SCOPUA Journal of Applied Statistical Research (JASR)

Vol.2, Issue 2, June 2026
DOI: https://doi.org/10.64060/JASR2V2i4


# Bayesian Monitoring of Waiting-Time Processes using Exponential Distribution

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Received: 14 March 2026 / Revised: 04 May 2026 / Accepted: 10 May 2026 / Published online: 17 May 2026



ABSTRACT

The Bayesian control chart is an advanced statistical tool for monitoring process stability by incorporating prior knowledge with observed data. This study proposes a Bayesian control chart for monitoring waiting-time processes assuming an exponential distribution with a gamma conjugate prior. Posterior distributions of the process parameter are derived, and the posterior mean and standard deviation are used to construct dynamic control limits. A comprehensive simulation study is conducted by varying sample sizes and prior hyperparameters to evaluate the performance of the proposed chart. The results indicate that Bayesian control limits are generally narrower and more responsive to process shifts compared to classical control charts, leading to faster detection of out-of-control conditions. However, this increased sensitivity is associated with a higher false alarm rate, reflecting a trade-off between detection speed and in-control stability. The applicability of the proposed method is further demonstrated using real-life data, which supports the simulation findings. The study highlights the effectiveness of Bayesian approaches in process monitoring, particularly in situations with limited or uncertain data, while also emphasizing the importance of appropriate prior selection and control limit design. Future work may focus on developing probability-based or asymmetric control limits tailored for non-normal distributions.

**Keywords***:*Control chart; Bayesian control chart; Gamma Distribution; Exponential Distribution

## 1. Introduction

The two primary categories for control charting systems are classical and Bayesian. In contrast to traditional control structures, control charting structures based on Bayesian approaches are considered when the process parameters are unclear. Usually, previous knowledge is used to quantify parametric uncertainty, which the Bayesian setup effectively manages. The practical challenge with the Bayesian technique is that it requires more precise and easily accessible information about the process structure.

A control chart (CC) is a statistical technique designed to verify and then track the statistical stability of a process, often permitting graphical application. A control chart's primary function is to keep an eye on the process throughout the manufacturing phase (Abujiya and Muttlak, 2004). The manufacturing process keeps producing goods if the procedure is managed, according to the control chart. In this case, no more action is needed. The required actions are taken by the quality engineers to regain control of the process. Whether any movement in the process is depicted on the control chart (Hajej et al., 2021). The control chart serves as a tool for determining when corrective action is necessary. Consequently, the CC serves as the tool for assessing the necessity of corrective actions (Roberts, 1966). The CL, UCL, and LCL are the three main components of a univariate Shewhart chart (Bakir, 2004). Statistical methods for quality control are essential in many industrial and service sectors. Control of statistical quality is a subfield of manufacturing data mainly concerned with SPC, capacity analysis, design of experiments, and acceptance sampling. To avoid the process of overreacting or underreacting, one of the primary functions of control plots is to differentiate between differences resulting from assignable sources and dissimilarities resulting from random causes (Ali et al., 2016). The collection of data points about a particular process or product attribute is the foundation of control charts. These data points might be metrics like weights, counts, or dimensions, and they are usually collected regularly (Koutras et al., 2007). Control limits specify the range in which the process should function normally. In general, they are set to three standard deviations of the process mean. Using data collected about the process under observation, control charts are created. Product weights, dimensions, defect counts, and any other pertinent quality attribute measures might be included in this data (Sullivan and Jones, 2002). More information on the control chart can be seen in (Aslam et al., 2015), (Riaz and Ali, 2016) and (Abid et al., 2018).

A prior distribution represents the beliefs, knowledge, or uncertainty about the parameters of a statistical model before observing any data (Gelman, 2002). The parameters of interest were random variables with a prior distribution (PD). PD was determined by the information available at the time (Singh et al., 2013). The prior appropriately could need a lot of data. Therefore, a major influence on the Bayesian approach's accuracy might come from the quality of the priors (Touw, 2009). The Bayesian analysis focuses on the fact that how the prior distribution affects the posterior (Ghaderinezhad and Ley, 2019). The Bayesian method bases its decision on the appropriate control strategy on the posterior likelihood, upgraded after each sample that the process is out of control by the application of the Bayes theorem (Makis, 2008). The economic-statistical design of the Bayesian control chart is based on a predictive distribution with two types of informative and non-informative prior distributions (Seirani et al., 2023). A Bayesian chart is comparable to a Shewhart chart in which the bounds are quickly modified following each sample to reflect the most recent information available. Using Bayesian approaches, the decision-maker can include historical knowledge about the process's current state, which might come from expert opinion, past observations, or possibly other unrelated data (Celano et al., 2008). At each phase, the decision-maker obtains the likelihood that the process will eventually stabilize. This framework draws inspiration from the real options methodology (Moltchanova, 2017). The flexibility of Bayesian control charts allows them to adjust to varying sample sizes and amounts of previous knowledge. When there is a lack of expert knowledge or previous data, Bayesian control charts can nevertheless offer valuable insights by utilizing the information at hand to increase monitoring accuracy (Rasay et al., 2024).

Most existing research (Kalisetti et al., 2024) on control charts has primarily focused on classical estimation methods for parameter determination and chart construction. However,(Supharakonsakun, 2024) comparatively limited attention has been devoted to Bayesian estimation approaches that incorporate prior knowledge through suitable prior distributions. (Ali et al., 2025) Evaluating the effectiveness of classical and Bayesian control charts. (Supharakonsakun, 2024) introduced an improved classical control chart for monitoring nonconformities via the empirical Bayes approach. (Labha et al., 2025) discussed a Bayesian Exponentially Weighted Moving Average (EWMA) control chart designed for monitoring the shape parameter of lifetime Pareto distribution processes. Recent developments, such as the study by (Nafisah et al., 2025), proposed a variable sample size-based EWMA control chart with an exponential scaling mechanism for production process monitoring. (Joghee and Mathew, 2025) developed process shifts in the case of Six-Sigma based Bayesian Control Charts. Then, (Kanwal and Abbas, 2025) introduced a comparative analysis of quantile-based control charts for the French distribution, highlighting the increasing interest in exploring alternative statistical methodologies for process monitoring.

In this context, the objective of the present work is to develop a Bayesian control chart for the exponential distribution based on a gamma prior, and to evaluate its performance by comparing it with the traditional classical method. The exponential distribution is widely used in reliability analysis and lifetime data modeling, making it a natural candidate for quality control applications where failure times or inter-arrival times are of interest. The gamma prior, on the other hand, provides a mathematically convenient and flexible choice for Bayesian analysis, allowing incorporation of prior beliefs about the process parameter. This study not only aims to demonstrate the advantages of Bayesian approaches in terms of incorporating prior information but also to provide deeper insights into the efficiency and robustness of Bayesian control charts in real-world applications.

Although the Gamma–Exponential conjugate analysis is well known in Bayesian inference, its explicit use for constructing adaptive Shewhart-type control charts for exponential lifetime data has received very limited attention. Existing SPC literature primarily relies on classical estimators, and very few studies provide a complete Bayesian framework with dynamically updated limits. The present work fills this gap by integrating posterior updating into control limit calculation and systematically evaluating the effect of prior parameters and sample size on chart performance.

## 2. Methodology

### *2.1 Exponential Distribution*

The single-parameter exponential distribution serves as an educational tool for demonstrating parameter estimation methods (Elfessi and Reineke, 2001). The exponential density serves as a fundamental starting point for studying more general distributions because its analytical adjustments remain easy to perform. Using mathematical operations because they are straightforward enables us to explore and better understand the relationship between classical and Bayesian estimation methods (Rasheed and Abd, 2023).

The random variable follows a one-parameter Exponential Distribution where θ (θ>0) represents the parameter value in the associated Probability Density function for the continuous random variable X.

$$f(x,\theta)=\theta e^{-\theta x};\ 0< x<\infty \tag{1}$$

### *2.2 Gamma Distribution*

The GD is a continuous probability distribution. The probability of waiting for a firm amount of time before an event occurs. The gamma distribution predicts the wait time until future events. It has a connection to the normal distribution, the ED, and the Chi-squared distribution. "Γ" represents the Gamma function. A GD random variable θ with shape parameter "a" and rate parameter "b" (Son and Oh, 2006).

$$\theta \sim \Gamma(a,b) = Gamma(a,b) \tag{2}$$

In this study, the utilization of an informative conjugate prior is deemed suitable for the Bayesian approach. The corresponding PDF in the shape-rate parameterization is

$$f\left(\theta \mid a,b\right) = \frac{b^a}{\Gamma(a)}\theta^{a-1}e^{-b\theta}; \qquad \theta > 0 \ and \ a,b > 0 \tag{3}$$

The posterior distribution can be derived as follows:

$$P(\theta/x) = \frac{p(\theta).L(x/\theta)}{\int_\theta p(\theta).L(x/\theta)d\theta} \tag{4}$$

Where $L(x;\theta)$ represents the likelihood function of the Exponential probability function. Therefore, the Posterior distribution can be presented as follows:

$$f(\theta/x) = \frac{\beta^\alpha}{\Gamma(\alpha)}\theta^{\alpha-1}e^{-\beta\theta} \tag{5}$$

The Posterior distribution of the parameter $\theta$ is a gamma distribution, denoted as Gamma (a+n, $b+\sum_{i=1}^{n}x_i$ ). It can be expressed in the following form:

$$f(\theta/x) = \frac{(b+\sum_{i=1}^{n}x_i)^{a+n}}{\Gamma(a+n)}\theta^{(a+n)-1}e^{-(b+\sum_{i=1}^{n}x_i)\theta} \tag{6}$$

is the posterior distribution of the Gamma Prior Distribution for the case of Exponential distribution.

***2.3 Bayes Estimation***

The estimation of the parameter θ of the posterior distribution is determined such as Posterior Mean and Posterior Variance

2.3.1 Posterior Mean

The Posterior mean is defined as

$$mean = \mu = E(\theta/x) \tag{7}$$

As we know that the formula of expectation is

$$E(\theta/x) = \int_0^\infty \theta.f(\theta/x)d\theta$$

$$E(\theta/x) = \int_0^\infty \theta.\frac{\beta^\alpha}{\Gamma(\alpha)}\theta^{\alpha-1}e^{-\beta\theta}d\theta$$

$$E(\theta/x) = \frac{\beta^\alpha}{\Gamma(\alpha)}\int_0^\infty (\frac{v}{\beta})^{(\alpha+1)-1}e^{-v}\frac{dv}{\beta}$$

$$E(\theta/x) = \frac{\alpha}{\beta} \tag{8}$$

Hence, the mean of the posterior distribution is

$$where \ \alpha = a+n \quad and \quad \beta = b+\sum_{i=1}^{n}x_i$$

$$E(\theta/x) = \frac{a+n}{b+\sum_{i=1}^{n}x_i} \tag{9}$$

2.3.2 Posterior Variance

The Posterior Variance is defined as

$$\operatorname{var}(\theta) = \sigma^2 = E(\theta^2/x) \tag{10}$$

As we know that the formula of variance is

$$\operatorname{var}(\theta/x) = E(\theta^2/x) - [E(\theta/x)]^2$$

$$E(\theta^2/x) = \int_0^\infty \theta^2 . f(\theta/x) d\theta$$

Now,

$$E(\theta^2/x) = \int_0^\infty \theta^2 . \frac{\beta^\alpha}{\Gamma(\alpha)} \theta^{\alpha-1} e^{-\beta\theta} d\theta$$

$$E(\theta^2/x) = \frac{(a+n+1)a+n}{(b+\sum_{i=1}^{n} x_i)^2}$$

Put this in the variance formula,

$$var(\theta/x) = \frac{\alpha(\alpha+1)}{\beta^2} - \left[\frac{\alpha}{\beta}\right]^2$$

Hence, the variance of the posterior distribution is

$$var(\theta/x) = \frac{a+n}{(\mathrm{b}+\sum_{i=1}^{n} x_i)^2} \tag{11}$$

3.3.3 Posterior Standard Deviation

The Standard Deviation of posterior distribution is

$$S.D(\theta/x) = \sqrt{\frac{a+n}{(\mathrm{b}+\sum_{i=1}^{n} x_i)^2}} \tag{12}$$

### *2.4 Control Limits*

Establishing control limits is typically based on estimating the Posterior mean and posterior standard deviation. The control limit of the posterior distribution is defined as follows:

2.4.1 Lower Control Limit

The Lower Control Limit of Posterior distribution is defined as

$$LCL = mean - 3\,S.D \tag{13}$$

$$LCL = \mu - 3\sigma$$

$$LCL = \frac{a+n}{\mathrm{b}+\sum_{i=1}^{n} x_i} - 3\sqrt{\frac{a+n}{(\mathrm{b}+\sum_{i=1}^{n} x_i)^2}} \tag{14}$$

2.4.2 Upper Control Limit

The Upper Control Limit of Posterior distribution is defined as

$$UCL = mean + 3\,S.D \tag{15}$$

$$UCL = \mu + 3\sigma$$

$$UCL = \frac{a+n}{\mathrm{b}+\sum_{i=1}^{n} x_i} + 3\sqrt{\frac{a+n}{(\mathrm{b}+\sum_{i=1}^{n} x_i)^2}} \tag{16}$$

2.4.3 Central Limit

The Central Limit of Posterior distribution is defined as

$$CL = mean$$

$$CL = \frac{a+n}{\mathrm{b}+\sum_{i=1}^{n} x_i} \tag{17}$$

In the proposed Bayesian control chart, the monitoring statistic is the posterior distribution of the exponential rate parameter θ rather than the predictive distribution of future observations. This choice is intentional and consistent with the class of parameter-based Bayesian control charts discussed in earlier studies (Makis, 2008; Celano et al., 2008; Seirani et al., 2023). For lifetime and reliability-type data, the rate parameter θ directly reflects the underlying failure behavior of the process; therefore, tracking its evolution provides a more stable and interpretable signal of process changes compared to charts based solely on raw observations, which may exhibit high variability. Posterior-based monitoring naturally incorporates prior knowledge and new information at every sampling stage, producing smoother control limits and reducing the likelihood of false alarms, particularly when sample sizes are small or moderate. By updating θ after each subgroup, the proposed chart focuses on detecting genuine shifts in the process failure rate, which aligns with the practical objectives of reliability-oriented process monitoring. The implications of this design choice and its relevance to exponential-based processes are now explicitly discussed to clarify the advantages of parameter-driven monitoring within a Bayesian framework.

**Algorithm for Simulation Study**

**Pseudo-Code**

**Step 1:** Select parameter values
- Choose rate parameter θ = 1, 1.5, 2.
- Choose sample sizes n = 25, 50, 100, 150
- Choose prior parameters α = 1, 1.5, 2 and β = 0.5, 1, 1.5, 2.

**Step 2:** Generate data
- Generate random samples X1,X2,...,Xn ∼ Exponential (θ)

**Step 3:** Specify prior distribution
- Assume θ ∼ Gamma (α,β)

**Step 4:** Compute posterior distribution
- Update parameters:
  - $\alpha* = \alpha+n$
  - $\beta* = \beta+\sum Xi$

**Step 5:** Compute posterior estimates
- Posterior Mean:

$$\mu=\alpha*/\beta*$$

- Posterior Variance:

$$\sigma^2 = \alpha*/ (\beta*)^2$$

- Posterior Standard Deviation:

$$\sigma = \sqrt{\sigma^2}$$

**Step 6:** Construct control limits
- Central Line (CL): μ
- Upper Control Limit (UCL): μ + 3σ
- Lower Control Limit (LCL): μ − 3σ

**Step 7:** Repeat simulation
- Repeat Steps 2–6 for all combinations of parameters

**Step 8:** Compare results
- Compare Bayesian control limits with classical control limits
- Evaluate performance using ARL measures

The flowchart of the methodology applied in the simulation study is shown in Figure 1.

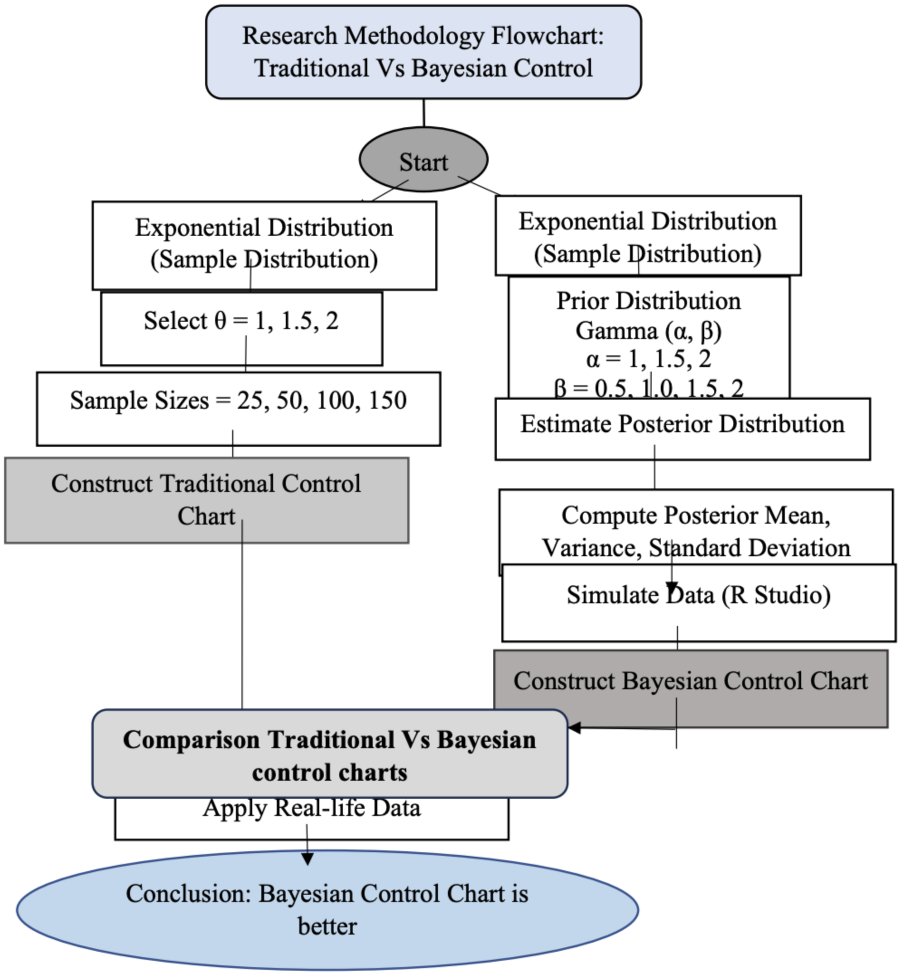


**Figure 1:** Flowchart of the methodologyy applied in the simulation study

## 3 Results

This study aims to develop and evaluate a Bayesian control chart specifically for the exponential distribution using a gamma prior. The research seeks to establish a more flexible and insightful approach to quality management by integrating Bayesian statistics with traditional control chart methodologies. This research aims to bridge the gap between traditional quality control methods and modern Bayesian statistical techniques by integrating prior knowledge and real-time data to enhance decision-making in process monitoring. Through comprehensive simulation studies utilizing R software, the research compares the performance of these Bayesian control charts with traditional control charts derived from frequentist methods. The numerical simulation study considers parameter values of $n = 25, 50, 100, 150$, and $\theta = 1, 1.5, 2.0$. The hyper parameter values of the gamma prior for the proposed Bayesian method are set to $(\alpha, \beta) = (1, 1.5, \text{and } 2.0)$ and $(0.5, 1.0, 1.5, \text{and } 2.0)$. These parameter values are applied in the proposed Bayesian method to determine the control limit such as UCL, LCL, and CL.

Although the control limits in this study are constructed using the conventional mean ± 3 standard deviation approach, it is important to note that the exponential distribution is positively skewed. Therefore, symmetric control limits may not be fully efficient for such distributions. The use of 3-sigma limits in this study is primarily intended to ensure comparability with classical control charts. Future research may consider the development of probability-based or quantile-based control limits that better accommodate the asymmetric nature of the exponential distribution.

**Table 1:** Control Limits for the Posterior Distribution of Exponential Distribution with $\theta = 1$ by using gamma prior with $\alpha=1$ at different values of $\beta$

| $\alpha = 1.5$, $\theta = 1$ | | | | | | | |
|---|---|---|---|---|---|---|---|
| $\beta$ | n | $\mu$ | $\sigma^2$ | $\sigma$ | CL | UCL | LCL |
| 0.5 | 25 | 1.0981 | 0.0263 | 0.1622 | 1.0981 | 1.5849 | 0.6113 |
| | 50 | 1.1274 | 0.0378 | 0.1945 | 1.1274 | 1.7112 | 0.5437 |
| | 100 | 1.1672 | 0.0376 | 0.1939 | 1.1672 | 1.7489 | 0.5854 |
| | 150 | 1.1695 | 0.0467 | 0.2162 | 1.1695 | 1.8182 | 0.5208 |
| 1 | 25 | 1.0743 | 0.0252 | 0.1587 | 1.0743 | 1.5505 | 0.5980 |
| | 50 | 1.1030 | 0.0362 | 0.1903 | 1.1030 | 1.6741 | 0.5319 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | 100 | 1.1418 | 0.0359 | 0.1897 | 1.1418 | 1.7110 | 0.5727 |
| | 150 | 1.1441 | 0.0447 | 0.2115 | 1.1441 | 1.7787 | 0.5095 |
| 1.5 | 25 | 1.0514 | 0.0241 | 0.1553 | 1.0514 | 1.5176 | 0.5853 |
| | 50 | 1.0795 | 0.0347 | 0.1863 | 1.0795 | 1.6385 | 0.5206 |
| | 100 | 1.1176 | 0.0344 | 0.1856 | 1.1176 | 1.6746 | 0.5605 |
| | 150 | 1.1198 | 0.0428 | 0.2070 | 1.1198 | 1.7409 | 0.4987 |
| 2 | 25 | 1.0296 | 0.0231 | 0.1521 | 1.0296 | 1.4860 | 0.5731 |
| | 50 | 1.0571 | 0.0332 | 0.1824 | 1.0571 | 1.6044 | 0.5097 |
| | 100 | 1.0943 | 0.0330 | 0.1818 | 1.0943 | 1.6398 | 0.5488 |
| | 150 | 1.0965 | 0.0411 | 0.2027 | 1.0965 | 1.7047 | 0.4883 |

Table 1 displays the control limits central line (CL), upper control limit (UCL), and lower control limit (LCL) for the posterior distribution of an exponential distribution with a rate parameter θ =1, under the assumption of a gamma prior with shape parameter α =1. The analysis spans various values of the rate parameter β (0.5, 1, 1.5, and 2) and different sample sizes n of 25, 50, 100, and 150. The results show that as β increases, the CL generally increases as well, indicating that the central tendency of the posterior distribution shifts upwards. The process control expands the Upper Control Limit upward as it achieves greater stability at the Lower Control Limit. The CL value equals 1.0564 under β=2 and n=25 and its LCL stands at 0.3430 and the UCL reaches 2.0854. The process control shows tighter regulation and generates more accurate capability estimations when working with higher β values based on these numerical values. The control limits narrow down when a larger sample size enters the analysis which shows that statistical stability increases as information from data points adds to posterior distribution calculation.

