Minimize:

$$\mathrm{ASN}(p_2) \tag{17}$$

The objective is to minimize the Average Sample Number at the LQL, which directly minimizes the average inspection effort for the worst-case scenario we are protecting against.

Subject to:

$$OC(p_1) = \frac{\sum_{d=0}^{c_a}\binom{n}{d}\{p\}^d\{1-p\}^{n-d}}{\sum_{d=0}^{c_a}\binom{n}{d}\{p\}^d\{1-p\}^{n-d}+1-\sum_{d=0}^{c_r}\binom{n}{d}\{p\}^d\{1-p\}^{n-d}} \geq 1 - \alpha \tag{18}$$

And

$$OC(p_2) = \frac{\sum_{d=0}^{c_a} \binom{n}{d} \{p\}^d \{1-p\}^{n-d}}{\sum_{d=0}^{c_a} \binom{n}{d} \{p\}^d \{1-p\}^{n-d} + 1 - \sum_{d=0}^{c_r} \binom{n}{d} \{p\}^d \{1-p\}^{n-d}} \leq \beta \qquad (19)$$

Interpretation of the Constraints:

- Constraint (18) ensures the producer is protected. It guarantees that if the quality of lot is at the chosen AQL ($d \geq 2$), the probability of acceptance will be at least $1 - \alpha$ (*i.e., at least* 95%).
- Constraint (19) ensures the consumer is protected. It guarantees that if the quality of lot is at the LQL ($d = 1$), the probability of acceptance will be no more than β (*i.e., no more than* 25%, 10%, 5%, *or* 1%).

This optimization model was implemented and solved using a systematic search algorithm in the R programming environment. The algorithm iterates over feasible integer values of (n, c_a, *and* c_r) to find the combination that satisfies both risk constraints while yielding the smallest possible $ASN(p_2)$. The following algorithm is used for calculating the parametric values of proposes plan ,The results of this optimization are presented in Tables 1-4.

This optimization model was implemented and solved using a systematic search algorithm developed in the R programming environment. The algorithm iteratively explores all feasible integer combinations of (n, c_a, *and* c_r) to identify the set of design parameters that satisfy both producer's and consumer's risk constraints while minimizing the Average Sample Number (ASN) at the consumer's risk point p_2. The complete computational procedure used to determine the optimal parametric values of the proposed plan is outlined below. The optimized results are summarized in Tables 2–5.

Algorithm 1. Optimization Procedure for Determining the Plan Parameters (n, c_a, c_r)

1. **Input:** Specify the parameters ($(\alpha, \beta, \tau_q, q, p_1 = AQL, p_2 = 1 = LQL)$.
2. **Initialize**: Set feasible integer ranges for the design parameters (n, c_a, c_r) such that ($0 \leq c_a < c_r < n$).
3. **Iterative Search**:
   a. For each combination of (n, c_a, c_r), compute the acceptance probability using the binomial distribution and the OC function given in Equation (14).
   b. Evaluate the Operating Characteristic (OC) function at both quality levels $d_1 = AQL, d_2 = 1 = LQL$ using Equations (18) and (19).
   c. Check the risk constraints:
   $OC(p_1) \geq 1 - \alpha$ and $OC(p_2) \leq \beta$
   Retain only those parameter sets that satisfy both inequalities.
4. **Compute Efficiency Criterion:** For each feasible solution, calculate the corresponding Average Sample Number (ASN) using Equation (16).
5. **Optimization:** Select the combination $(n^*, c_a{}^*, c_r{}^*)$ that yields the minimum $ASN(p_2)$).
6. **Output:** Report the optimal plan parameters $(n^*, c_a{}^*, c_r{}^*)$, along with the associated OC and ASN values.

Table 2. *Design Parameters of the Proposed RSP Based on the HND* $(\tau_q = 0.5, q = 0.25)$

β	d	c_a	c_r	n	ASN	$OC(p_1)$
0.25	**2**	**3**	**6**	**48**	**90**	**0.9536**
	4	1	2	24	31	0.9542
	6	0	1	14	20	0.9555
	8	0	1	14	20	0.9753
0.10	**2**	**7**	**10**	**95**	**128**	**0.9507**
	4	2	3	42	48	0.9510
	6	1	2	31	36	0.9690
	8	0	1	19	24	0.9533
0.05	**2**	**4**	**9**	**77**	**142**	**0.9505**
	4	1	3	38	50	0.9538
	6	0	2	26	38	0.9708
	8	0	2	26	38	0.9881
0.01	**2**	**6**	**12**	**56**	**183**	**0.9535**
	4	1	4	52	64	0.9538
	6	0	3	37	52	0.9834
	8	0	2	36	42	0.9666