**Table 2:** Control Limits for the Posterior Distribution of Exponential Distribution with θ=1 by using gamma prior with α =1.5 at different values of β

| α =1.5, θ =1 | | | | | | | |
|---|---|---|---|---|---|---|---|
| β | n | μ | $\sigma^2$ | σ | CL | UCL | LCL |
| 0.5 | 25 | 1.0981 | 0.0263 | 0.1622 | 1.0981 | 1.5849 | 0.6113 |
| | 50 | 1.1274 | 0.0378 | 0.1945 | 1.1274 | 1.7112 | 0.5437 |
| | 100 | 1.1672 | 0.0376 | 0.1939 | 1.1672 | 1.7489 | 0.5854 |
| | 150 | 1.1695 | 0.0467 | 0.2162 | 1.1695 | 1.8182 | 0.5208 |
| 1 | 25 | 1.0743 | 0.0252 | 0.1587 | 1.0743 | 1.5505 | 0.5980 |
| | 50 | 1.1030 | 0.0362 | 0.1903 | 1.1030 | 1.6741 | 0.5319 |
| | 100 | 1.1418 | 0.0359 | 0.1897 | 1.1418 | 1.7110 | 0.5727 |
| | 150 | 1.1441 | 0.0447 | 0.2115 | 1.1441 | 1.7787 | 0.5095 |
| 1.5 | 25 | 1.0514 | 0.0241 | 0.1553 | 1.0514 | 1.5176 | 0.5853 |
| | 50 | 1.0795 | 0.0347 | 0.1863 | 1.0795 | 1.6385 | 0.5206 |
| | 100 | 1.1176 | 0.0344 | 0.1856 | 1.1176 | 1.6746 | 0.5605 |
| | 150 | 1.1198 | 0.0428 | 0.2070 | 1.1198 | 1.7409 | 0.4987 |
| 2 | 25 | 1.0296 | 0.0231 | 0.1521 | 1.0296 | 1.4860 | 0.5731 |
| | 50 | 1.0571 | 0.0332 | 0.1824 | 1.0571 | 1.6044 | 0.5097 |
| | 100 | 1.0943 | 0.0330 | 0.1818 | 1.0943 | 1.6398 | 0.5488 |
| | 150 | 1.0965 | 0.0411 | 0.2027 | 1.0965 | 1.7047 | 0.4883 |

A gamma prior with α=1.5 expands upon the previous analysis format in Table 2. The higher α value makes the prior distribution more specific, leading to changes in the resulting control limits. The CL UCL and LCL values increase substantially in Table 2 when compared to Table 4.1. The CL value under these parameters expands to 1.0875 compared to 1.0564 seen before in the previous table while the UCL and LCL values also grow correspondingly. The posterior distribution tightens its concentration near the prior mean value as α increases resulting in higher control limit values. The control limits demonstrate direct correlation with sample size because broader population data results in narrower boundaries. An appropriate prior selection must be based on the specific analytical context as demonstrated by this table. Using the more informative prior α=1.5 allows higher control limits that could suit situations needing process capability overestimation yet demands thorough examination of applied prior information.

**Table 3:** Control Limits for the Posterior Distribution of Exponential Distribution with θ = 1 by using gamma prior with α =2.0 at different values of β

| α =2.0, θ =1 | | | | | | | |
|---|---|---|---|---|---|---|---|
| β | n | μ | $\sigma^2$ | σ | CL | UCL | LCL |
| 0.5 | 25 | 1.1197 | 0.0268 | 0.1638 | 1.1197 | 1.6113 | 0.6281 |
| | 50 | 1.1493 | 0.0385 | 0.1964 | 1.1493 | 1.7386 | 0.5600 |
| | 100 | 1.1894 | 0.0383 | 0.1957 | 1.1894 | 1.7767 | 0.6021 |
| | 150 | 1.1917 | 0.0476 | 0.2181 | 1.1917 | 1.8465 | 0.5369 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | 25 | 1.0954 | 0.0257 | 0.1603 | 1.0954 | 1.5763 | 0.6144 |
| | 50 | 1.1243 | 0.0369 | 0.1921 | 1.1243 | 1.7009 | 0.5478 |
| | 100 | 1.1636 | 0.0366 | 0.1915 | 1.1636 | 1.7381 | 0.5890 |
| | 150 | 1.1659 | 0.0455 | 0.2135 | 1.1659 | 1.8064 | 0.5253 |
| 1.5 | 25 | 1.0721 | 0.0246 | 0.1569 | 1.0721 | 1.5428 | 0.6014 |
| | 50 | 1.1005 | 0.0353 | 0.1880 | 1.1005 | 1.6647 | 0.5362 |
| | 100 | 1.1389 | 0.0351 | 0.1874 | 1.1389 | 1.7012 | 0.5765 |
| | 150 | 1.1411 | 0.0436 | 0.2089 | 1.1411 | 1.7681 | 0.5141 |
| 2 | 25 | 1.0498 | 0.0236 | 0.1536 | 1.0498 | 1.5107 | 0.5889 |
| | 50 | 1.0776 | 0.0339 | 0.1841 | 1.0776 | 1.6301 | 0.5250 |
| | 100 | 1.1152 | 0.0336 | 0.1835 | 1.1152 | 1.6658 | 0.5645 |
| | 150 | 1.1173 | 0.0418 | 0.2046 | 1.1173 | 1.7313 | 0.5034 |

Table 3 demonstrates how making the gamma prior more informative through α=2.0 produces control limits. Experiment results show how both β values and sample sizes affect control limit outcomes when controlling with strong prior knowledge. The CL, UCL, and LCL values continue to rise as α increases. Compared to the previous tables, the posterior distribution is becoming increasingly centered on a higher mean, reflecting the stronger influence of the prior. The results from this table highlight the balance between prior information and data-driven estimates. In practical applications, this balance could determine whether a more conservative (wider control limits) or a more optimistic (narrower control limits) approach is taken, depending on the level of confidence in the prior information.

**Table 4:** Control Limits for the Posterior Distribution of Exponential Distribution with θ=1.5 by using gamma prior with α =1.0 at different values of β

| α =1.0, θ =1.5 | | | | | | | |
|---|---|---|---|---|---|---|---|
| β | n | μ | $\sigma^2$ | σ | CL | UCL | LCL |
| 0.5 | 25 | 1.5971 | 0.0568 | 0.2383 | 1.5971 | 2.3122 | 0.8820 |
| | 50 | 1.6402 | 0.0817 | 0.2859 | 1.6402 | 2.4979 | 0.7824 |
| | 100 | 1.6985 | 0.0812 | 0.2849 | 1.6985 | 2.5534 | 0.8437 |
| | 150 | 1.7020 | 0.1009 | 0.3177 | 1.7020 | 2.6552 | 0.7487 |
| 1 | 25 | 1.5462 | 0.0532 | 0.2307 | 1.5462 | 2.2385 | 0.8539 |
| | 50 | 1.5879 | 0.0766 | 0.2768 | 1.5879 | 2.4183 | 0.7575 |
| | 100 | 1.6444 | 0.0761 | 0.2758 | 1.6444 | 2.4721 | 0.8168 |
| | 150 | 1.6478 | 0.0946 | 0.3076 | 1.6478 | 2.5706 | 0.7249 |
| 1.5 | 25 | 1.4984 | 0.0500 | 0.2236 | 1.4984 | 2.1693 | 0.8275 |
| | 50 | 1.5389 | 0.0719 | 0.2682 | 1.5389 | 2.3436 | 0.7341 |
| | 100 | 1.5936 | 0.0714 | 0.2673 | 1.5936 | 2.3957 | 0.7915 |
| | 150 | 1.5969 | 0.0888 | 0.2981 | 1.5969 | 2.4912 | 0.7025 |
| 2 | 25 | 1.4535 | 0.0470 | 0.2169 | 1.4535 | 2.1043 | 0.8027 |
| | 50 | 1.4928 | 0.0677 | 0.2602 | 1.4928 | 2.2734 | 0.7121 |
| | 100 | 1.5459 | 0.0672 | 0.2593 | 1.5459 | 2.3239 | 0.7678 |
| | 150 | 1.5490 | 0.0836 | 0.2891 | 1.5490 | 2.4166 | 0.6814 |

Table 4 presents the focus of an exponential distribution with a higher rate parameter θ=1.5 while maintaining a gamma prior with α=1.0. The control limits are calculated across various values of β and sample sizes. The increase in θ naturally leads to higher CL, UCL, and LCL values across all β and n combinations. The effect of β remains consistent with previous tables, with higher β values leading to narrower control limits. The control limits become more accurate when samples increase in size. The control limits need adjustment to handle processes with exponential distribution and higher rate parameter values because this distribution type produces additional variability.

**Table 5:** Control Limits for the Posterior Distribution of Exponential Distribution with θ=1.5 by using gamma prior with α =1.5 at different values of β

| α =1.5, θ =1.5 | | | | | | | |
|---|---|---|---|---|---|---|---|
| β | n | μ | $\sigma^2$ | σ | CL | UCL | LCL |
| 0.5 | 25 | 1.6291 | 0.0579 | 0.2407 | 1.6291 | 2.3513 | 0.9069 |
| | 50 | 1.6726 | 0.0833 | 0.2886 | 1.6726 | 2.5387 | 0.8066 |
| | 100 | 1.7315 | 0.0827 | 0.2877 | 1.7315 | 2.5947 | 0.8684 |
| | 150 | 1.7350 | 0.1029 | 0.3207 | 1.7350 | 2.6974 | 0.7726 |
| 1 | 25 | 1.5772 | 0.0543 | 0.2330 | 1.5772 | 2.2764 | 0.8780 |
| | 50 | 1.6193 | 0.0781 | 0.2794 | 1.6193 | 2.4578 | 0.7809 |
| | 100 | 1.6764 | 0.0775 | 0.2785 | 1.6764 | 2.5120 | 0.8408 |
| | 150 | 1.6797 | 0.0964 | 0.3105 | 1.6797 | 2.6114 | 0.7480 |
| | 25 | 1.5285 | 0.0510 | 0.2258 | 1.5285 | 2.2061 | 0.8508 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1.5 | 50 | 1.5693 | 0.0733 | 0.2708 | 1.5693 | 2.3818 | 0.7567 |
| | 100 | 1.6246 | 0.0728 | 0.2699 | 1.6246 | 2.4344 | 0.8148 |
| | 150 | 1.6278 | 0.0905 | 0.3009 | 1.6278 | 2.5308 | 0.7249 |
| 2 | 25 | 1.4827 | 0.0480 | 0.2191 | 1.4827 | 2.1400 | 0.8253 |
| | 50 | 1.5223 | 0.0690 | 0.2627 | 1.5223 | 2.3105 | 0.7341 |
| | 100 | 1.5759 | 0.0685 | 0.2618 | 1.5759 | 2.3615 | 0.7904 |
| | 150 | 1.5791 | 0.0852 | 0.2919 | 1.5791 | 2.4549 | 0.7032 |

The use of a gamma prior with α=1.5 leads to the results in Table 5. The control limits become wider because they combine the effects between a higher rate parameter with an informative prior distribution. The control limits from Table 5 exceed those from Table 4 at all levels of β values. A stronger informative alpha=1.5 value produces a posterior distribution that concentrates more tightly around the mean which increases the control limits. The influence of sample size on the control limits is consistent with previous findings. As n increases, the control limits become narrower, indicating increased stability in the posterior estimates. This table underscores the importance of carefully selecting the prior distribution, particularly in processes with higher variability (higher θ).

**Table 6:** Control Limits for the Posterior Distribution of Exponential Distribution with θ=1.5 by using gamma prior with α =2.0 at different values of β.

| α =2.0, θ =1.5 | | | | | | | |
|---|---|---|---|---|---|---|---|
| β | n | μ | $\sigma^2$ | σ | CL | UCL | LCL |
| 0.5 | 25 | 1.6611 | 0.0591 | 0.2431 | 1.6611 | 2.3905 | 0.9318 |
| | 50 | 1.7051 | 0.0849 | 0.2914 | 1.7051 | 2.5793 | 0.8308 |
| | 100 | 1.7645 | 0.0843 | 0.2904 | 1.7645 | 2.6358 | 0.8933 |
| | 150 | 1.7681 | 0.1048 | 0.3238 | 1.7681 | 2.7394 | 0.7966 |
| 1 | 25 | 1.6082 | 0.0553 | 0.2353 | 1.6082 | 2.3143 | 0.9021 |
| | 50 | 1.6507 | 0.0795 | 0.2821 | 1.6507 | 2.4971 | 0.8043 |
| | 100 | 1.7083 | 0.0790 | 0.2811 | 1.7083 | 2.5518 | 0.8648 |
| | 150 | 1.7117 | 0.0982 | 0.3134 | 1.7117 | 2.6521 | 0.7712 |
| 1.5 | 25 | 1.5585 | 0.0520 | 0.2280 | 1.5585 | 2.2428 | 0.8742 |
| | 50 | 1.5997 | 0.0747 | 0.2734 | 1.5997 | 2.4200 | 0.7795 |
| | 100 | 1.6555 | 0.0742 | 0.2724 | 1.6555 | 2.4730 | 0.8381 |
| | 150 | 1.6588 | 0.0922 | 0.3038 | 1.6588 | 2.5702 | 0.7474 |
| 2 | 25 | 1.5118 | 0.0489 | 0.2212 | 1.5118 | 2.1756 | 0.8481 |
| | 50 | 1.5518 | 0.0703 | 0.2652 | 1.5518 | 2.3475 | 0.7561 |
| | 100 | 1.6059 | 0.0698 | 0.2643 | 1.6059 | 2.3989 | 0.8130 |
| | 150 | 1.6091 | 0.0868 | 0.2947 | 1.6091 | 2.4932 | 0.7250 |

Table 6 presents the analysis with a more informative prior (α=2.0) while keeping θ=1.5. The control limits for different β values (0.5, 1.0, 1.5, and 2.0) and sample sizes reflect the increasing influence of the prior as α increases. As expected, the CL, UCL, and LCL values are higher compared to the previous tables. The increased α parameter results in a posterior distribution that is more strongly influenced by the prior, leading to higher and more stable control limits. The results from this table suggest that in situations where a highly informative prior is used, the resulting control limits will be higher and more stable, reflecting greater confidence in the prior information.

**Table 7:** Control Limits for the Posterior Distribution of Exponential Distribution with θ=2.0 by using gamma prior with α =1.0 at different values of β

| α =1.0, θ =2 | | | | | | | |
|---|---|---|---|---|---|---|---|
| β | n | μ | $\sigma^2$ | σ | CL | UCL | LCL |
| 0.5 | 25 | 2.1063 | 0.0988 | 0.3143 | 2.1063 | 3.0494 | 1.1632 |
| | 50 | 2.1632 | 0.1421 | 0.3770 | 2.1632 | 3.2945 | 1.0319 |
| | 100 | 2.2402 | 0.1412 | 0.3758 | 2.2402 | 3.3676 | 1.1127 |
| | 150 | 2.2447 | 0.1756 | 0.4190 | 2.2447 | 3.5019 | 0.9875 |
| 1 | 25 | 2.0187 | 0.0907 | 0.3012 | 2.0187 | 2.9225 | 1.1148 |
| | 50 | 2.0732 | 0.1306 | 0.3613 | 2.0732 | 3.1574 | 0.9890 |
| | 100 | 2.1470 | 0.1297 | 0.3601 | 2.1470 | 3.2275 | 1.0664 |
| | 150 | 2.1513 | 0.1613 | 0.4016 | 2.1513 | 3.3562 | 0.9464 |
| 1.5 | 25 | 1.9381 | 0.0836 | 0.2892 | 1.9381 | 2.8058 | 1.0703 |
| | 50 | 1.9904 | 0.1203 | 0.3469 | 1.9904 | 3.0313 | 0.9495 |
| | 100 | 2.0612 | 0.1195 | 0.3458 | 2.0612 | 3.0986 | 1.0238 |
| | 150 | 2.0654 | 0.1486 | 0.3855 | 2.0654 | 3.222 | 0.9086 |
| | 25 | 1.8636 | 0.0773 | 0.2781 | 1.8636 | 2.6980 | 1.0292 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 2 | 50 | 1.9139 | 0.1113 | 0.3336 | 1.9139 | 2.9148 | 0.9130 |
| | 100 | 1.9820 | 0.1105 | 0.3325 | 1.9820 | 2.9796 | 0.9845 |
| | 150 | 1.9860 | 0.1374 | 0.3707 | 1.9860 | 3.0984 | 0.8737 |

Table 7 presents the control limits for an exponential distribution with an even higher rate parameter θ =2.0 while maintaining a gamma prior with α=1.0. The calculation of control limits relies on different β values and sample sizes. The parameters show their biggest values in all measured regions as the value of θ rises. The increasing β values in control limits shows a pattern that matches earlier findings because wider β values create more constrained boundaries. The control limits demonstrate direct correlation to sample size as larger sample quantities automatically generate narrower control bands. The LCL becomes 1.3732 when β equals 1.5 and the sample size is 150 while the LCL equates to 0.2425 when using n=25. The control limits in this table illustrate why distribution-specific adjustments need to be applied to control methodologies.

**Table 8:** Control Limits for the Posterior Distribution of Exponential Distribution with θ=2.0 by using gamma prior with α =1.5 at different values of β

| α =1.5, θ =2 | | | | | | | |
|---|---|---|---|---|---|---|---|
| β | n | μ | $\sigma^2$ | σ | CL | UCL | LCL |
| 0.5 | 25 | 2.1486 | 0.1008 | 0.3175 | 2.1486 | 3.1011 | 1.1960 |
| | 50 | 2.2060 | 0.1449 | 0.3807 | 2.2060 | 3.3482 | 1.0638 |
| | 100 | 2.2837 | 0.1439 | 0.3794 | 2.2837 | 3.4220 | 1.1454 |
| | 150 | 2.2882 | 0.1789 | 0.4230 | 2.2882 | 3.5575 | 1.0190 |
| 1 | 25 | 2.0592 | 0.0925 | 0.3043 | 2.0592 | 2.9721 | 1.1463 |
| | 50 | 2.1142 | 0.1331 | 0.3648 | 2.1142 | 3.2088 | 1.0195 |
| | 100 | 2.1887 | 0.1322 | 0.3636 | 2.1887 | 3.2796 | 1.0977 |
| | 150 | 2.1930 | 0.1644 | 0.4054 | 2.1930 | 3.4095 | 0.9766 |
| 1.5 | 25 | 1.9769 | 0.0853 | 0.2921 | 1.9769 | 2.8534 | 1.1005 |
| | 50 | 2.0297 | 0.1227 | 0.3503 | 2.0297 | 3.0807 | 0.9788 |
| | 100 | 2.1012 | 0.1218 | 0.3491 | 2.1012 | 3.1486 | 1.0539 |
| | 150 | 2.1054 | 0.1515 | 0.3892 | 2.1054 | 3.2733 | 0.9376 |
| 2 | 25 | 1.9010 | 0.0789 | 0.2809 | 1.9010 | 2.7438 | 1.0582 |
| | 50 | 1.9518 | 0.1134 | 0.3368 | 1.9518 | 2.9623 | 0.9412 |
| | 100 | 2.0205 | 0.1127 | 0.3357 | 2.0205 | 3.0277 | 1.0134 |
| | 150 | 2.0246 | 0.1401 | 0.3743 | 2.0246 | 3.1475 | 0.9016 |

Table 8 illustrates the same situation with θ = 2.0, but it applies an informative prior (α = 1.5). Simulation data demonstrate that control limits adapt their positions based on the combination of higher rate parameters, along with more informative priors and their β values (0.5, 1.0, 1.5, 2.0), and sample sizes. Because of the greater α value the control limits exceed those in Table 4.7. The impact of β on narrowing the control limits is consistent with previous observations, but the limits are overall more stable, particularly at higher sample sizes. The CL exhibiting high values reflects the dominant influence of the larger rate parameter as the UCL and LCL display reduced variability with increasing β.