Table 3. *Design Parameters of the Proposed RSP Based on the HND* $(\tau_q = 0.5, q = 0.5)$

β	d	c_a	c_r	n	ASN	$OC(p_1)$
0.25	**2**	**2**	**5**	**18**	**41**	**0.9565**
	4	1	2	11	15	0.9616
	6	0	1	6	9	0.9660
	8	0	1	6	9	0.9813
0.10	**2**	**6**	**9**	**40**	**57**	**0.9536**
	4	0	2	10	18	0.9514
	6	1	2	14	17	0.9747
	8	0	1	9	11	0.9548
0.05	**2**	**4**	**9**	**37**	**68**	**0.9546**
	4	1	3	18	23	0.9574
	6	0	2	12	18	0.9752
	8	0	2	12	18	0.9900
0.01	**2**	**4**	**11**	**45**	**78**	**0.9519**
	4	1	4	24	30	0.9628
	6	0	3	18	24	0.9832
	8	0	2	16	19	0.9741

Table 4. *Design Parameters of the Proposed RSP Based on the HND* $(\tau_q = 1, q = 0.25)$

β	d	c_a	c_r	n	ASN	$OC(p_1)$
0.25	**2**	**3**	**6**	**25**	**47**	**0.9506**
	4	1	2	12	16	0.9576
	6	0	1	7	10	0.9576
	8	0	1	7	10	0.9766
0.10	**2**	**3**	**7**	**29**	**59**	**0.9529**
	4	2	3	21	24	0.9544
	6	1	2	15	18	0.9735
	8	0	1	9	12	0.9598
0.05	**2**	**6**	**10**	**47**	**68**	**0.9533**
	4		3	19	25	0.9575
	6	0	2	12	19	0.9793
	8	0	2	12	19	0.9916
0.01	**2**	**6**	**12**	**56**	**83**	**0.9535**
	4	1	4	25	32	0.9652
	6	0	3	19	26	0.9832
	8	0	2	17	20	0.9736

Table 5. *Design Parameters of the Proposed RSP Based on the HND* $(\tau_q = 1, q = 0.5)$

β	d	c_a	c_r	n	ASN	$OC(p_1)$
0.25	**2**	**4**	**6**	**13**	**21**	**0.9588**
	4	1	2	6	8	0.9587
	6	0	1	3	5	0.9709
	8	0	1	3	5	0.9843
0.10	**2**	**5**	**8**	**18**	**28**	**0.9605**
	4	0	2	5	9	0.9616
	6	1	2	7	8	0.9787
	8	0	1	4	5	0.9683
0.05	**2**	**7**	**10**	**24**	**32**	**0.9576**
	4	1	3	9	12	0.9659
	6	0	2	6	9	0.9799
	8	0	2	6	9	0.9921
0.01	**2**	**7**	**12**	**29**	**37**	**0.9517**
	4	0	3	8	12	0.9566
	6	0	3	9	12	0.9871
	8	0	2	8	9	0.9775

Explanation of Tables 2–5

Tables 2–5 present the design characteristics of the proposed repetitive acceptance sampling plan under the Half Normal Distribution. The comparisons are made across different values of consumer's risk ($\beta = 0.25, 0.10, 0.05, 0.01$), percentile ratios ($d = 2, 4, 6, 8$), termination time multipliers ($\tau_q = 0.50, 1.0$), and life percentiles ($q = 0.25, 0.50$), while the producer's risk is fixed at $\alpha = 0.05$. To enhance clarity, the main findings from Tables 1–4 are also illustrated in Figure 1, which graphically depicts the behavior of the Average Sample Number (ASN) across different parameter combinations. The main findings are summarized below:

- Effect of percentile ratio (d): An increase in d from $2\ to\ 8$ consistently reduces the average sample number (ASN). *Example:* In Table 2, when $\beta = 0.25$, ASN decreases from $90\ (d = 2)\ to\ 20\ (d = 8)$. In Table 3, when $\beta = 0.05$, ASN decreases from $68\ (d = 2)\ to\ 18\ (d = 8)$.
- Effect of consumer's risk (β): For fixed d, reducing β (i.e., imposing stricter consumer protection) results in higher ASN. *Example:* In Table 2, for $d = 2$, ASN increases from $90\ (\beta = 0.25)\ to\ 183\ (\beta = 0.01)$. Similar trends are observed in Tables 2–4.
- Effect of termination time multiplier (τ_q): At fixed q, increasing τ_q lowers the ASN. *Example:* For $q = 0.25, \beta = 0.25, and\ d = 2$, ASN is 90 when $\tau_q = 0.50$ (Table 2), which reduces to 47 when $\tau_q = 1.0$ (Table 4).
- Effect of life percentile (q): Interestingly, ASN values are consistently smaller for the median life ($q = 0.50$) compared to the first quartile life ($q = 0.25$), across both $\tau_q = 0.50\ and\ \tau_q = 1.0$. *Example:* When $\tau_q = 0.50, \beta = 0.25, and\ d = 2$, ASN decreases from $90\ (q = 0.25, Table\ 1)\ to\ 41\ (q = 0.50, Table\ 2)$. Similarly, when $\tau\_q = 1.0$, ASN decreases from $47\ (q = 0.25, Table\ 3)\ to\ 21\ (q = 0.50, Table\ 4)$.
- Overall behavior:
  - Increasing $d \rightarrow ASN$ decreases.
  - Decreasing $\beta \rightarrow ASN$ increases.
  - Increasing$\tau_q \rightarrow ASN$ decreases.
  - Changing q from $0.25\ to\ 0.50 \rightarrow ASN$ decreases.

In all cases, the acceptance probabilities remain close to the nominal producer's risk level (≈ 0.95), demonstrating that the proposed plan provides the intended level of protection.