**Table 9:** Control Limits for the Posterior Distribution of Exponential Distribution with θ=2.0 by using gamma prior with α =2.0 at different values of β

| α =2.0 , θ =2 | | | | | | | |
|---|---|---|---|---|---|---|---|
| β | n | μ | $\sigma^2$ | σ | CL | UCL | LCL |
| 0.5 | 25 | 2.1908 | 0.1028 | 0.3206 | 2.1908 | 3.1527 | 1.2289 |
| | 50 | 2.2487 | 0.1477 | 0.3843 | 2.2487 | 3.4018 | 1.0957 |
| | 100 | 2.3272 | 0.1467 | 0.3830 | 2.3272 | 3.4763 | 1.1781 |
| | 150 | 2.3318 | 0.1823 | 0.4270 | 2.3318 | 3.6129 | 1.0506 |
| 1 | 25 | 2.0997 | 0.0944 | 0.3072 | 2.0997 | 3.0215 | 1.1778 |
| | 50 | 2.1552 | 0.1356 | 0.3683 | 2.1552 | 3.2602 | 1.0501 |
| | 100 | 2.2304 | 0.1347 | 0.3671 | 2.2304 | 3.3316 | 1.1291 |
| | 150 | 2.2347 | 0.1675 | 0.4092 | 2.2347 | 3.4626 | 1.0069 |
| 1.5 | 25 | 2.0158 | 0.0870 | 0.2950 | 2.0158 | 2.9008 | 1.1307 |
| | 50 | 2.0691 | 0.1250 | 0.3536 | 2.0691 | 3.13003 | 1.0082 |
| | 100 | 2.1413 | 0.1242 | 0.3524 | 2.1413 | 3.1985 | 1.0840 |
| | 150 | 2.1455 | 0.1543 | 0.3929 | 2.1455 | 3.3243 | 0.9667 |
| 2 | 25 | 1.9384 | 0.0804 | 0.2836 | 1.9384 | 2.7894 | 1.0873 |
| | 50 | 1.9896 | 0.1156 | 0.3400 | 1.9896 | 3.0098 | 0.9694 |
| | 100 | 2.0590 | 0.1148 | 0.3388 | 2.0590 | 3.0757 | 1.0424 |
| | 150 | 2.0631 | 0.1427 | 0.3778 | 2.0631 | 3.1966 | 0.9295 |

Table 9 represents the analysis considering an exponential distribution with θ=2.0 and a highly informative gamma prior with α=2.0. The control limits for various β values (0.5, 1.0, 1.5, and 2.0) and sample sizes provide insight into the combined effect of a high rate parameter and a strong prior. This table highlights the importance of considering both the prior information and the process variability when setting control limits. In scenarios where both the prior and the process are highly variable, the resulting control limits will be higher and more stable, reflecting the increased uncertainty in the process estimates. The combination of high θ and α results in the highest CL, UCL, and LCL values observed across all tables.

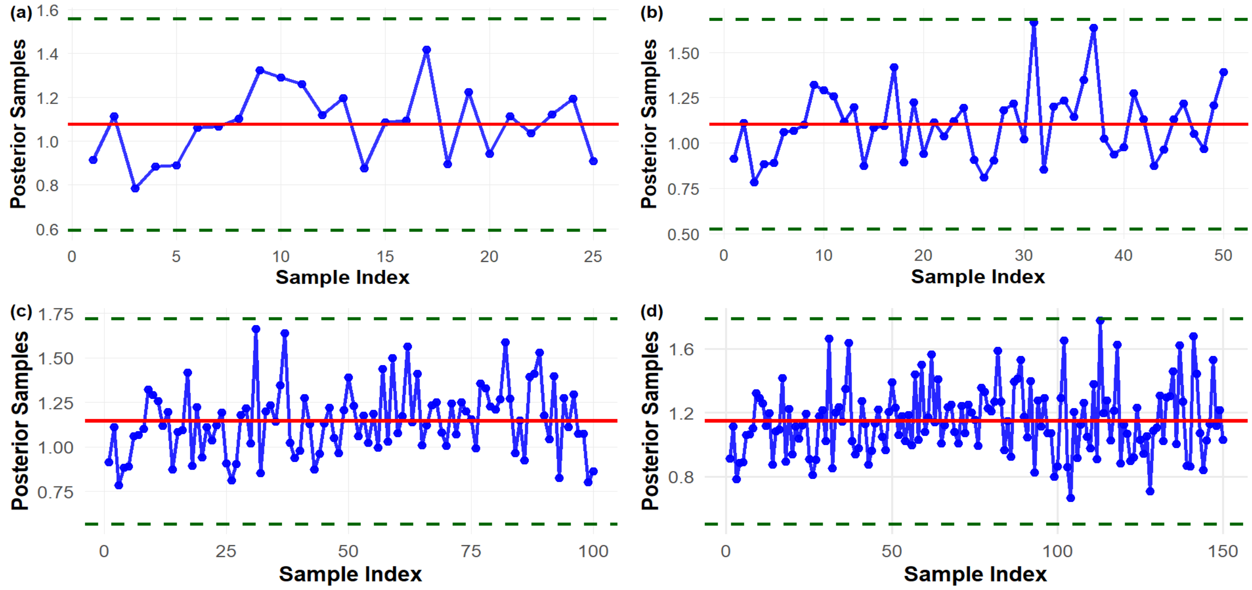


**Figure 2:** Bayesian Control Chart for the Posterior distribution of Exponential distribution with θ =1 by using Gamma Prior with α =1 and β =0.5 at different sample sizes.

Figure 2 illustrates the Bayesian control charts for the posterior distribution of the exponential process with θ=1, α=1, and β=0.5 across different sample sizes. The results indicate that all posterior sample points lie within the control limits, suggesting that the process remains in statistical control. As the sample size increases, the dispersion of the posterior samples becomes more visible, reflecting increased variability due to larger data representation; however, no significant out-of-control signals are observed.

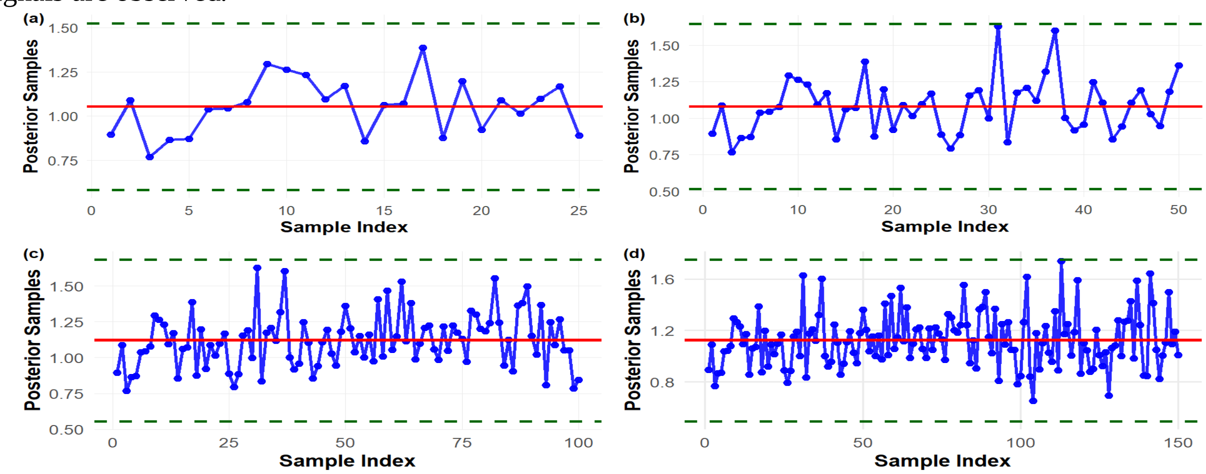


**Figure 3:** Bayesian Control Chart for the Posterior distribution of Exponential distribution with θ =1 by using Gamma Prior with α =1 and β =1.0 at different sample sizes.

Figure 3 presents the control charts with an increased prior parameter β = 1.0. Compared to Figure 2, the control limits appear slightly more stable, reflecting the influence of a stronger prior. The posterior samples remain within the control limits for all sample sizes, indicating consistent process stability. The incorporation of a more informative prior improves the robustness of the monitoring scheme.

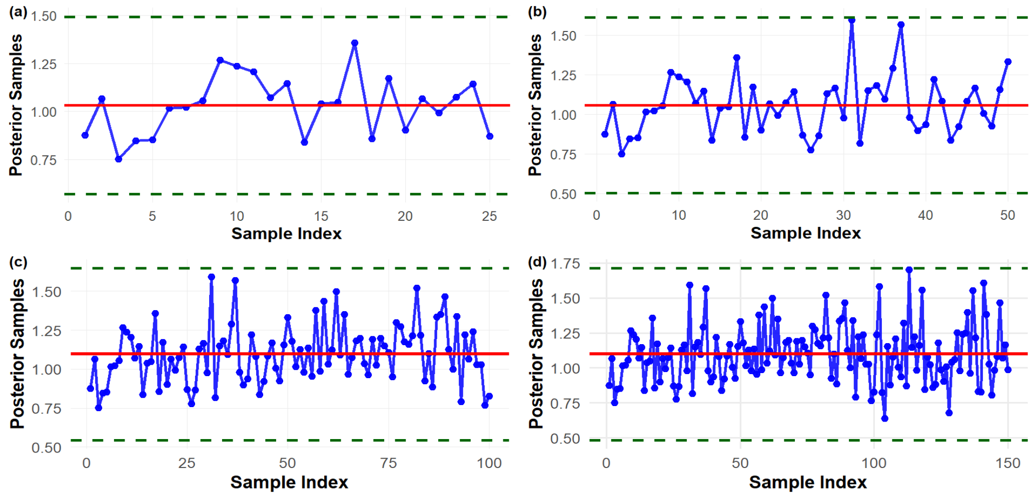


**Figure 4:** Bayesian Control Chart for the Posterior distribution of Exponential distribution with θ =1 by using Gamma Prior with α =1 and β =1.5 at different sample sizes.

Figure 4 shows the control charts for β = 1.5. The process continues to remain within control limits across all sample sizes. However, slight increases in variability are observed for larger samples. The control limits remain well-defined, demonstrating the effectiveness of Bayesian updating in maintaining stability under moderately informative prior assumptions.

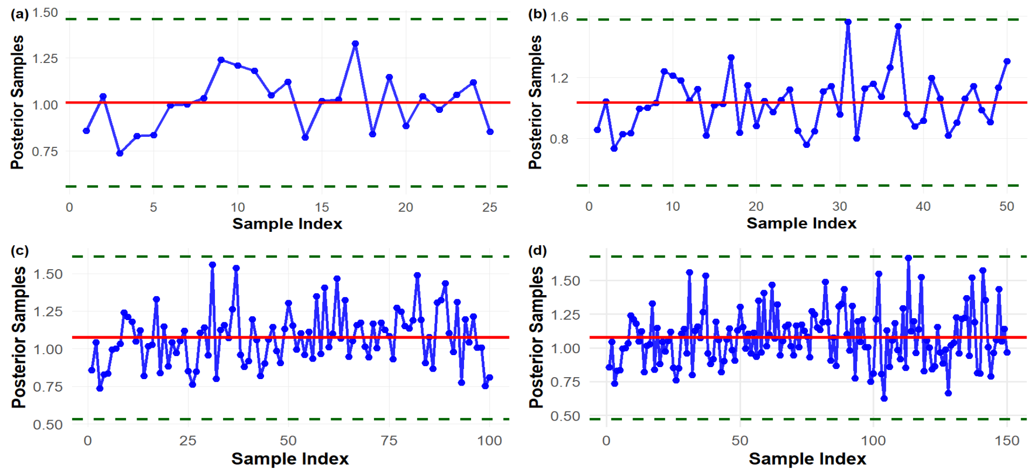


**Figure 5:** Bayesian Control Chart for the Posterior distribution of Exponential distribution with θ =1 by using Gamma Prior with α =1 and β =2.0 at different sample sizes.

Figure 5 represents the case with β = 2.0, indicating a more informative prior. The control limits become relatively more stable, and posterior samples are tightly clustered around the central line. This reflects the stronger influence of prior information, leading to improved consistency in monitoring and reduced variability in the posterior estimates.

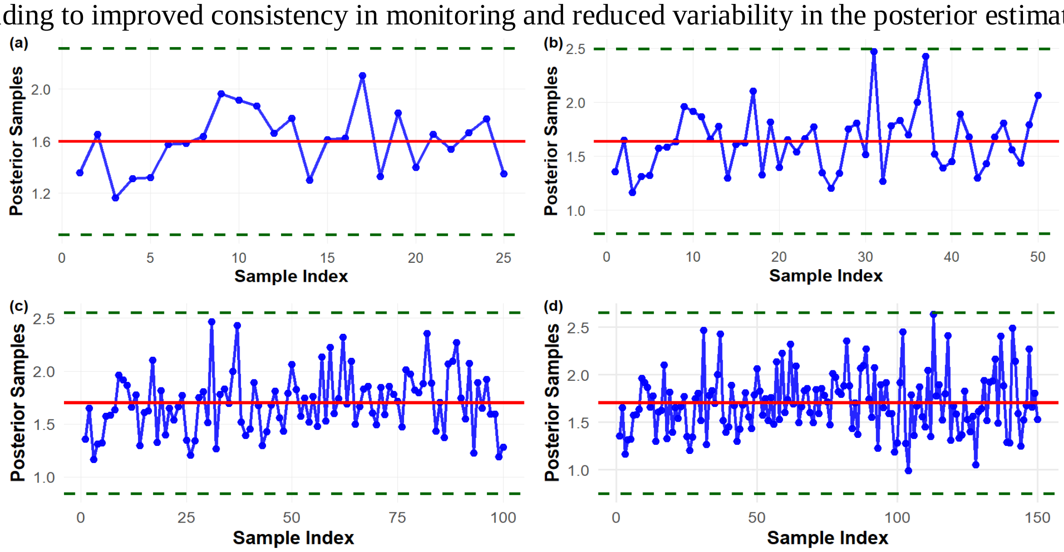

**Figure 6:** Bayesian Control Chart for the Posterior distribution of Exponential distribution with θ =1.5 by using Gamma Prior with α =1.0 and β =0.5 at different sample sizes.

Figure 6 presents the control charts for a higher rate parameter θ = 1.5. Compared to earlier figures, the variability of posterior samples increases, particularly for larger sample sizes. Despite this increase, most observations remain within control limits, indicating that the process is still stable, although more sensitive to fluctuations.

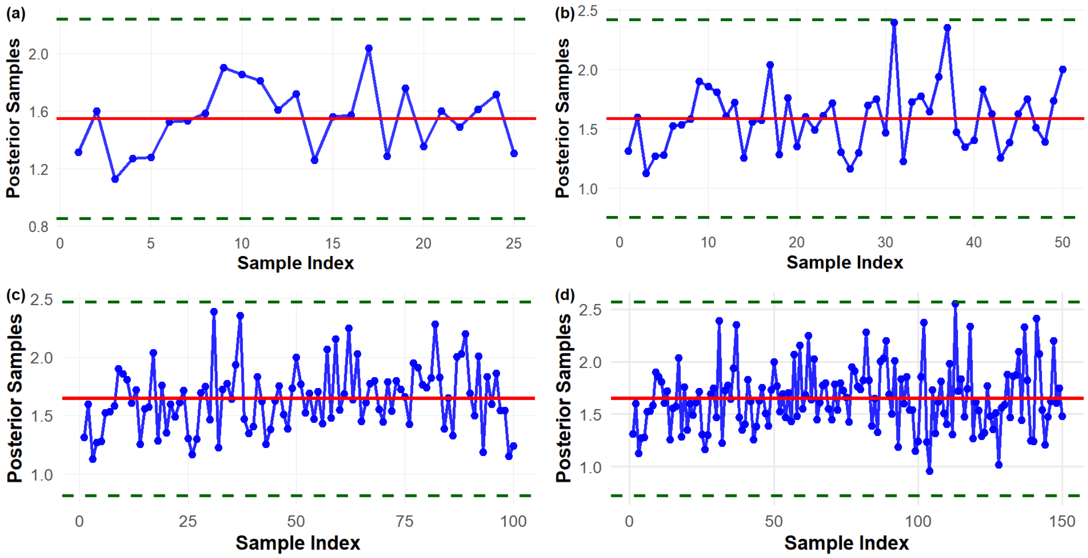


**Figure 7:** Bayesian Control Chart for the Posterior distribution of Exponential distribution with θ =1.5 by using Gamma Prior with α =1.0 and β =1.0 at different sample sizes.

Figure 7 shows the impact of increasing β to 1.0 for θ = 1.5. The control limits effectively capture process variation, with fewer points approaching the limits. This suggests that incorporating a stronger prior enhances the stability of the monitoring process and reduces extreme deviations.

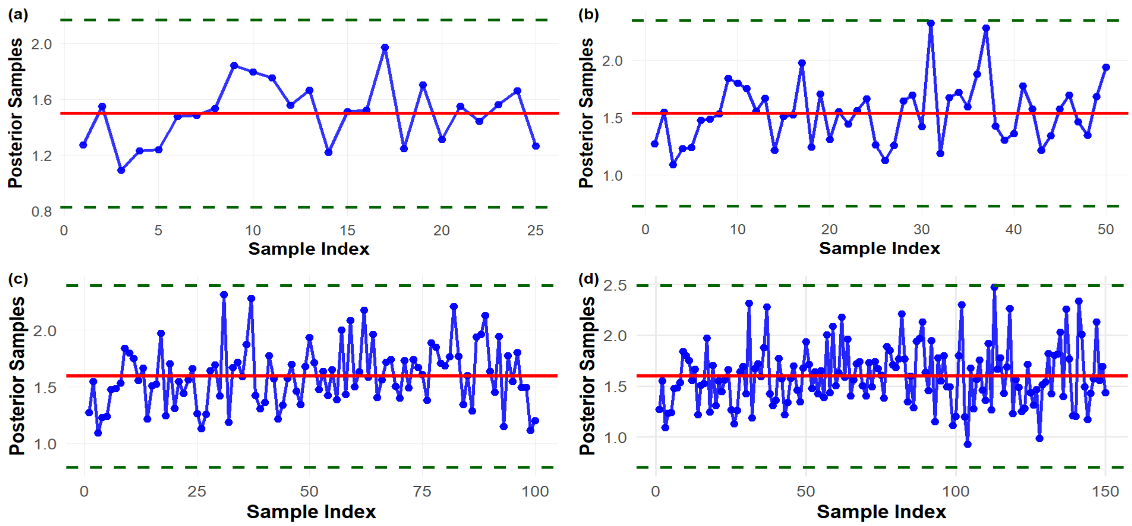


**Figure 8:** Bayesian Control Chart for the Posterior distribution of Exponential distribution with θ =1.5 by using Gamma Prior with α =1.0 and β =1.5 at different sample sizes.

Figure 8 demonstrates increased variability in posterior samples, especially for larger sample sizes. Some observations approach the control limits, indicating potential sensitivity to parameter changes. This suggests that the process may require closer monitoring under these parameter settings.

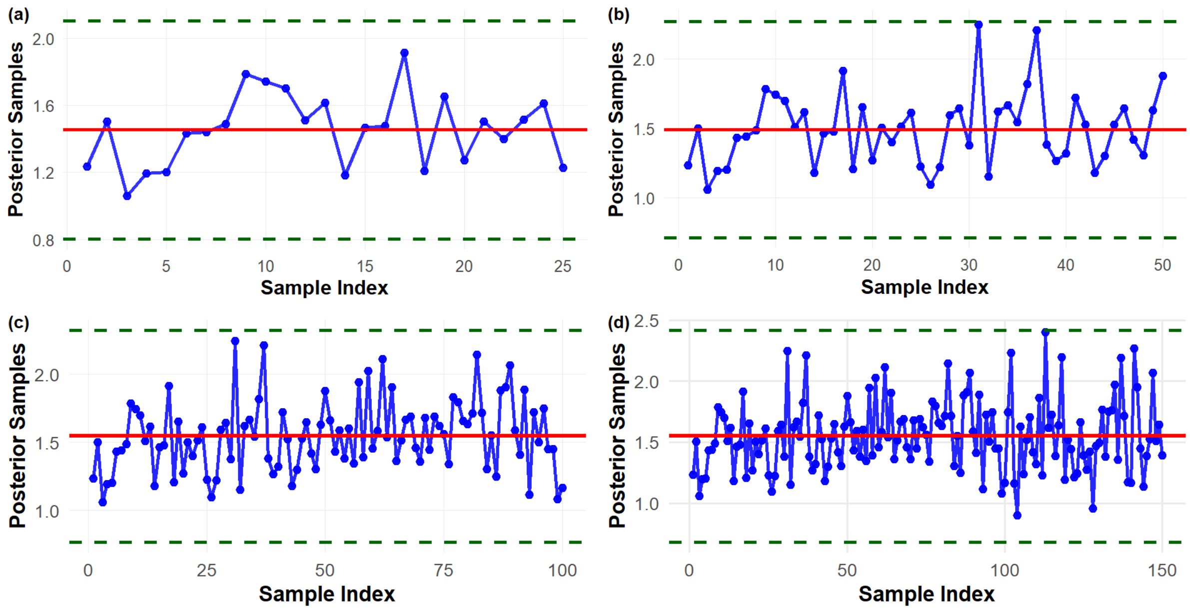


**Figure 9:** Bayesian Control Chart for the Posterior distribution of Exponential distribution with θ =1.5 by using Gamma Prior with α =1.0 and β =2.0 at different sample sizes.

Figure 9 illustrates the case with β = 2.0, where the prior is more informative. The control limits are more stable, and the posterior samples show reduced dispersion compared to Figure 8. This highlights the stabilizing effect of stronger prior information in Bayesian monitoring.

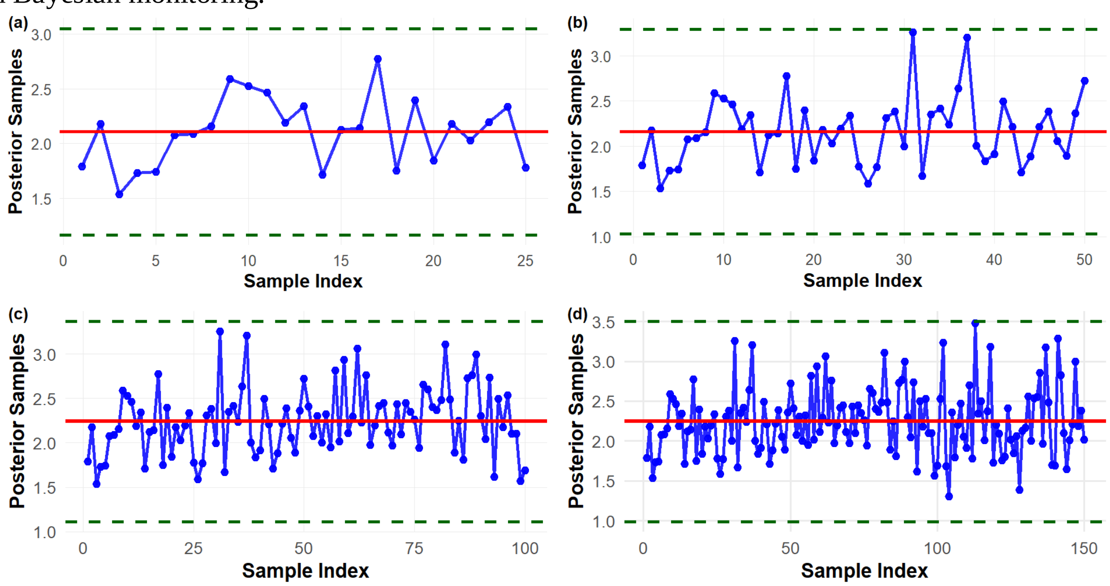


**Figure 10:** Bayesian Control Chart for the Posterior distribution of Exponential distribution with θ =2.0 by using Gamma Prior with α =1.0 and β =0.5 at different sample sizes.

Figure 10 presents the control charts for θ = 2.0, indicating a higher process rate. The variability of posterior samples increases significantly, and observations are more widely spread. Despite this, the control limits still capture most of the observations, demonstrating the adaptability of the Bayesian control chart.

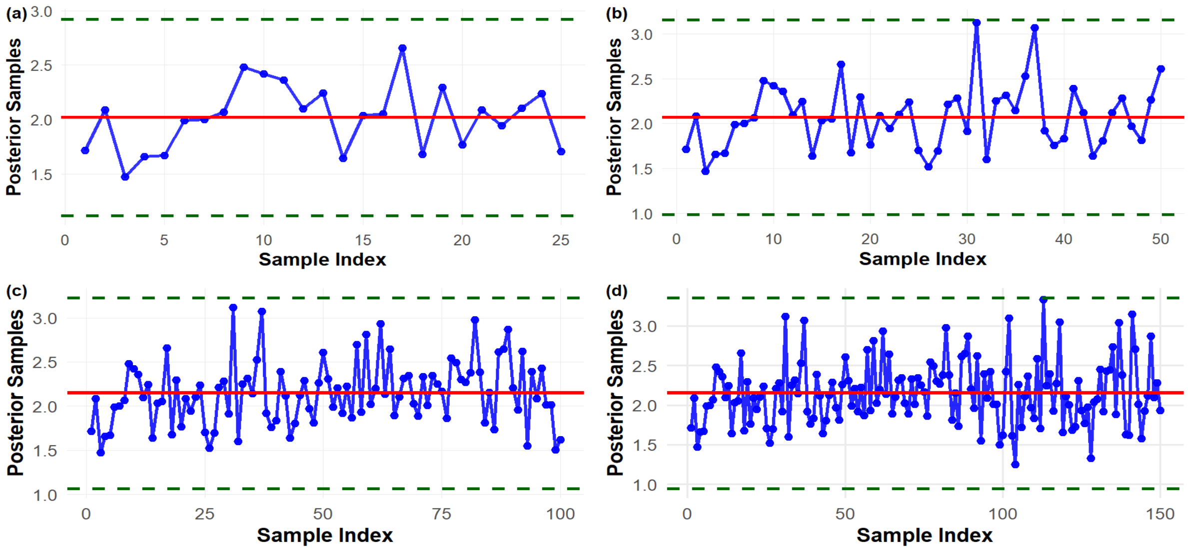


**Figure 11:** Bayesian Control Chart for the Posterior distribution of Exponential distribution with θ =2.0 by using Gamma Prior with α =1.0 and β =1.0 at different sample sizes.

Figure 11 shows a scenario where posterior samples are more dispersed and tend to cluster near control limits. This indicates increased process variability and reduced stability. The monitoring system becomes more sensitive, and distinguishing between normal variation and process shifts becomes more challenging.

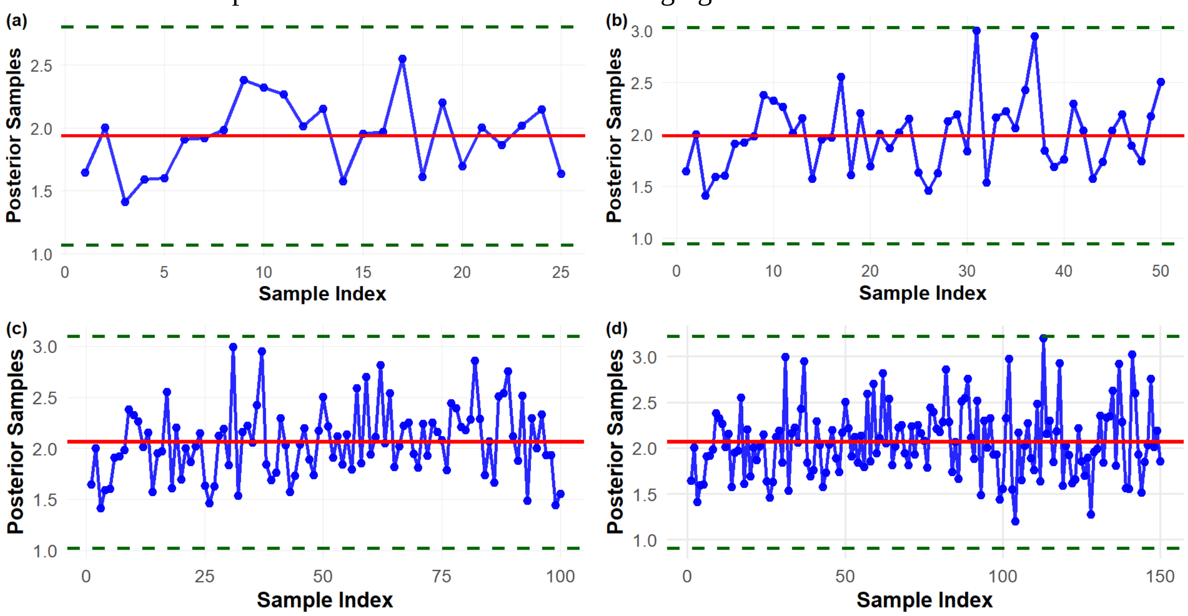


**Figure 12:** Bayesian Control Chart for the Posterior distribution of Exponential distribution with θ =2.0 by using Gamma Prior with α =1.0 and β =1.5 at different sample sizes.

Figure 12 reflects improved stability compared to Figure 11 due to stronger prior influence. The control limits become narrower, and posterior samples are better contained within the limits, indicating enhanced monitoring performance.

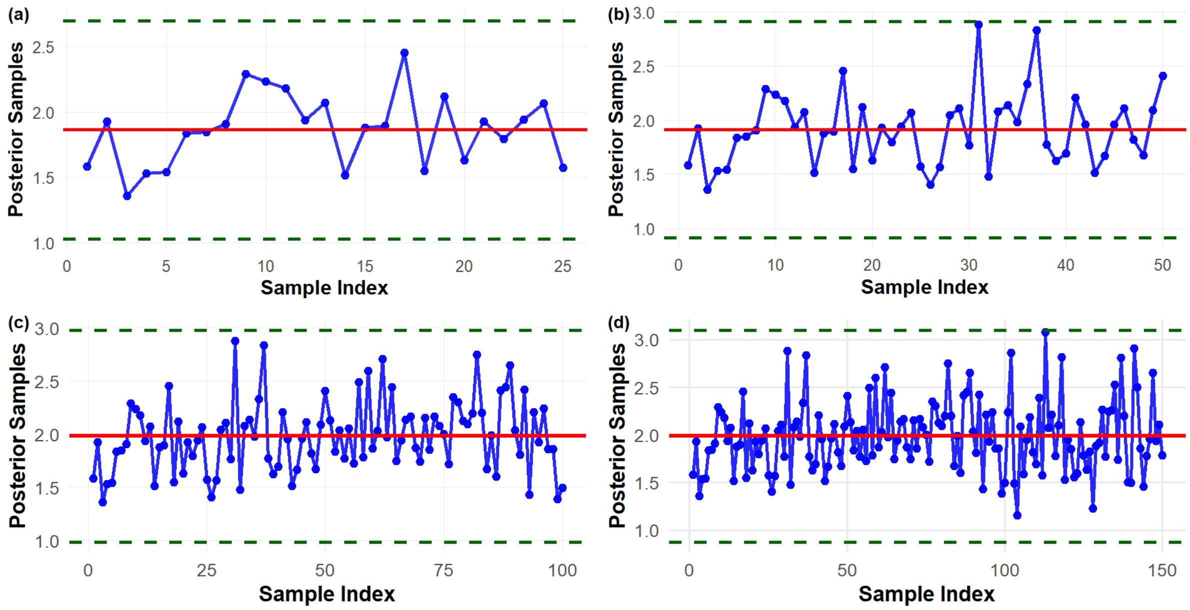


**Figure 13:** Bayesian Control Chart for the Posterior distribution of Exponential distribution with θ =2.0 by using Gamma Prior with α =1.0 and β =2.0 at different sample sizes.

Figure 13 demonstrates that with increasing prior strength (β=2.0), the control limits become more consistent and stable. Although variability is still present due to higher θ, the process remains largely under control, confirming the effectiveness of Bayesian control charts under strong prior assumptions.

## 4. Comparison of Bayesian Control Charts with Classical Control Charts based on Exponential Distribution

In this section, we compare the performance of the control chart and the Bayesian Control Charts. Bayesian control charts differ from traditional control charts primarily in their adaptability and ability to handle uncertainty. While traditional control charts use a fixed, frequentist approach with static control limits based on historical data, Bayesian control charts dynamically update control limits using prior knowledge and new data, adjusting the posterior distribution with each new observation. Bayesian control charts are computationally more complex and require knowledge of Bayesian methods, making them harder to interpret than the more straightforward traditional charts.

**Table 10**: Comparison of Classical and Bayesian Control Limits and Average Run Lengths ($ARL_0$, $ARL_1$) for the Exponential Distribution with in-control rate θ = 1 and shift θ = 1.5 (n = 50, α = 1, β = 0.5).

| Method | CL | LCL | UCL | ARL0 | ARL1 | ARL0/ARL1 |
|---|---|---|---|---|---|---|
| Classical | 1.0192 | 0.6050 | 1.4334 | 109.740 | 1.553 | 70.6632 |
| Bayesian | 0.9002 | 0.5300 | 1.2703 | 15.445 | 1.098 | 14.0664 |

Table 10 represents compares Classical and Bayesian control charts for an exponential process with in-control rate θ=1, a shift to θ=1.5, and sample size n=50. The Classical control chart yields a much larger in-control Average Run Length (ARL0=109.740) than the Bayesian chart (ARL0=15.445), indicating stronger resistance to false alarms. Under out-of-control conditions, both methods detect the shift rapidly; however, the Bayesian chart shows faster detection (ARL1=1.098) compared to the Classical chart (ARL1=1.553). The ratio ARL0/ARL1 is higher for the Classical approach, reflecting its conservative nature. Overall, the Bayesian control chart demonstrates greater sensitivity to process shifts, while the Classical chart emphasizes in-control stability.

The $ARL_0$ values obtained in this study are lower than the conventional benchmark of approximately 370 used in classical SPC. This is because the control limits were not optimized to achieve a specific $ARL_0$, but were constructed using the posterior mean ± 3 standard deviation approach. As a result, the proposed Bayesian control chart exhibits higher sensitivity to process shifts, as reflected in lower $ARL_1$ values, but at the cost of an increased false alarm rate. Therefore, the proposed method should be interpreted as a comparative and methodological contribution rather than a finalized industrial design. For practical applications, future research should focus on calibrating control limits using ARL-based or probability-based approaches to achieve desired in-control performance.

**Table 11:** Control Limits and ARL Performance Measures ($ARL_0$, $ARL_1$, and $ARL_0/ARL_1$) for the Bayesian Control Chart of the Exponential Distribution with Gamma Prior for Different Values of α and β.

| α | β | n | μ | $\sigma^2$ | $\sigma$ | CL | LCL | UCL | ARL0 | ARL1 | ARL0/ARL1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.5 | 25 | 0.906 | 0.020 | 0.142 | 0.906 | 0.481 | 1.331 | 30.911 | 1.19 | 25.98 |
| 1 | 0.5 | 50 | 0.913 | 0.017 | 0.132 | 0.913 | 0.517 | 1.310 | 24.192 | 1.163 | 20.80 |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.5 | 100 | 0.889 | 0.016 | 0.127 | 0.889 | 0.507 | 1.272 | 16.59 | 1.105 | 15.01 |
| 1 | 1 | 25 | 0.890 | 0.016 | 0.127 | 0.890 | 0.509 | 1.270 | 19.103 | 1.14 | 16.76 |
| 1 | 1 | 50 | 0.879 | 0.014 | 0.120 | 0.879 | 0.518 | 1.240 | 13.948 | 1.087 | 12.83 |
| 1 | 1 | 100 | 0.878 | 0.017 | 0.130 | 0.878 | 0.487 | 1.270 | 18.4 | 1.119 | 16.44 |
| 1 | 1.5 | 25 | 0.867 | 0.019 | 0.139 | 0.867 | 0.451 | 1.282 | 25.451 | 1.181 | 21.55 |
| 1 | 1.5 | 50 | 0.872 | 0.011 | 0.105 | 0.872 | 0.557 | 1.188 | 9.634 | 1.046 | 9.21 |
| 1 | 1.5 | 100 | 0.862 | 0.014 | 0.118 | 0.862 | 0.509 | 1.216 | 12.294 | 1.073 | 11.46 |
| 1 | 2 | 25 | 0.886 | 0.018 | 0.134 | 0.886 | 0.485 | 1.287 | 32.222 | 1.204 | 26.76 |
| 1 | 2 | 50 | 0.872 | 0.015 | 0.123 | 0.872 | 0.503 | 1.241 | 19.363 | 1.13 | 17.14 |
| 1 | 2 | 100 | 0.867 | 0.016 | 0.128 | 0.867 | 0.483 | 1.252 | 20.73 | 1.134 | 18.28 |
| 1.5 | 0.5 | 25 | 0.922 | 0.010 | 0.101 | 0.922 | 0.618 | 1.227 | 9.049 | 1.041 | 8.69 |
| 1.5 | 0.5 | 50 | 0.913 | 0.018 | 0.134 | 0.913 | 0.510 | 1.316 | 22.171 | 1.17 | 18.95 |
| 1.5 | 0.5 | 100 | 0.915 | 0.018 | 0.135 | 0.915 | 0.510 | 1.319 | 22.577 | 1.153 | 19.58 |
| 1.5 | 1 | 25 | 0.883 | 0.011 | 0.104 | 0.883 | 0.570 | 1.197 | 8.229 | 1.036 | 7.94 |
| 1.5 | 1 | 50 | 0.869 | 0.018 | 0.134 | 0.869 | 0.467 | 1.272 | 16.213 | 1.12 | 14.48 |
| 1.5 | 1 | 100 | 0.889 | 0.015 | 0.123 | 0.889 | 0.518 | 1.259 | 14.581 | 1.1 | 13.26 |
| 1.5 | 1.5 | 25 | 0.853 | 0.012 | 0.111 | 0.853 | 0.520 | 1.186 | 7.638 | 1.05 | 7.27 |
| 1.5 | 1.5 | 50 | 0.901 | 0.012 | 0.108 | 0.901 | 0.578 | 1.225 | 11.989 | 1.097 | 10.93 |
| 1.5 | 1.5 | 100 | 0.877 | 0.016 | 0.127 | 0.877 | 0.496 | 1.257 | 16.113 | 1.097 | 14.69 |
| 1.5 | 2 | 25 | 0.856 | 0.022 | 0.149 | 0.856 | 0.410 | 1.301 | 34.038 | 1.236 | 27.54 |
| 1.5 | 2 | 50 | 0.876 | 0.016 | 0.125 | 0.876 | 0.501 | 1.251 | 19.106 | 1.126 | 16.97 |
| 1.5 | 2 | 100 | 0.887 | 0.020 | 0.141 | 0.887 | 0.463 | 1.311 | 35.835 | 1.253 | 28.60 |
| 2 | 0.5 | 25 | 0.907 | 0.019 | 0.138 | 0.907 | 0.494 | 1.320 | 19.197 | 1.154 | 16.64 |
| 2 | 0.5 | 50 | 0.881 | 0.009 | 0.093 | 0.881 | 0.603 | 1.159 | 4.607 | 1.018 | 4.53 |
| 2 | 0.5 | 100 | 0.909 | 0.017 | 0.131 | 0.909 | 0.516 | 1.302 | 16.766 | 1.118 | 15.00 |
| 2 | 1 | 25 | 0.901 | 0.014 | 0.117 | 0.901 | 0.550 | 1.253 | 11.913 | 1.07 | 11.13 |
| 2 | 1 | 50 | 0.889 | 0.019 | 0.139 | 0.889 | 0.472 | 1.306 | 22.281 | 1.148 | 19.41 |
| 2 | 1 | 100 | 0.886 | 0.016 | 0.125 | 0.886 | 0.510 | 1.262 | 14.181 | 1.083 | 13.09 |
| 2 | 1.5 | 25 | 0.884 | 0.014 | 0.119 | 0.884 | 0.527 | 1.241 | 12.287 | 1.065 | 11.54 |
| 2 | 1.5 | 50 | 0.895 | 0.016 | 0.128 | 0.895 | 0.510 | 1.280 | 17.918 | 1.146 | 15.64 |
| 2 | 1.5 | 100 | 0.899 | 0.011 | 0.104 | 0.899 | 0.588 | 1.210 | 10.088 | 1.058 | 9.53 |
| 2 | 2 | 25 | 0.859 | 0.009 | 0.095 | 0.859 | 0.575 | 1.143 | 5.781 | 1.021 | 5.66 |
| 2 | 2 | 50 | 0.876 | 0.024 | 0.154 | 0.876 | 0.415 | 1.337 | 45.123 | 1.29 | 34.98 |
| 2 | 2 | 100 | 0.896 | 0.016 | 0.126 | 0.896 | 0.519 | 1.273 | 20.537 | 1.136 | 18.08 |

The results presented in Table 11 should be interpreted as a comparative analysis between classical and Bayesian control charts under a unified framework. Although both methods exhibit relatively low $ARL_0$ values, indicating higher false alarm rates, the comparison remains valid as both approaches are evaluated under identical conditions. The findings highlight that the Bayesian control chart provides faster detection of process shifts (lower $ARL_1$) compared to the classical chart. However, due to the elevated false alarm rate, these results may have limited practical applicability without further calibration. Future research should focus on designing control limits that achieve desirable $ARL_0$ performance to enhance both reliability and practical significance.

**Real-Life Application**

The following skewed to right a set of data, discussed by (Nassar and Nada, 2011), represents the monthly actual tax revenue (in 1000 million Egyptian pounds) in Egypt between January 2006 and November 2010 (Murthy et al., 2004). The values are and given below and histogram is shown in Figure 16.

5.9, 20.4, 14.9, 16.2, 17.2, 7.8, 6.1, 9.2, 10.2, 9.6, 13.3, 8.5, 21.6, 18.5, 5.1, 6.7, 17, 8.6, 9.7, 39.2, 35.7, 15.7, 9.7, 10, 4.1, 36, 8.5, 8, 9.2, 26.2, 21.9, 16.7, 21.3, 35.4, 14.3, 8.5, 10.6, 19.1,20.5, 7.1, 7.7, 18.1, 16.5, 11.9, 7, 8.6, 12.5, 10.3, 11.2, 6.1, 8.4, 11,11.6,11.9, 5.2, 6.8, 8.9, 7.1, 10.8.