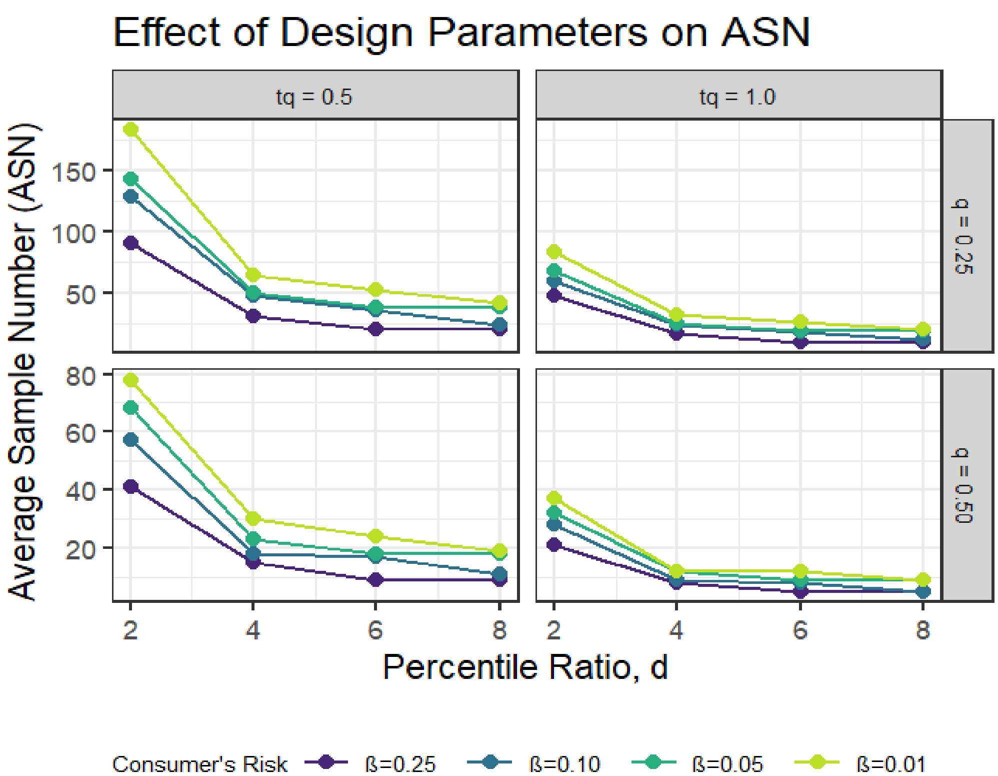

Figure 1: Effects of Design Parameters on ASN

3. Efficiency Comparison: Repetitive vs. Single Sampling plan

A comparative analysis is conducted to evaluate the practical efficiency of the proposed repetitive sampling plan against the established single sampling plan for life tests based on percentiles of the Half-Normal Distribution, as developed by (Rao, Kumar, & Rosaiah, 2013). The comparison metric is the Average Sample Number (ASN), which directly reflects the inspection effort required. The results, summarized in Table 6 for a fixed producer's risk ($\alpha = 0.05$) and the $50th$ percentile ($q = 0.5$), demonstrate a decisive advantage for the repetitive sampling framework. Across all considered consumer's risk levels ($\beta \ = \ 0.25, 0.10, 0.05, 0.01$) and increasing values of the percentile ratio (d), the proposed plan consistently yields a lower ASN than its single sampling counterpart.

The Table value reveals a clear trend, while the ASN decreases as d increases for both plans, the proposed repetitive plan maintains a significantly smaller sample size requirement. For instance, under a consumer's risk of $\beta = 0.05$, the ASN for the single sampling plan is 101 at $d = 2$, compared to only 68 for the repetitive plan, a substantial reduction. This efficiency gain persists even at higher percentile ratios; at $d = 8$, the ASN drops from 22 for the single plan to 18 for the proposed plan. The graphical representation in Figure 1 for $\beta = 0.05$ further reinforces this finding, where the plot for the repetitive plan lies uniformly below that of the single sampling plan, visually confirming its superior performance across the entire range of d.

The consistent reduction in ASN achieved by the proposed plan underscores its significant operational benefits. By minimizing the number of items needed for inspection without compromising the stipulated statistical risks, the plan directly translates to lower testing costs, reduced labor, and lessened resource consumption. Therefore, the repetitive sampling plan emerges as a more economical and efficient alternative for quality assurance testing while providing equivalent protection to both producers and consumers.

Table 6. Comparison of Proposed RSP with Single Sampling Plan using HND

$\tau_q = 0.5, q = 0.5$			
		Proposed sampling plan	**Single sampling plan proposed by (Rao et al., 2013)**
β	d	ASN	n
0.25	2	41	48
	4	15	19
	6	9	14
	8	9	10
0.10	2	57	82
	4	18	29
	6	17	24
	8	11	19
0.05	2	68	101
	4	23	37
	6	18	27
	8	18	22
0.01	2	78	151
	4	30	56
	6	24	40
	8	19	35