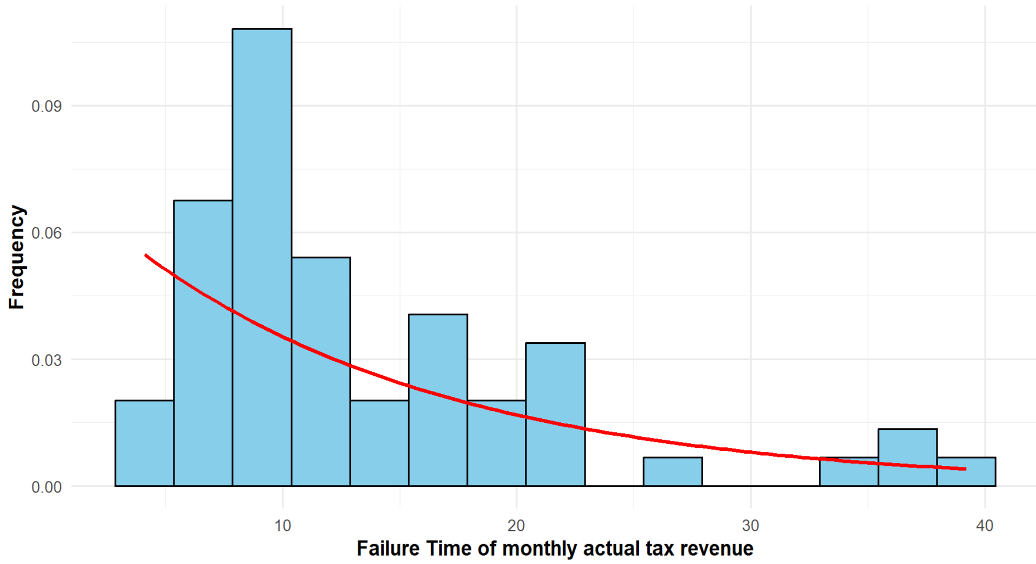


**Figure 14:** Histogram of Failure Time of monthly actual tax revenue.

This figure shows that the distribution is positively skewed. A classical control chart was constructed using the sample mean and standard deviation. Although the underlying data follows an exponential distribution, the control limits were approximated using the three-sigma approach for comparison purposes.

**Table 12**: Classical Control Chart Parameters and ARL Performance for Real Failure-Time Data

| UCL | LCL | ARL0 | ARL1 | ARL0/ARL1 |
|---|---|---|---|---|
| 37.6426 | 0 | 16.2939 | 6.4432 | 2.53 |

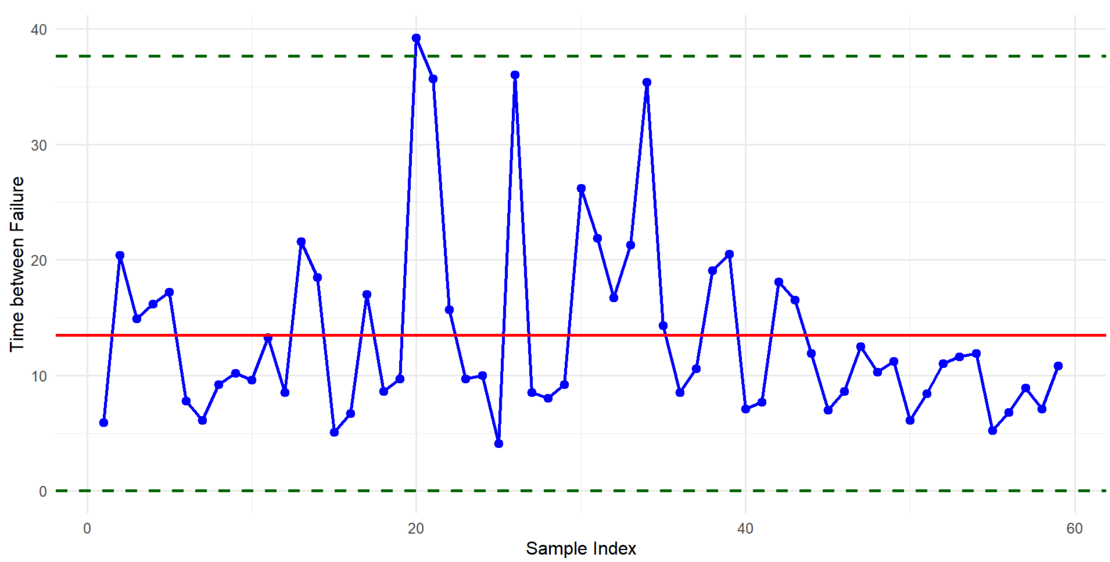


**Figure 15:** Control Chart of Failure Time of monthly actual tax revenue

The control chart for failure time for monthly actual tax revenue is shown in Figure 15. The control chart for the exponential distribution of failure times shows significant variability, with several points exceeding the upper control limit, indicating periods of unusually long times between failures. The classical $ARL_0$ value of 370 is based on the assumption of normality and the three-sigma rule. In this study, the process data follows an exponential distribution, which is inherently skewed. As a result, the probability of false alarm differs from the normal case, leading to a deviation in $ARL_0$. Therefore, $ARL_0$ is computed using the actual underlying distribution rather than relying on the normal approximation.

**Table 13**: Bayesian Control Chart Parameters and ARL Performance for Real Failure-Time Data

| α | β | CL | UCL | LCL | ARL0 | ARL1 | ARL0/ARL1 |
|---|---|---|---|---|---|---|---|
| 1 | 0.5 | 0.07429 | 0.1018 | 0.0480 | 1.0040 | 1.0029 | 1.00 |

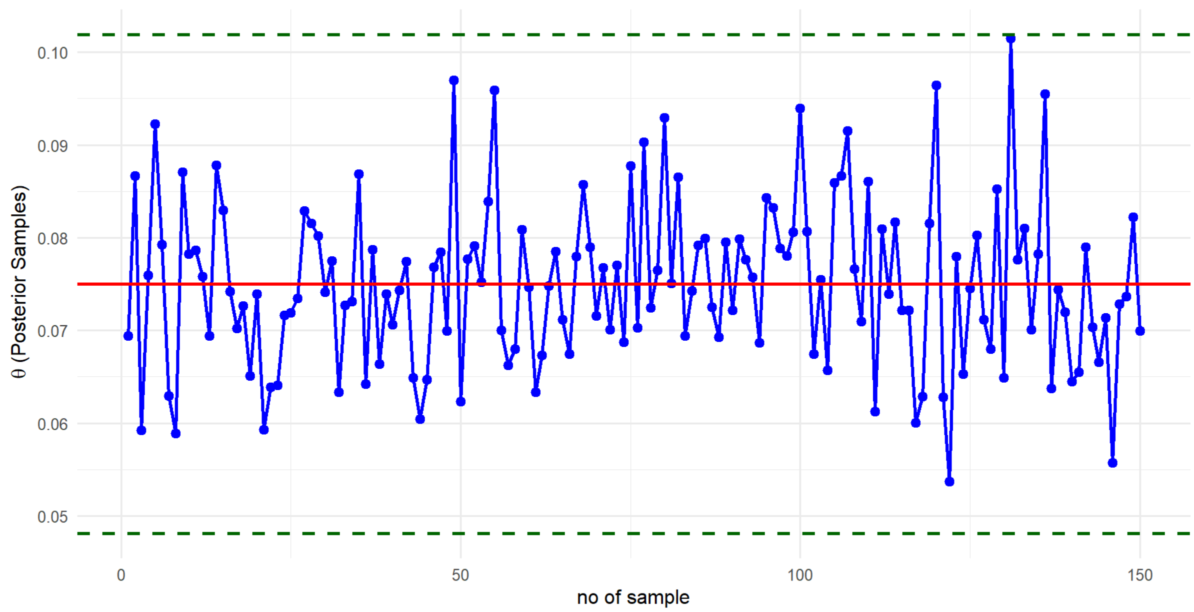


**Figure 16:** Bayesian Control Chart of Failure time of monthly actual tax revenue.

The Bayesian control chart for failure time for monthly actual tax revenue is shown in Figure 16. The Bayesian control chart shows that all posterior sample points fall within the control limits, indicating the process is statistically in control. The samples fluctuate randomly around the central line, reflecting only common cause variation. The narrower control limits highlight the Bayesian chart's higher sensitivity for detecting process shifts compared to classical charts.

**2nd Real-Life Application:**

The following data set obtained from a hospital center in the United States (Santiago and Smith, 2014). Instead of providing the information with the dates on which the infections were recorded, we have included the days in between the discharge of males in nosocomial urinary tract infections inpatients. The values are and given below.

0.57014,0.12014,0.27083,0.07431,0.11458,0.04514,0.15278,0.00347,0.13542,0.14583,0.12014,0.08,0.13889,0.04861,0.40347,0.14931,0.02778,0.12639,0.03333,0.32639,0.183,0.08681,0.64931,0.7083,0.33681,0.14931,0.15625,0.03819,0.01389,0.24653,0.24653,0.03819,0.04514,0.29514,0.46806,0.01736,0.11944,0.22222,1.08889,0.05208,0.29514,0.05208,0.12500,0.53472,0.02778,0.25000,0.15139,0.03472,0.40069,0.52569,0.23611,0.02500, 0.07986,0.35972.

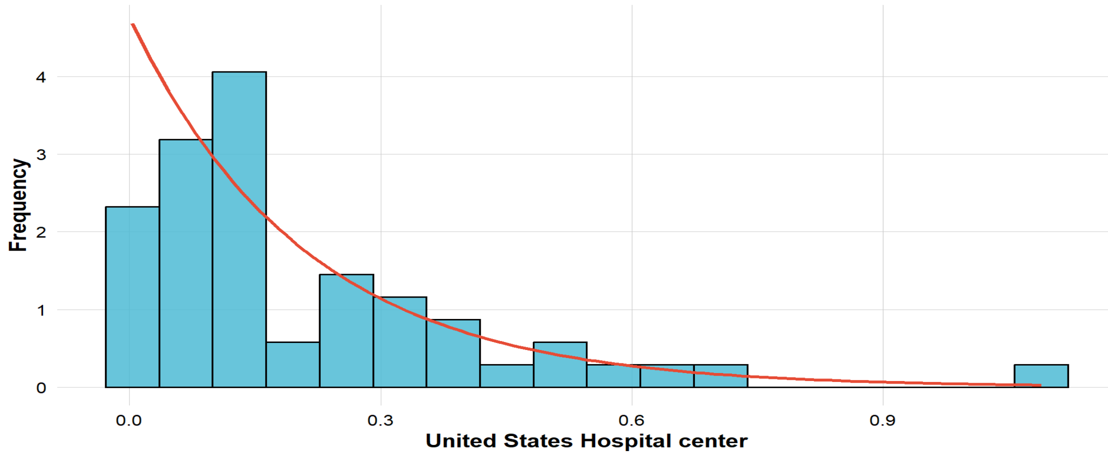


**Figure 17:** Histogram of Hospital center in the United States.

The histogram of the observed data exhibits a pronounced right-skewed distribution with a long tail, which is a key characteristic of the exponential distribution. The fitted exponential density curve shows a reasonable agreement with the empirical distribution.

**Table 14**: Classical Control Chart Parameters and ARL Performance for Real-Life Data

| CL | UCL | LCL | ARL0 | ARL1 | ARL0/ARL1 |
|---|---|---|---|---|---|
| 0.2101 | 0.8461 | 0 | 56.0820 | 14.1242 | 3.9706 |

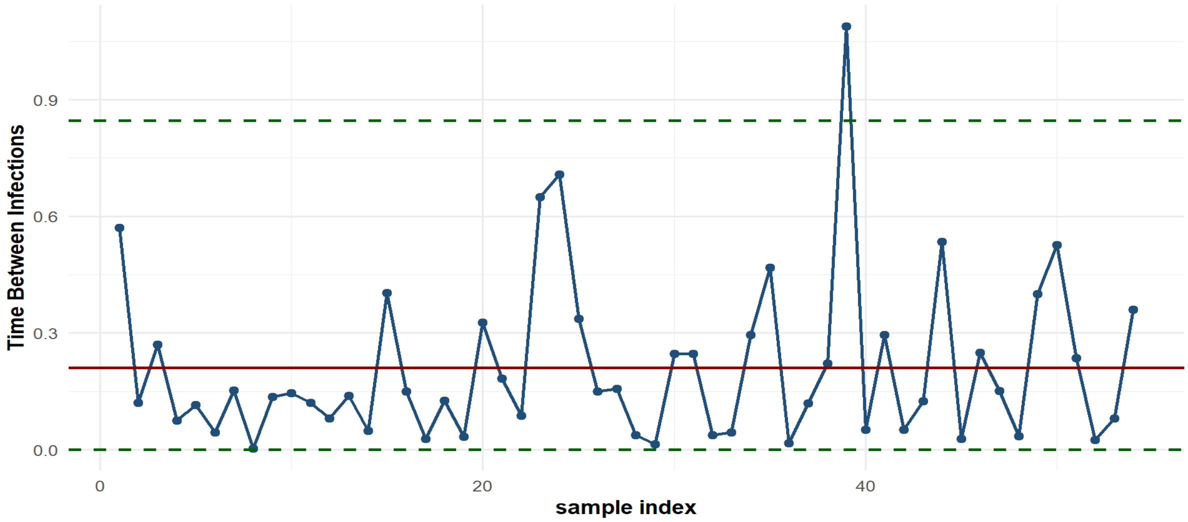


**Figure 18:** Classical Control chart of hospital center in the United States.

**Table 15**: Bayesian Control Chart Parameters and ARL Performance for Real- life Data

| α | β | CL | UCL | LCL | ARL0 | ARL1 | ARL0/ARL1 |
|---|---|---|---|---|---|---|---|
| 1 | 0.5 | 4.6181 | 6.3492 | 2.8871 | 1 | 1 | 1 |

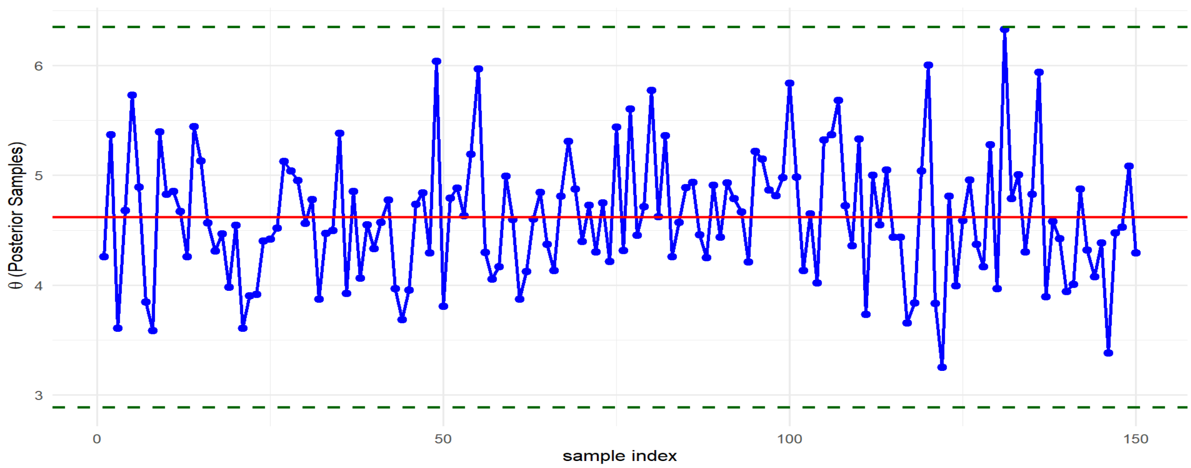


**Figure 19:** Bayesian Control chart of hospital center in the United States.

In the real data application, a relatively large number of observations fall outside the control limits. This is mainly due to the use of fixed mean ± 3 standard deviation limits, which are not calibrated to achieve a desired $ARL_0$. As a result, the Bayesian control chart exhibits higher sensitivity, leading to an increased number of signals. While some of these signals may represent true process variations, others may be false alarms. Therefore, the results should be interpreted with caution. For practical applications, it is recommended to design control limits using ARL-based or probability-based methods to improve reliability and reduce false alarm rates.

## 5 Concluding Remarks

This paper examines Bayesian control chart analysis of exponential distribution values under gamma prior conditions. The research evaluates statistical measures such as posterior mean and variance together with standard deviation and control limits through different parameter options and sample size variations. Products whose life data and time-to-failure follow the exponential distribution which has one single parameter θ. A gamma distribution serves as the prior due to its ability to connect with exponential distributions for posterior calculation purposes. The accuracy of Bayesian models depends heavily on the right selection of gamma distribution parameters alpha and beta because they determine the precise way posterior estimates are calculated. The evaluation with Bayesian methods uncovered that simulation along with direct methods generated superior control limit systems that performed better than traditional statistical approaches. In this study, the hyperparameters of the gamma prior distribution (α, β) are assumed and varied systematically rather than estimated from observed data. The selected values α = 1, 1.5, 2 and β = 0.5, 1, 1.5, 2 represent different levels of prior information, ranging from weak to moderately informative priors. This approach enables a sensitivity analysis of the Bayesian control chart under varying prior assumptions. In practical applications, hyperparameters can be determined using historical data, expert knowledge, or empirical Bayes methods. Although such estimation procedures may increase computational complexity, simplified approaches can be employed to ensure practical implementation. Process monitoring through Bayesian control charts becomes more powerful since these methods connect past knowledge with current operating information which results in enhanced adaptive control of process variations.

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## Declaration

**Conflict of Study:** The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

**Funding Statement:** This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

**Author Contribution Statement**: All the authors worked together as a cell group and designed the research; Analyzed and interpreted the data, and wrote the paper.

**Availability of Data and Material**: Data will be made available by the corresponding author on reasonable request.

**Ethical Approval**: Not Applicable

**Consent to Publish**: Not Applicable

**Consent to Participate**: Not Applicable

**Acknowledgement**: Not Applicable

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## Appendix

COMPLETE FIGURES OF TABLES 1 TO 9

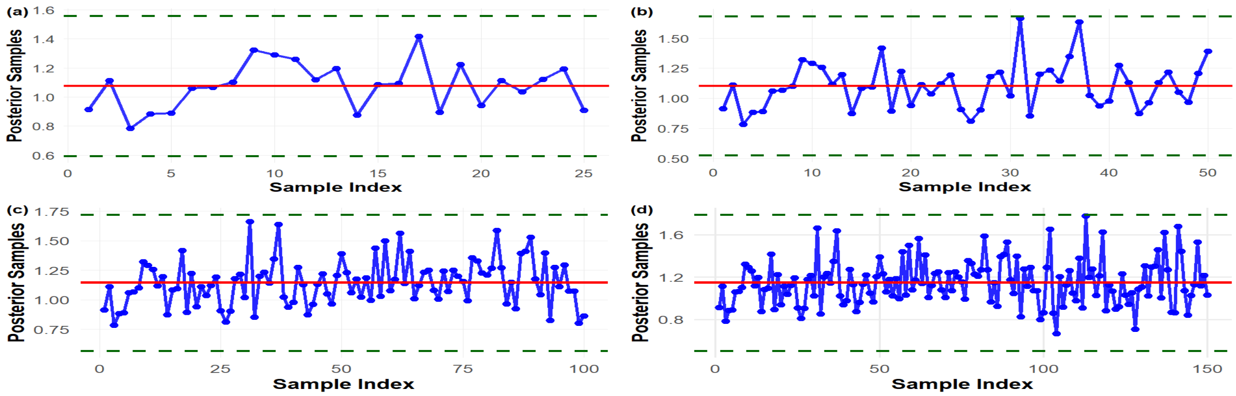


**Figure 2:** Bayesian Control Chart for the Posterior distribution of Exponential distribution with θ =1 by using Gamma Prior with α =1 and β =0.5 at different sample sizes.

Note: Figures 2–5 represent the graphical results corresponding to Table 1.

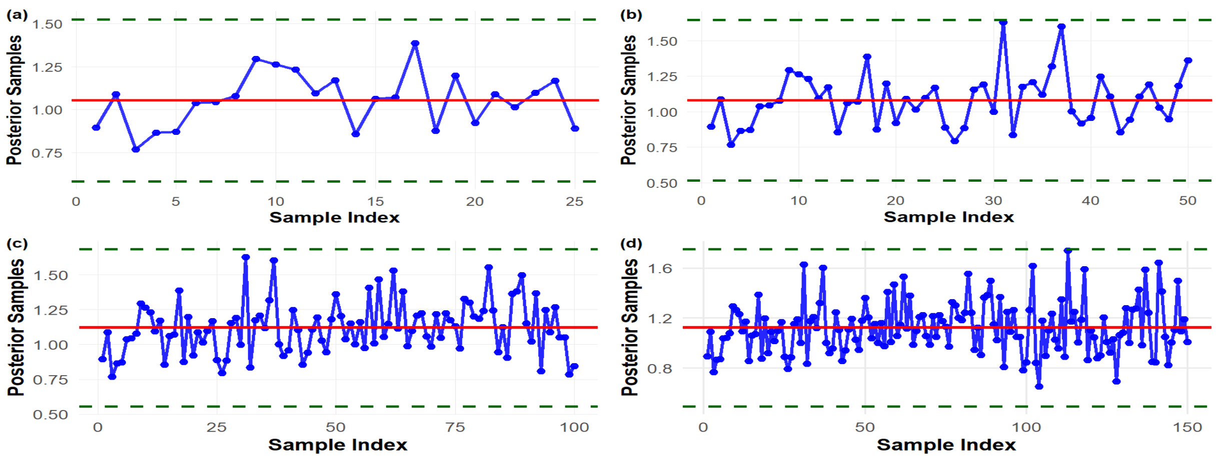


**Figure 3:** Bayesian Control Chart for the Posterior distribution of Exponential distribution with θ =1 by using Gamma Prior with α =1 and β =1.0 at different sample sizes.