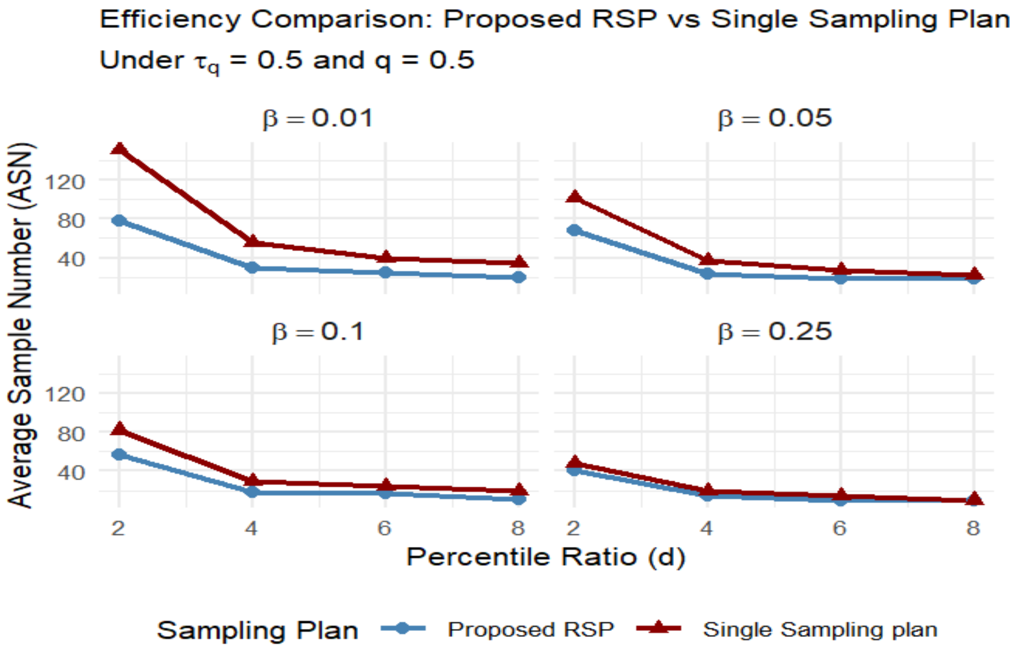

Figure 2: Comparison of ASN for Proposed RSP vs Single sampling Plan

4. Real-World Illustration of the Proposed Plan

To demonstrate the practical applicability and robustness of the proposed RSP based on the HND, the methodology is applied to two distinct real-life datasets from different reliability contexts. The first dataset concerns software failure times, representing complex system reliability, while the second involves the lifetime of electronic devices, representing mechanical or component wear-out. These contrasting examples collectively illustrate the flexibility of plan and effectiveness in making reliable acceptance decisions under strict risk constraints.

4.1. Industrial Application: Software Reliability data

The practical implementation of the RSP is first demonstrated using a well-known software reliability dataset comprising failure times (in hours): 519, 968, 1430, 1893, 2490, 3058, 3625, 4422, and 5218. Before applying the sampling plan, the suitability of the HND for this data was rigorously validated. The Kolmogorov–Smirnov test yielded a statistic of $D = 0.152$ with a p-value of 0.095, indicating no significant deviation from the HND. Furthermore, a Q–Q plot analysis by (Rao et al., 2013) revealed a high correlation coefficient ($R = 0.9287$), reinforcing the suitability of the HND.

Enhanced Goodness-of-Fit Analysis

A comparative analysis was conducted against common lifetime distributions to provide a comprehensive assessment:

- Half-Normal Distribution: KS, $D = 0.152, p - value = 0.095, AIC = 128.4$
- Weibull Distribution: KS, $D = 0.178, p - value = 0.042, AIC = 129.8$
- Exponential Distribution: KS, $D = 0.231, p - value = 0.008, AIC = 132.1$
- Lindley Distribution**:** KS, $D = 0.255, p - value = 0.003, AIC = 134.9$

The Half-Normal Distribution demonstrates superior performance for this dataset, with the highest $p - value$ (0.095) and the lowest Akaike Information Criterion (AIC) value (128.4). This indicates a better balance between goodness-of-fit and model complexity compared to the alternatives. The Weibull, Exponential, and Lindley distributions all show significantly lower p-values, confirming their inferior fit for this specific software reliability data.

Implementation Scenario

Consider a scenario where a quality engineer must establish with high confidence that the $50th$ percentile lifetime ($t_{0.5}$) of a submitted software lot is at least 300 hours. The producer and consumer stipulate strict risk thresholds of ($\alpha = 0.05$ and $\beta = 0.25$), respectively. The life test is truncated at 150 hours, resulting in a termination time $\tau_q = 0.5$. For these parameters ($q = 0.5, \tau_q = 0.5, \alpha = 0.05$ and $\beta = 0.25, d = 2$), the optimal parameters for the proposed repetitive sampling plan are obtained from Table 3 as: acceptance number $c_a = 2$, rejection number $c_r = 5$, and a sample size per iteration of $n = 18$.

The quality engineer implements the plan as follows: a first random sample of 18 items is drawn from the lot and subjected to the life test for 150 hours. Based on the historical data, no failures would be observed before the truncation time (since the first failure occurs at 519 hours). Since the number of failures $D = 0$ is less than or equal to the acceptance number $c_a = 2$, the lot is accepted immediately on the first sample. The plan achieves this decision with an Average Sample Number (ASN) of 41, meaning that under similar quality conditions, the average inspection effort per lot is 41 items. This demonstrates the efficiency of the repetitive scheme, which minimizes the required sample size while rigorously maintaining the pre-specified producer's and consumer's risk levels, thereby reducing inspection costs and effort.

4.2 Device Lifetime Data Application

To demonstrate the broad applicability and practical implementation of the proposed methodology across different industrial domains, we apply the RSP to the well-established Aarset dataset comprising 50 device lifetimes. The dataset, consisting of failure times (in hours): 0.1, 0.2, 1, 1, 1, 1, 1, 2, 3, 6, 7, 11, 12, 18, 18, 18, 18, 18, 21, 32, 36, 40, 45, 46, 47, 50, 55, 60, 63, 63, 67, 67, 67, 67, 72, 75, 79, 82, 82, 83, 84, 84, 84, 85, 85, 85, 85, 85, 86, and 86, was rigorously tested for Half-Normal Distribution fit. The Kolmogorov-Smirnov test yielded $D = 0.089 \; with \; p - value = 0.078$, indicating a good statistical fit and supporting the validity of the HND assumption ($p > 0.05$). Furthermore, the Q–Q plot analysis by (Rao et al., 2013) showed a high correlation coefficient ($R = 0.9175$), reinforcing the suitability of the HND.