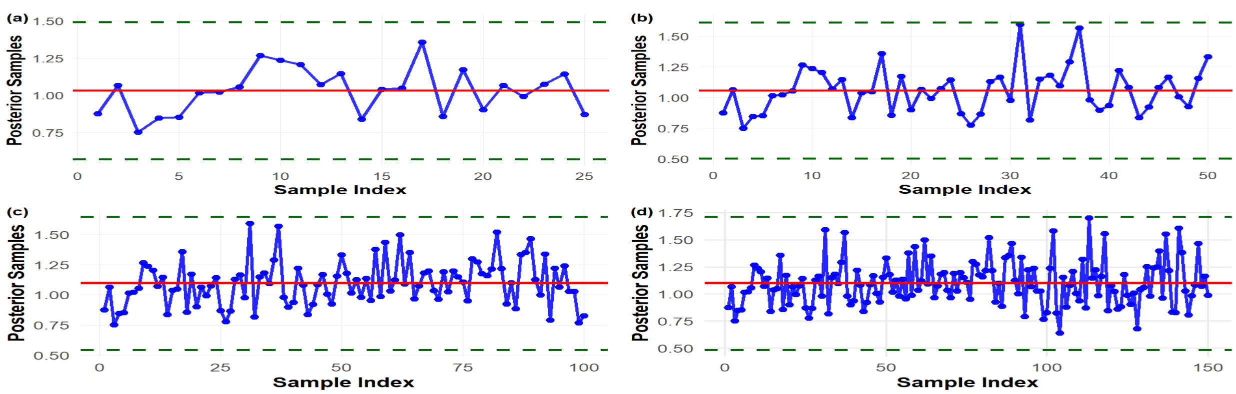


**Figure 4:** Bayesian Control Chart for the Posterior distribution of Exponential distribution with θ =1 by using Gamma Prior with α =1 and β =1.5 at different sample sizes.

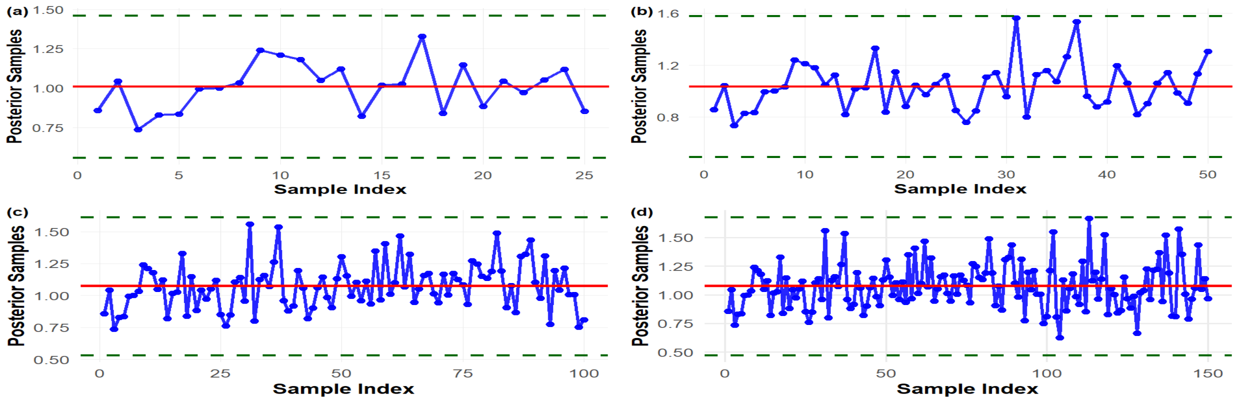


**Figure 5:** Bayesian Control Chart for the Posterior distribution of Exponential distribution with θ =1 by using Gamma Prior with α =1 and β =2.0 at different sample sizes.

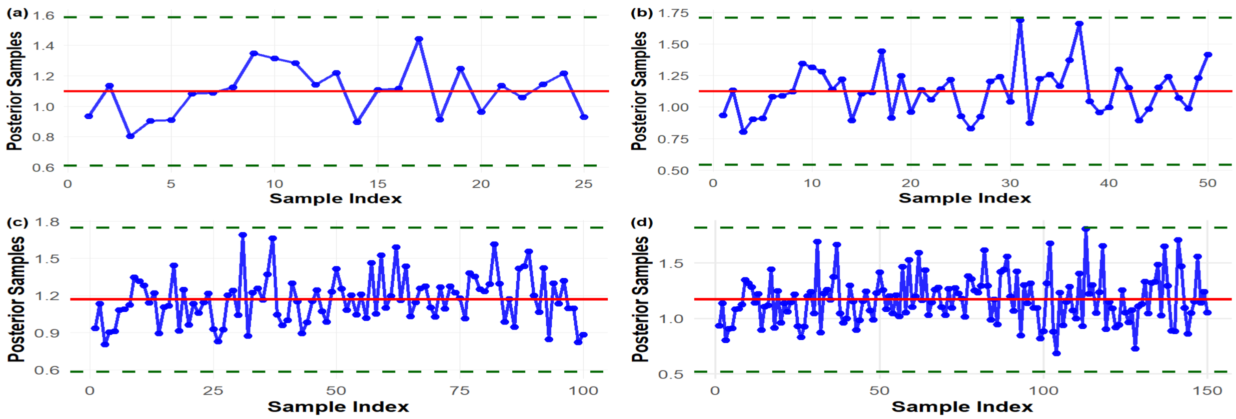


**Figure 6:** Bayesian Control Chart for the Posterior distribution of Exponential distribution with θ =1 by using Gamma Prior with α =1.5 and β = 0.5 at different sample sizes.

Note: Figures 6–9 represent the graphical results corresponding to Table 2.

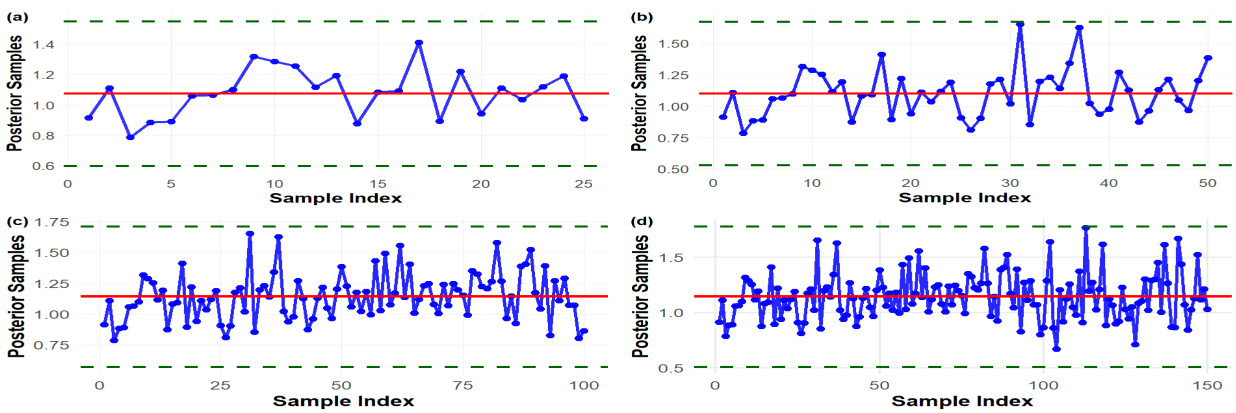


**Figure 7:** Bayesian Control Chart for the Posterior distribution of Exponential distribution with θ =1 by using Gamma Prior with α =1.5 and β = 1.0 at different sample sizes.

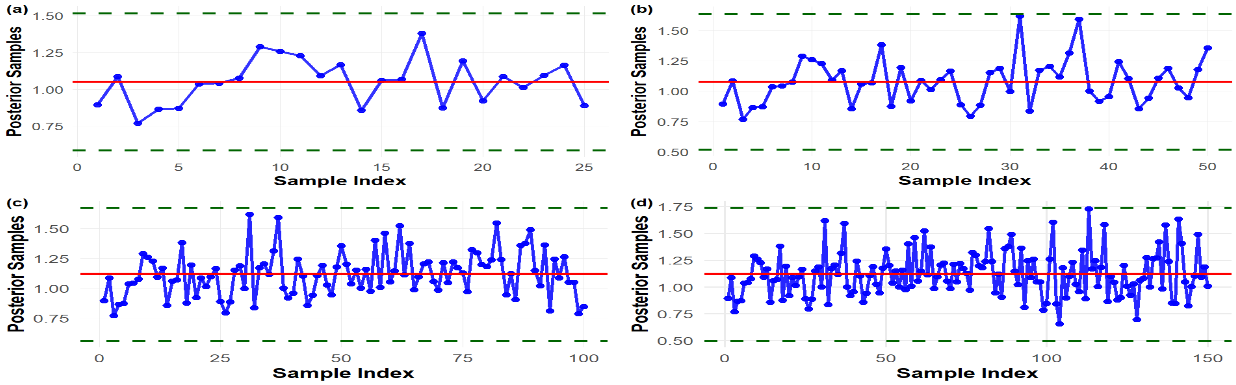


**Figure 8:** Bayesian Control Chart for the Posterior distribution of Exponential distribution with θ =1 by using Gamma Prior with α =1.5 and β = 1.5 at different sample sizes.

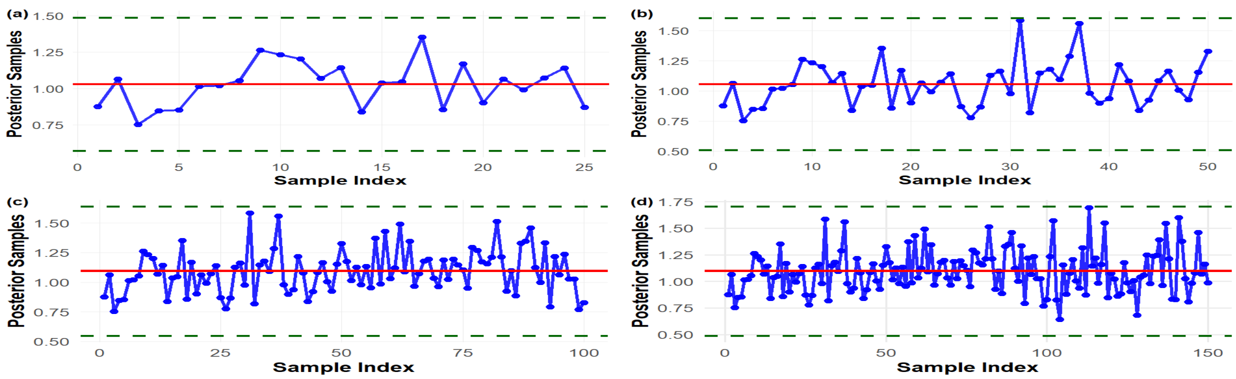


**Figure 9:** Bayesian Control Chart for the Posterior distribution of Exponential distribution with θ =1 by using Gamma Prior with α =1.5 and β = 2 at different sample sizes.

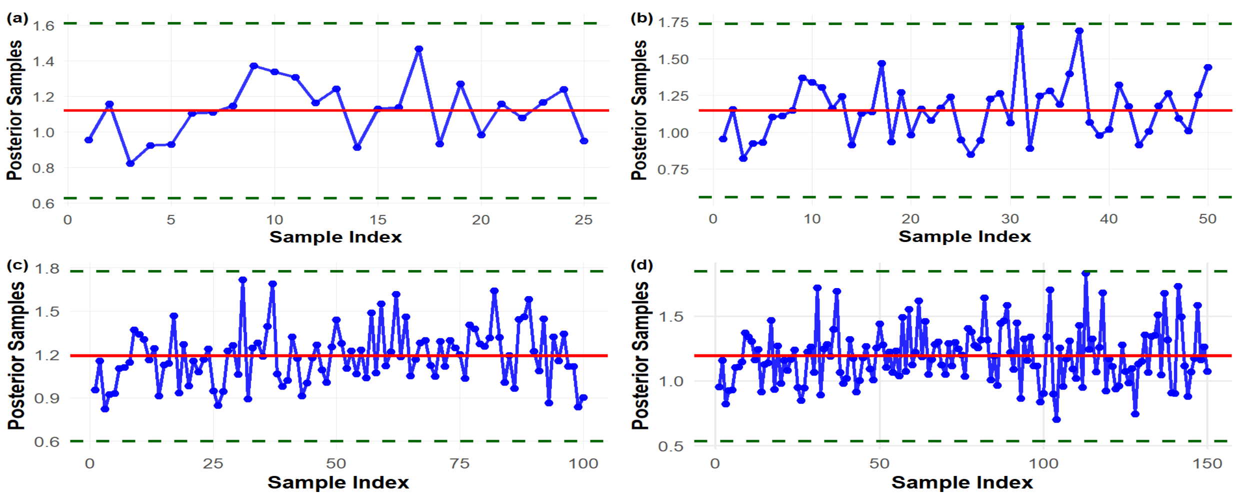


**Figure 10:** Bayesian Control Chart for the Posterior distribution of Exponential distribution with θ =1 by using Gamma Prior with α =2 and β = 0.5 at different sample sizes.

Note: Figures 10–13 represent the graphical results corresponding to Table 3.

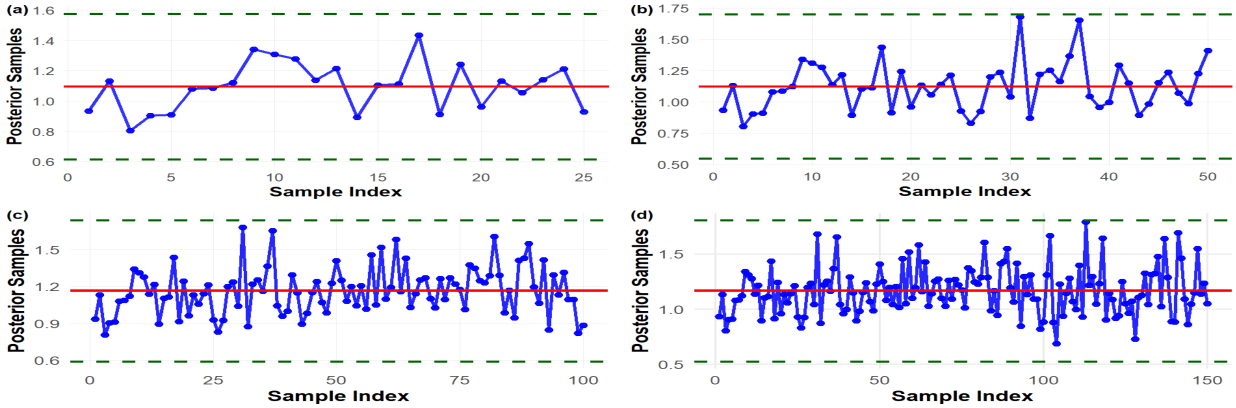


**Figure 11:** Bayesian Control Chart for the Posterior distribution of Exponential distribution with θ =1 by using Gamma Prior with α =2 and β = 1.0 at different sample sizes.

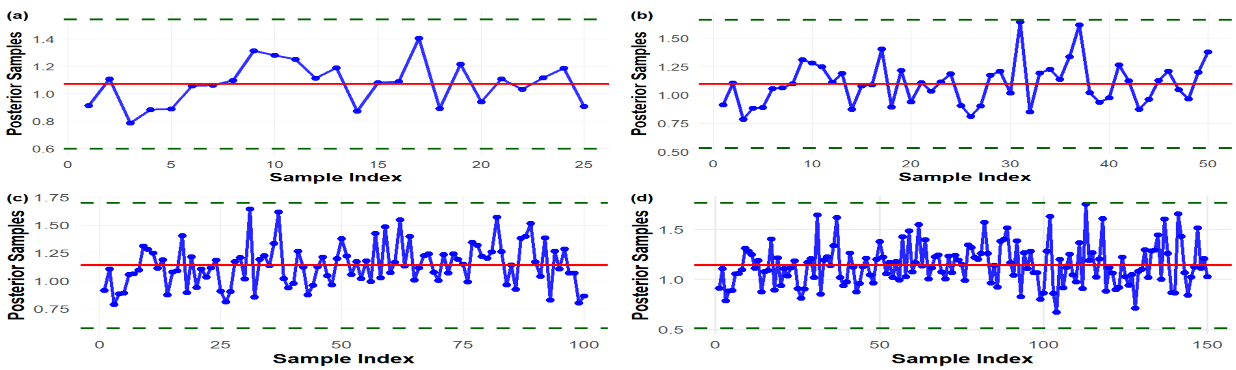


**Figure 12:** Bayesian Control Chart for the Posterior distribution of Exponential distribution with θ =1 by using Gamma Prior with α =2 and β = 1.5 at different sample sizes.

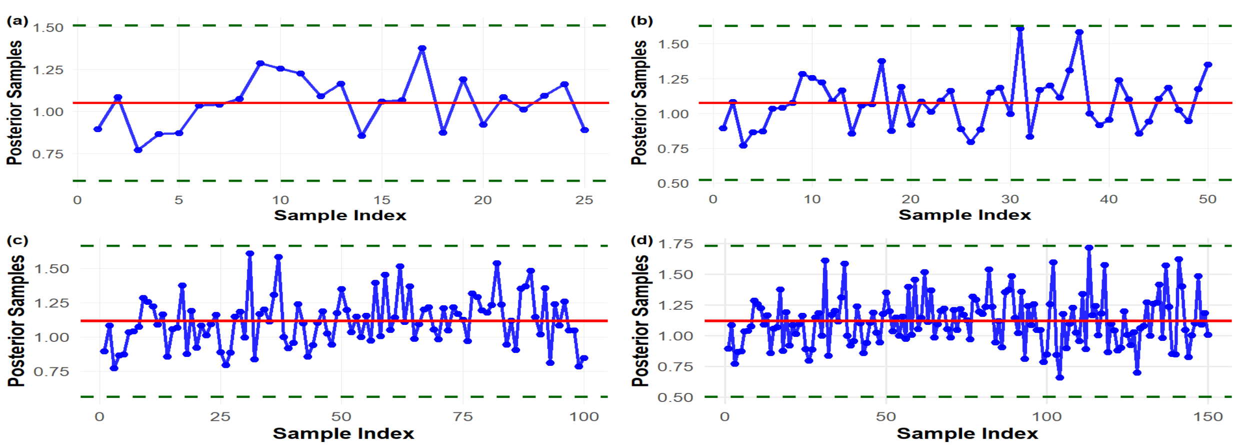


**Figure 13:** Bayesian Control Chart for the Posterior distribution of Exponential distribution with θ =1 by using Gamma Prior with α =2 and β = 2.0 at different sample sizes.

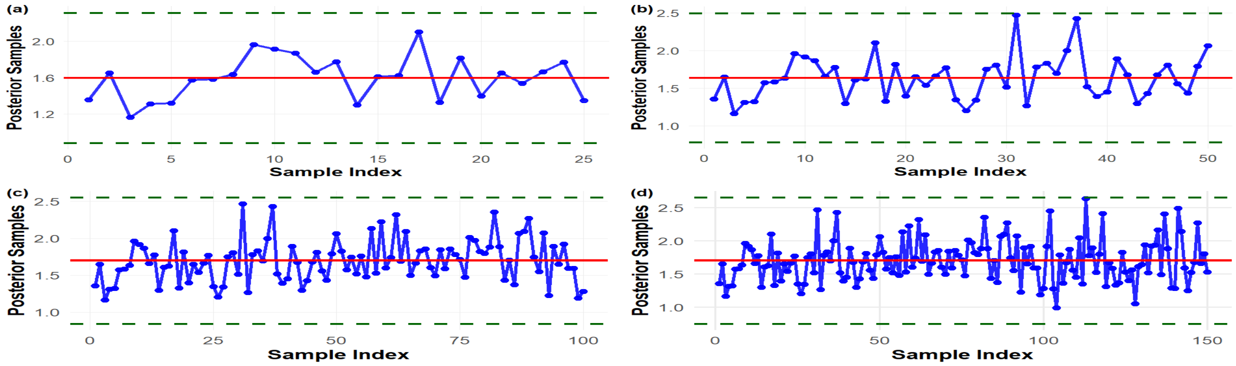


**Figure 14:** Bayesian Control Chart for the Posterior distribution of Exponential distribution with θ =1.5 by using Gamma Prior with α =1.0 and β = 0.5 at different sample sizes.

Note: Figures 14–17 represent the graphical results corresponding to Table 4.

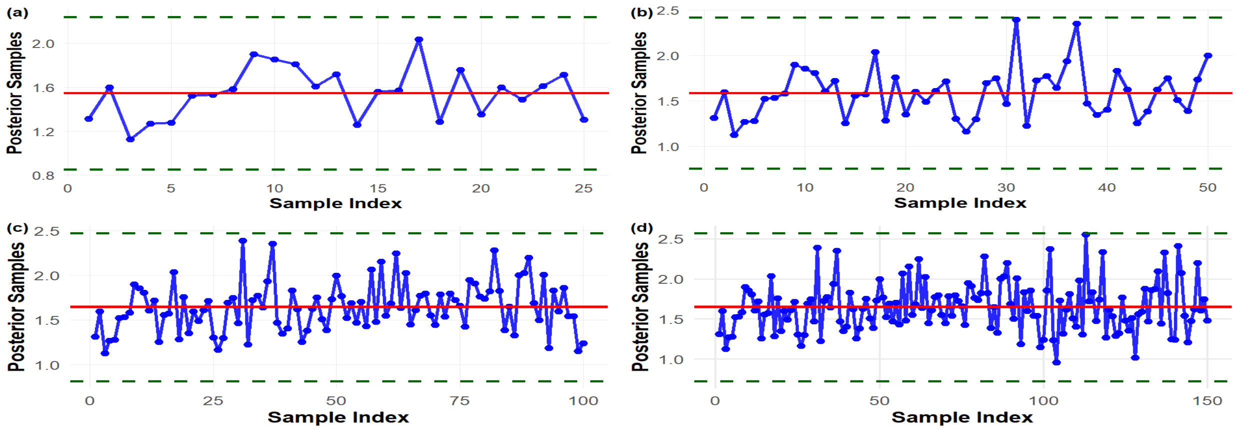


**Figure 15:** Bayesian Control Chart for the Posterior distribution of Exponential distribution with θ =1.5 by using Gamma Prior with α =1.0 and β = 1.0 at different sample sizes.

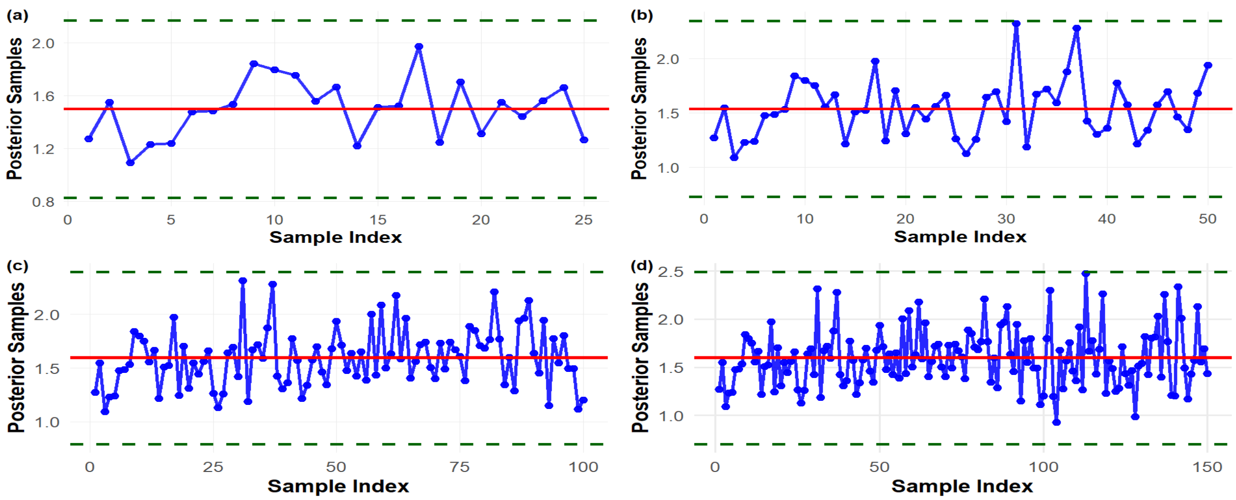


**Figure 16:** Bayesian Control Chart for the Posterior distribution of Exponential distribution with θ =1.5 by using Gamma Prior with α =1.0 and β = 1.5 at different sample sizes.

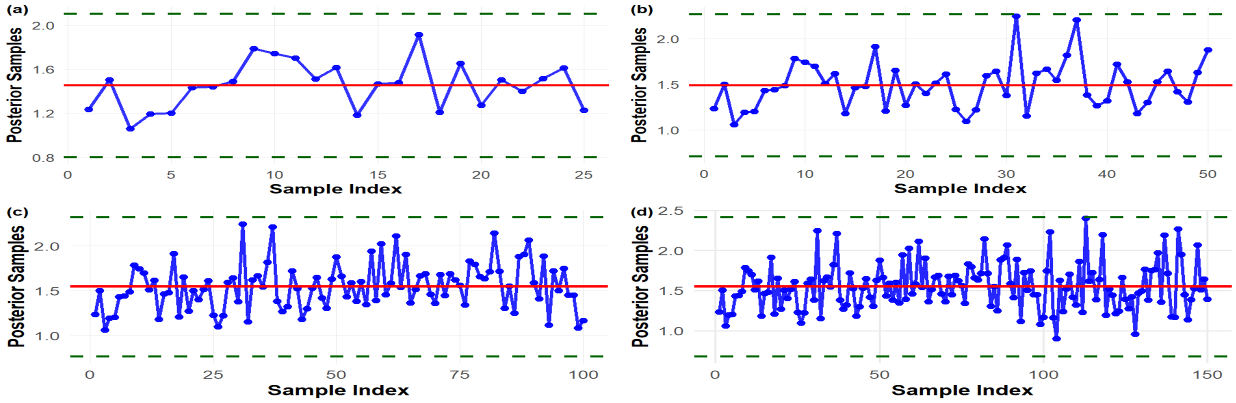


**Figure 17:** Bayesian Control Chart for the Posterior distribution of Exponential distribution with θ =1.5 by using Gamma Prior with α =1.0 and β = 2.0 at different sample sizes.

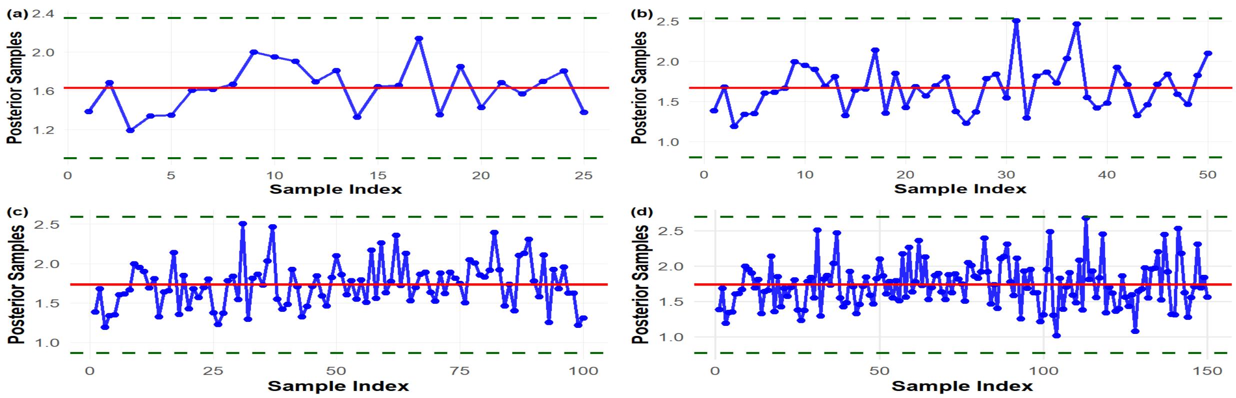


**Figure 18:** Bayesian Control Chart for the Posterior distribution of Exponential distribution with θ =1.5 by using Gamma Prior with α =1.5 and β = 0.5 at different sample sizes.

Note: Figures 18–21 represent the graphical results corresponding to Table 5.

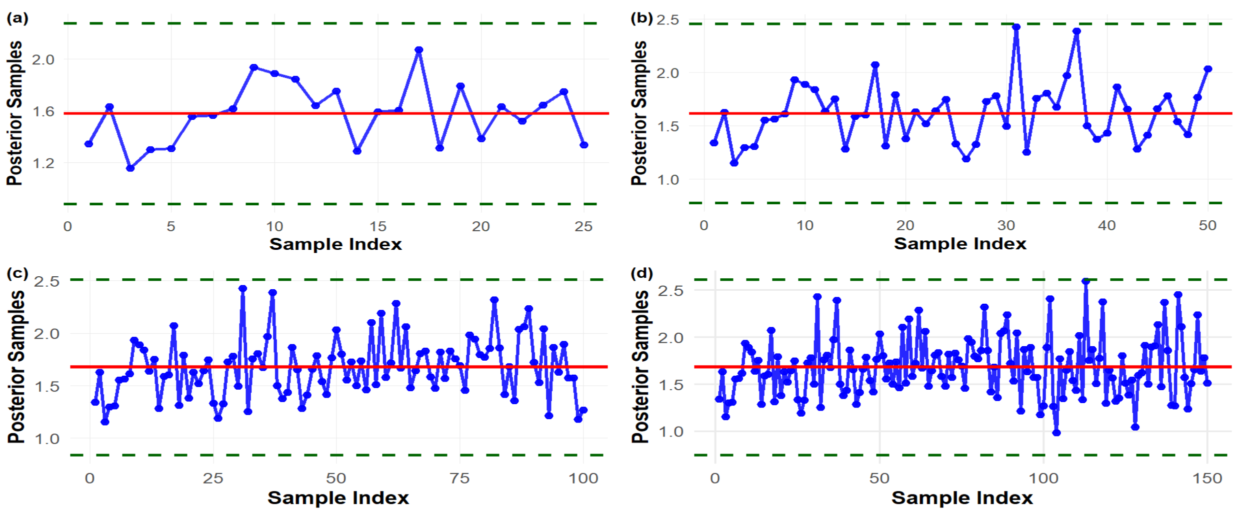


**Figure 19:** Bayesian Control Chart for the Posterior distribution of Exponential distribution with θ =1.5 by using Gamma Prior with α =1.5 and β = 1 at different sample sizes.

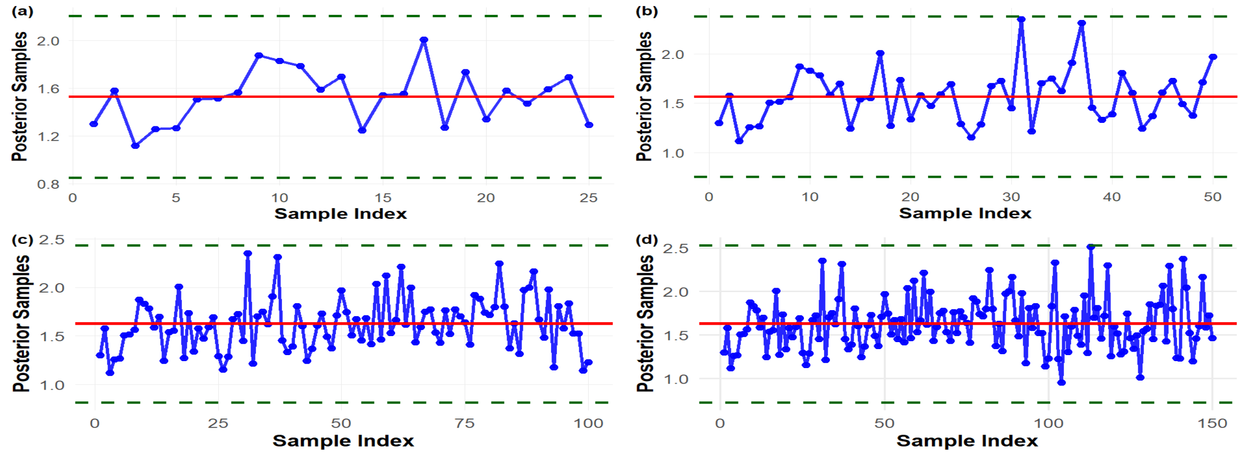


**Figure 20:** Bayesian Control Chart for the Posterior distribution of Exponential distribution with θ =1.5 by using Gamma Prior with α =1.5 and β = 1.5 at different sample sizes.

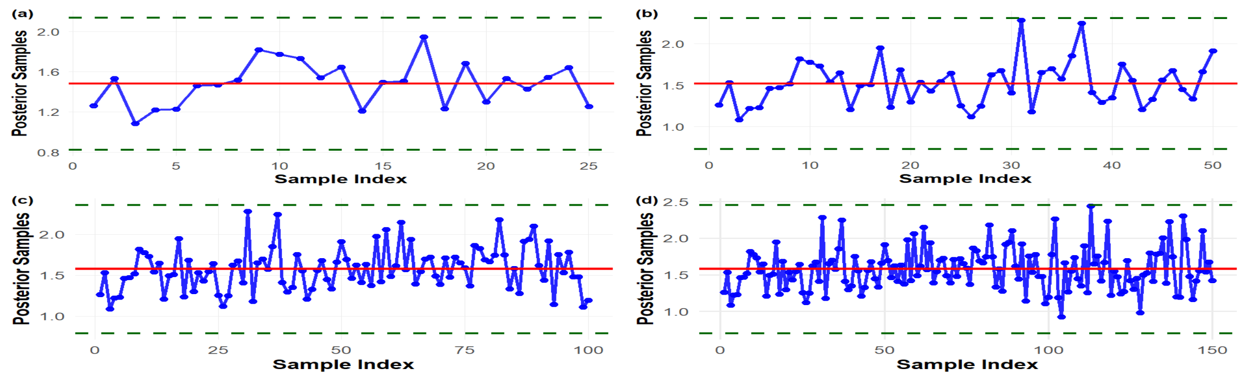


**Figure 21:** Bayesian Control Chart for the Posterior distribution of Exponential distribution with θ =1.5 by using Gamma Prior with α =1.5 and β = 2.0 at different sample sizes.

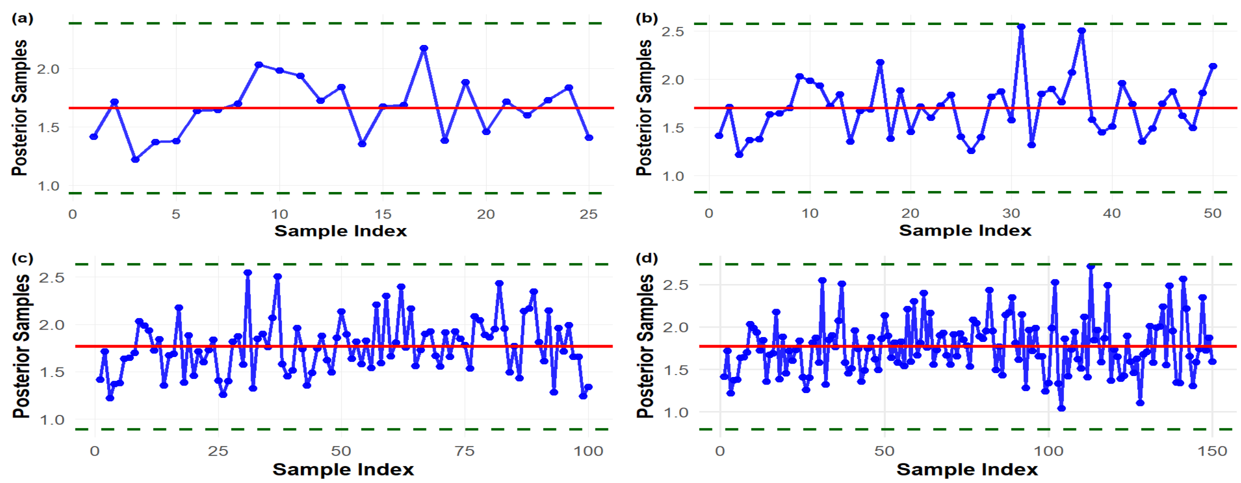


**Figure 22:** Bayesian Control Chart for the Posterior distribution of Exponential distribution with θ =1.5 by using Gamma Prior with α =2.0 and β = 0.5 at different sample sizes.

Note: Figures 22–25 represent the graphical results corresponding to Table 6.

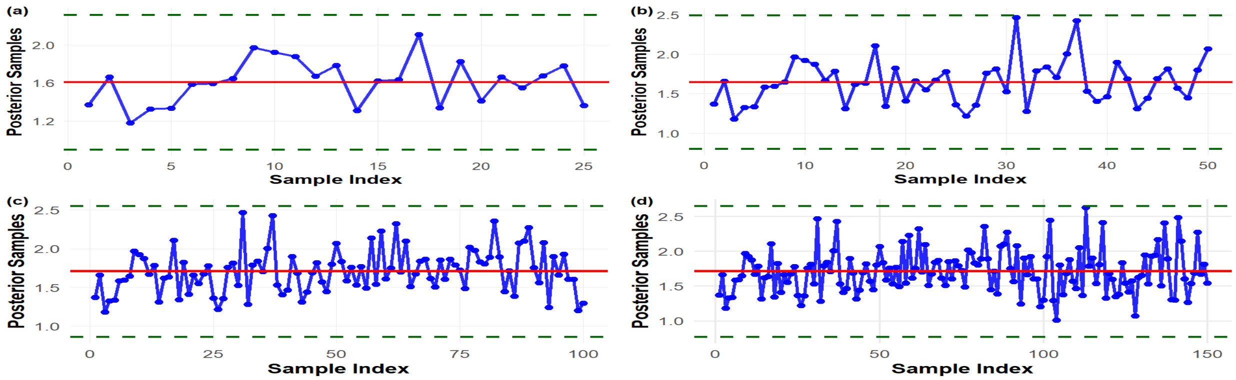


**Figure 23:** Bayesian Control Chart for the Posterior distribution of Exponential distribution with θ =1.5 by using Gamma Prior with α =2.0 and β = 1.0 at different sample sizes.

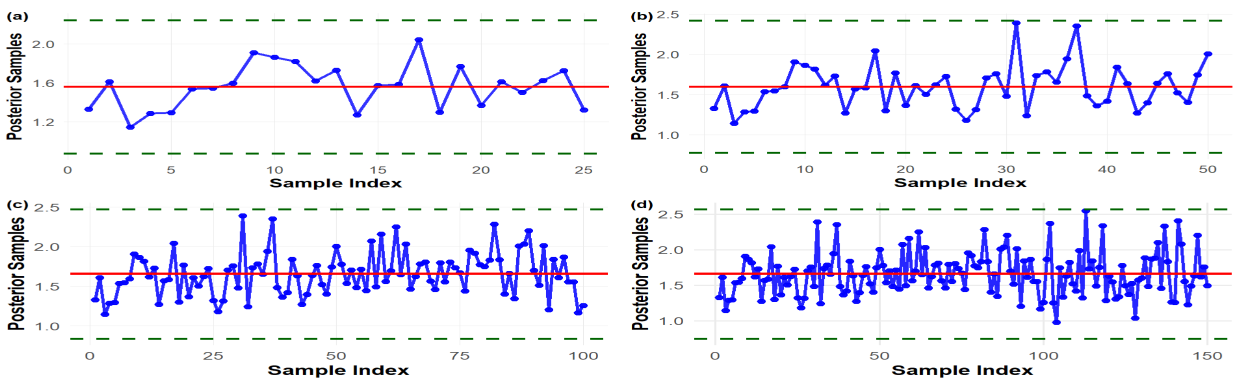


**Figure 24:** Bayesian Control Chart for the Posterior distribution of Exponential distribution with θ =1.5 by using Gamma Prior with α =2.0 and β = 1.5 at different sample sizes.

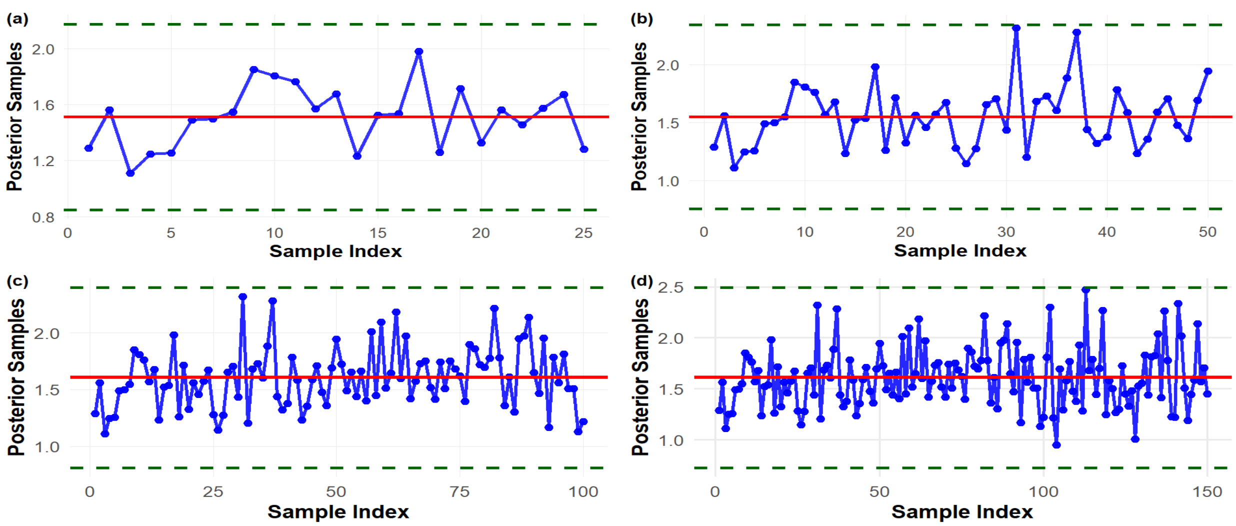


**Figure 25:** Bayesian Control Chart for the Posterior distribution of Exponential distribution with θ =1.5 by using Gamma Prior with α =2.0 and β = 2.0 at different sample sizes.

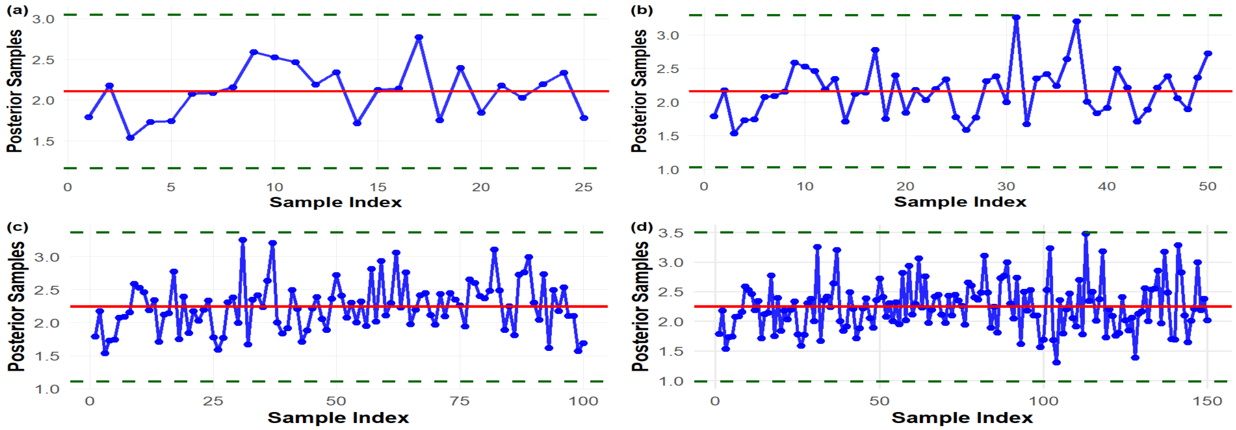


**Figure 26:** Bayesian Control Chart for the Posterior distribution of Exponential distribution with θ =2.0 by using Gamma Prior with α =1.0 and β = 0.5 at different sample sizes.