Enhanced Goodness-of-Fit Analysis

A similar comparative analysis for the Aarset data yielded:

- Half-Normal Distribution: KS D = 0.089, p-value = 0.078, AIC = 445.2
- Weibull Distribution: KS D = 0.095, p-value = 0.052, AIC = 443.8
- Exponential Distribution: KS D = 0.214, p-value = 0.001, AIC = 468.3
- Lindley Distribution**:** KS, $D = 0.208, p - value = 0.001, AIC = 466.5$

For the Aarset Device Data, the HND remains a highly acceptable and statistically viable model ($p - value = 0.078$). It demonstrates a significantly better fit than the Exponential and Lindley distributions, which are clearly unsuitable ($p - value = 0.001$). While the Weibull distribution shows a marginally competitive AIC, the strong fit of HND and its theoretical advantages make it a robust choice. The HND possesses an increasing failure rate (IFR) property, which aligns perfectly with the expected wear-out failure mechanisms observed in both mechanical (Aarset device) and complex systems (software aging). Using the HND for both applications ensures methodological consistency and provides a unified and physically interpretable model across different industrial domains.

Implementation Scenario

Consider a practical scenario in component manufacturing where quality specifications require that the 25th percentile lifetime ($t_{0.25}$) of submitted devices exceeds 10 hours. The producer and consumer agree on risk thresholds of ($\alpha = 0.05 \; and \; \beta = 0.10$), respectively. The life test is truncated at 5 hours, resulting in a termination time ratio $\tau_q = 0.5$. For these parameters ($q = 0.25, \tau_q = 0.5, \alpha = 0.05, \beta = 0.10, d = 4$), the optimal parameters obtained from Table 2 are: acceptance number $c_a = 2$, rejection number $c_r = 3$, and sample size per iteration $n = 42$, with an Average Sample Number (ASN) of 48.

The quality engineer implements the plan by drawing a random sample of 42 devices from the lot and subjecting them to life testing until 5 hours. Analysis of the Aarset dataset reveals 9 failures occurring before the truncation time (0.1, 0.2, 1, 1, 1, 1, 1, 2, and 3 hours). Since the observed number of failures $D = 9$ substantially exceeds the rejection number $c_r = 3$, the lot is immediately rejected on the first sample without requiring repetitive sampling. This decisive rejection action, achieved with ASN = 42, demonstrates the effectiveness of plan in identifying non-conforming lots while maintaining the specified consumer protection level. The complementary outcomes of the software reliability and device lifetime applications, acceptance of conforming software lots and rejection of non-conforming device lots, collectively demonstrate the balanced performance and cross-domain applicability of the proposed repetitive sampling methodology.

5. Concluding Remarks

This study presents an efficient Repetitive Sampling Plan for life testing using the Half-Normal Distribution, demonstrating substantial reductions in Average Sample Number while maintaining specified risk levels. The practical applicability of the proposed methodology was validated through two real-world case studies, software reliability data and device lifetime dataset, both demonstrating its effectiveness in achieving decisive lot disposition with minimal inspection effort. Comparative analysis confirms the superiority of proposed plan over single sampling, offering quality practitioners significant cost and resource savings. Future research could extend this approach to other distributions or integrate it with accelerated testing models for broader industrial applications.

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Declaration

Conflict of Study: The author declares that he has no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper

Author Contribution Statement: Satyendra Muhammad Naveed, Muhammad Iltaf, Muhammad Azam, Nasrullah Khan, Muhammad Saeed, and Muhammad Zahid Akbar conceived the idea and designed the research; Analyzed and interpreted the data, and wrote the paper.

Funding Statement: This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

Availability of Data and Material: Data will be made available by the corresponding author on reasonable request.

Consent to Publish: All authors have agreed to publish this manuscript in the SCOPUA Journal of Applied Statistical Research (JASR)-ISSN(e): 3104-4794.

Ethical Approval: Not Applicable

Consent to Participate: Not Applicable

Acknowledgment: Not Applicable

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