Note : Figures 26–29 represent the graphical results corresponding to Table 7.

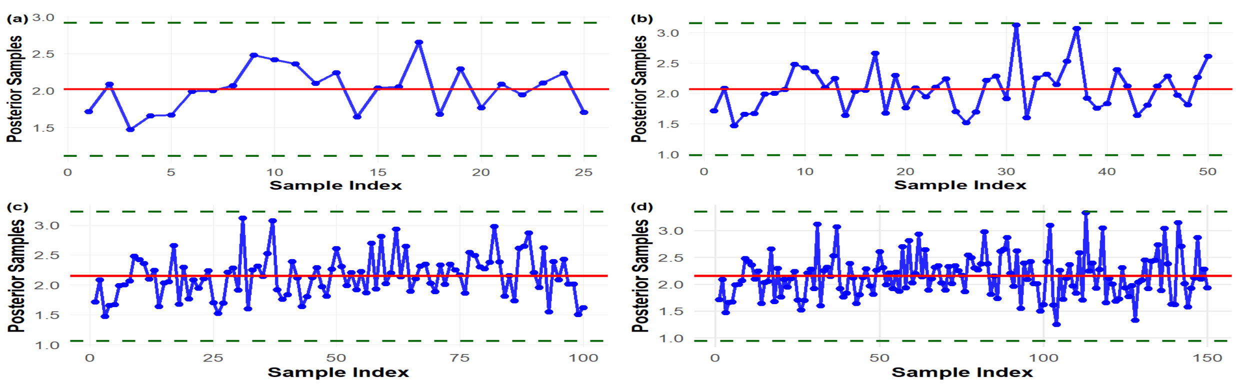


**Figure 27:** Bayesian Control Chart for the Posterior distribution of Exponential distribution with θ =2.0 by using Gamma Prior with α =1.0 and β = 1.0 at different sample sizes.

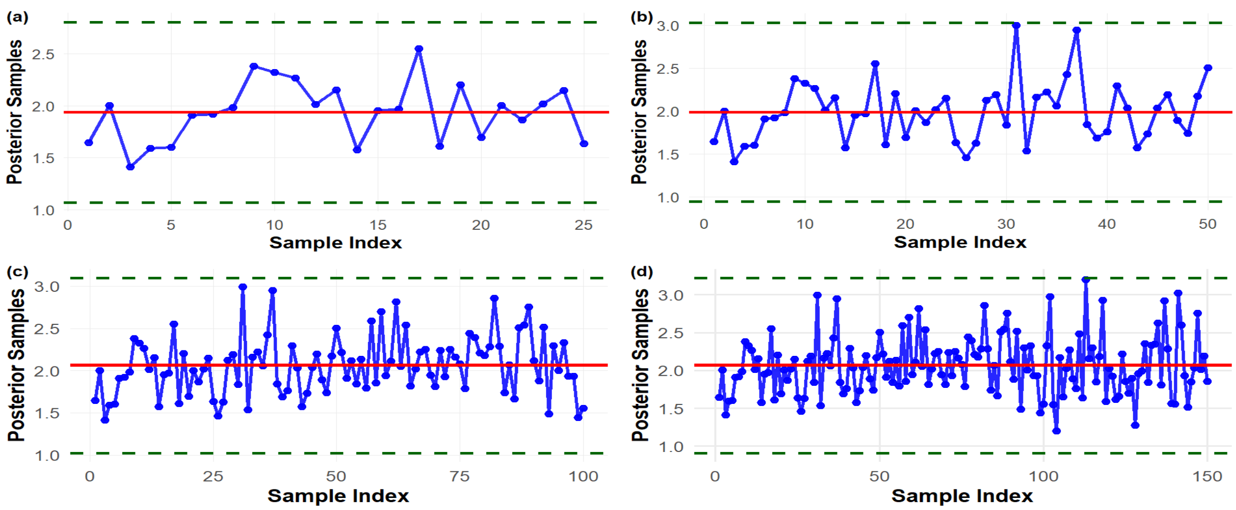


**Figure 28:** Bayesian Control Chart for the Posterior distribution of Exponential distribution with θ =2.0 by using Gamma Prior with α =1.0 and β = 1.5 at different sample sizes.

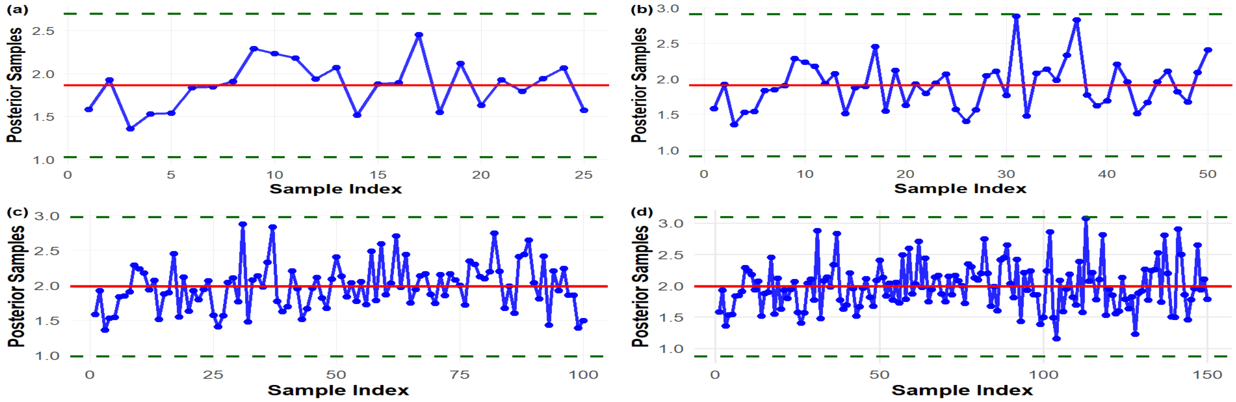


**Figure 29:** Bayesian Control Chart for the Posterior distribution of Exponential distribution with θ =2.0 by using Gamma Prior with α =1.0 and β = 2.0 at different sample sizes.

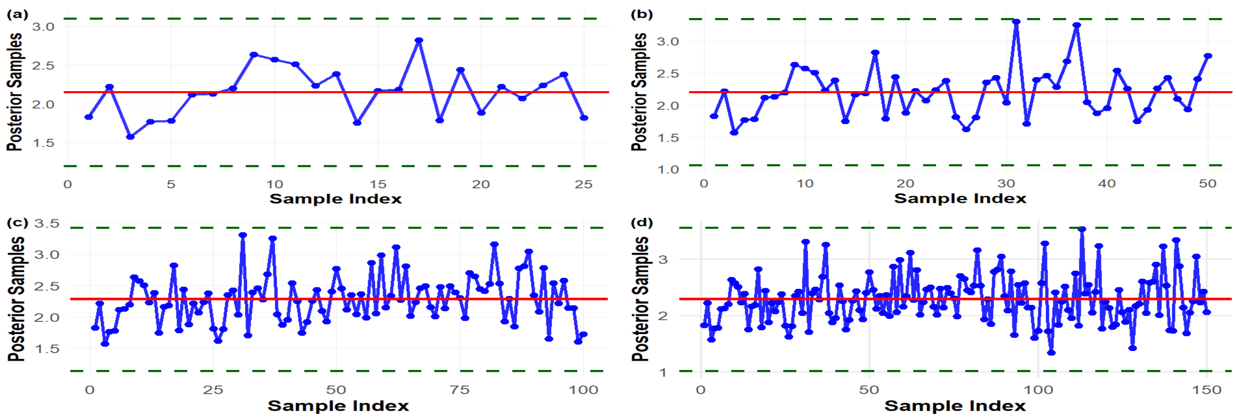


**Figure 30:** Bayesian Control Chart for the Posterior distribution of Exponential distribution with θ =2.0 by using Gamma Prior with α =1.5 and β = 0.5 at different sample sizes.

Note: Figures 30–33 represent the graphical results corresponding to Table 8.

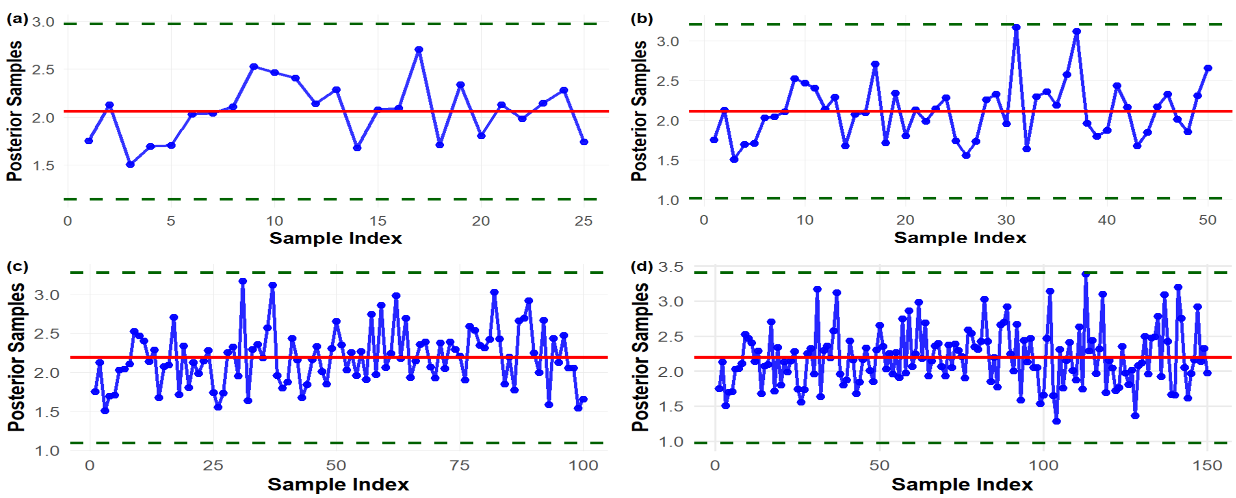


**Figure 31:** Bayesian Control Chart for the Posterior distribution of Exponential distribution with θ =2.0 by using Gamma Prior with α =1.5 and β = 1.0 at different sample sizes.

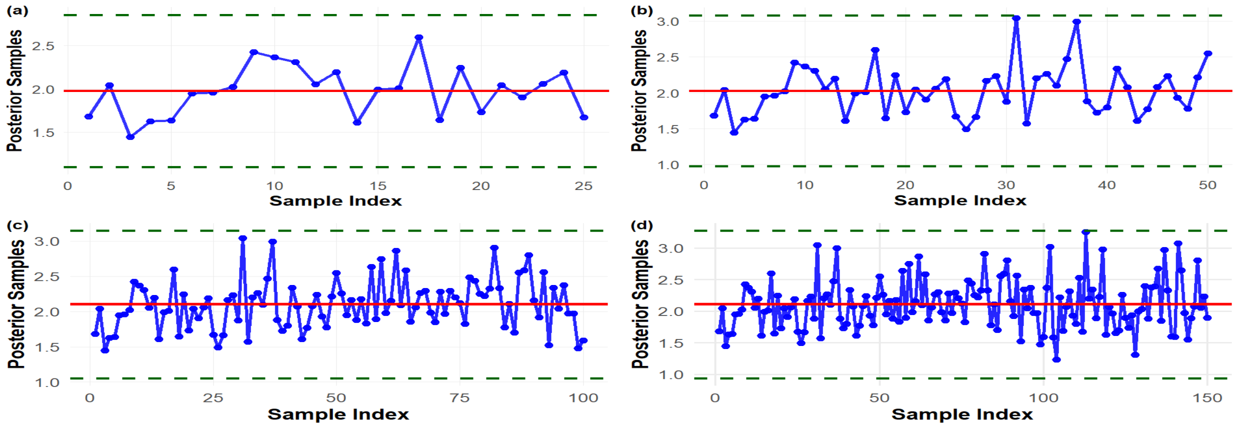


**Figure 32:** Bayesian Control Chart for the Posterior distribution of Exponential distribution with θ =2.0 by using Gamma Prior with α =1.5 and β = 1.5 at different sample sizes.

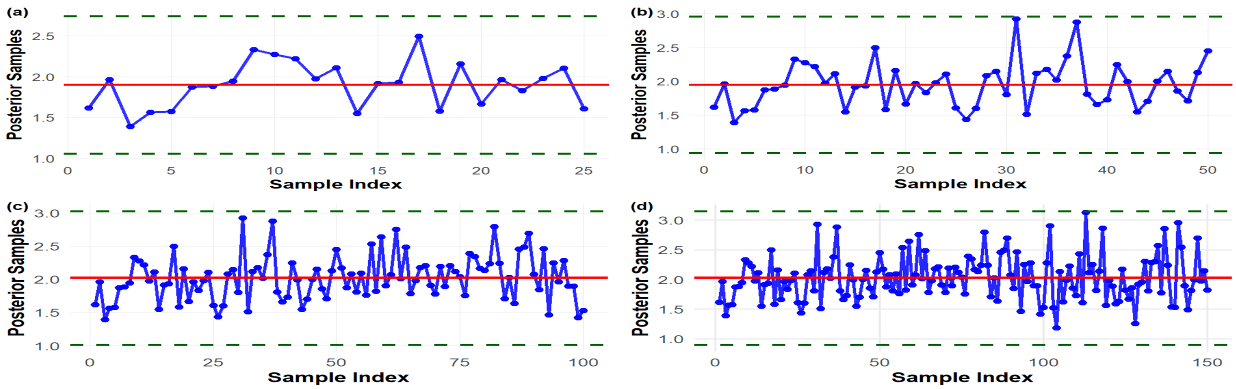


**Figure 33:** Bayesian Control Chart for the Posterior distribution of Exponential distribution with θ =2.0 by using Gamma Prior with α =1.5 and β = 2.0 at different sample sizes.

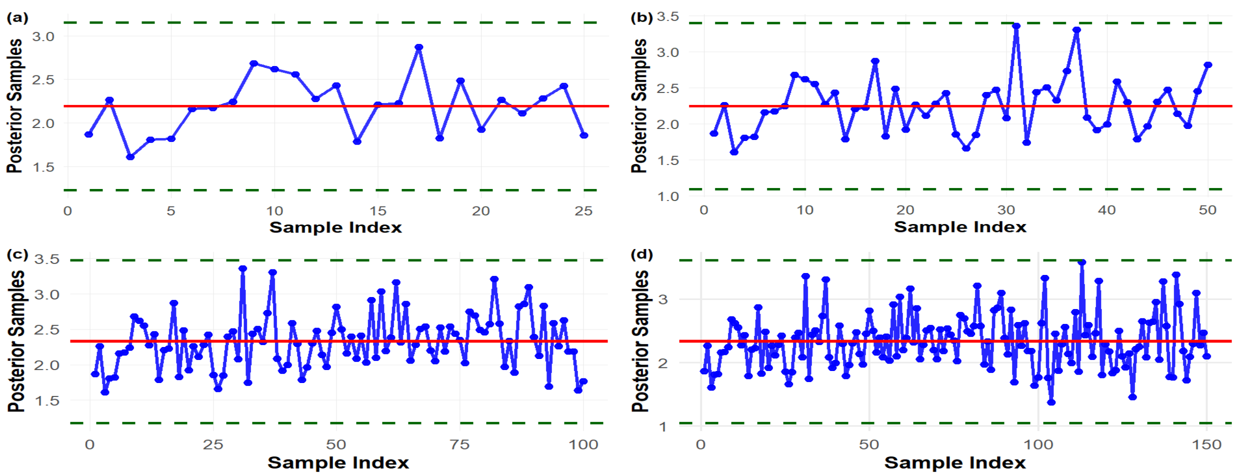


**Figure 34:** Bayesian Control Chart for the Posterior distribution of Exponential distribution with θ =2.0 by using Gamma Prior with α =2.0 and β = 0.5 at different sample sizes.

Note : Figures 34–37 represent the graphical results corresponding to Table 9.

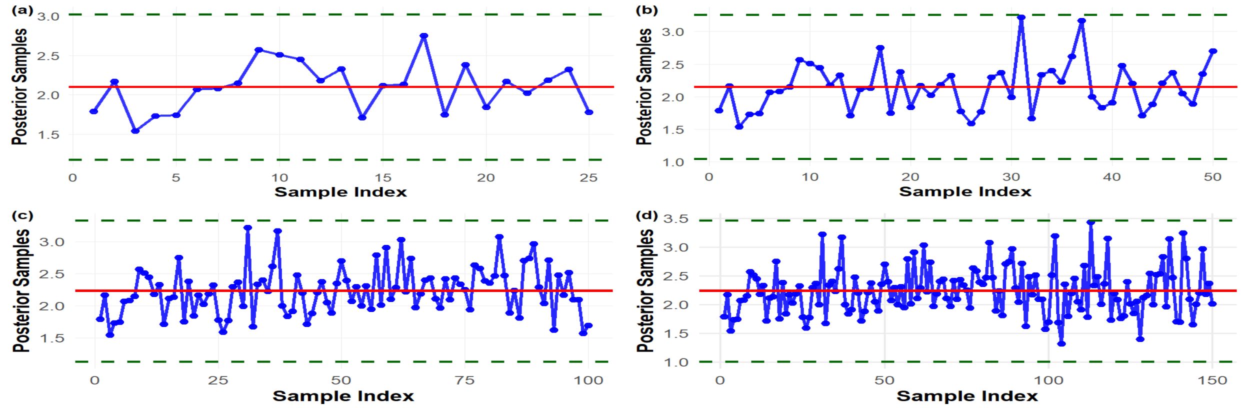


**Figure 35:** Bayesian Control Chart for the Posterior distribution of Exponential distribution with θ =2.0 by using Gamma Prior with α =2.0 and β = 1.0 at different sample sizes.

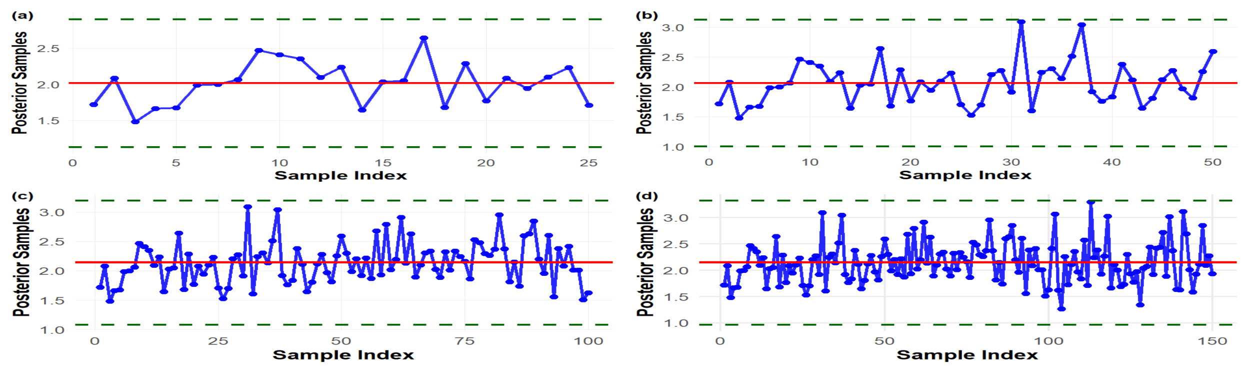


**Figure 36:** Bayesian Control Chart for the Posterior distribution of Exponential distribution with θ =2.0 by using Gamma Prior with α =2.0 and β = 1.5 at different sample sizes.

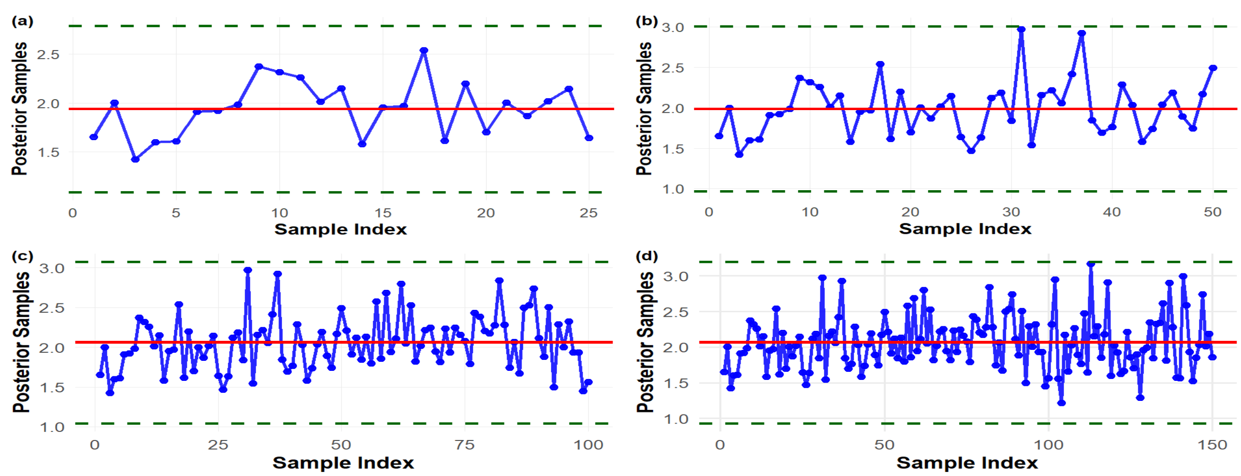


**Figure 37:** Bayesian Control Chart for the Posterior distribution of Exponential distribution with θ =2.0 by using Gamma Prior with α =2.0 and β = 2.0 at different sample sizes.