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A Multiple Dependent State Sampling Plan for Percentile Based Life Tests under the Half Normal Distribution with Applications to Software and Device Reliability

Muhammad Naveed [1*], Muhammad Iltaf [2], Muhammad Azam [3], Nasrullah Khan [4]

[1]Department of Statistics, Govt Graduate College for Boys, Gulberg, Lahore, Pakistan

[2]Department of Statistics, National College of Business Administration and Economics, Lahore, Pakistan.

[3]Department of Statistics and Computer Science, University of Veterinary and Animal Sciences, Lahore, 54000, Pakistan

[4]College of Statistical Science, University of the Punjab, Lahore, Pakistan

* Corresponding Email: mnaveedakbar09@gmail.com

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ABSTRACT

In modern reliability engineering, ensuring that product lifetimes meet specified percentile requirements is often more informative than relying on mean lifetime criteria, particularly for skewed or heavy-tailed failure distributions. This paper develops a novel multiple dependent state (MDS) sampling plan for truncated life tests, assuming that product lifetimes follow a half-normal distribution (HND). The proposed plan uses the percentile of the lifetime distribution as the quality benchmark, thereby providing enhanced protection against early failures. The operating characteristic (OC) function is derived, and an optimization framework is established to determine the minimum sample size (n) together with the acceptance (c_1) and rejection (c_2) numbers such that the producer's risk ($\alpha = 0.05$) and consumer's risk($\beta = 0.25, 0.10, 0.05, 0.01$) are simultaneously satisfied. Extensive tables of optimal plan parameters are presented for different truncation multipliers ($\delta = 0.5, 1.0$), percentile levels ($q = 0.25, 0.50$), and quality ratios ($d = 2,4,6,8$). The results show that the MDS plan substantially reduces the required sample size compared to conventional single sampling plans. For example, at $\beta = 0.25, d = 2, \delta = 0.5, q = 0.5$, the sample size decreases from 48 (single plan) to 28 (MDS), a reduction of 41.67%. Two real-world case studies, software failure times and electronic device lifetimes, demonstrate the practical applicability of the plan: a conforming software lot is accepted with minimal inspection, while a non-conforming device lot is swiftly rejected. The proposed MDS plan offers an efficient, flexible, and statistically rigorous tool for reliability engineers who must balance inspection cost with stringent quality assurance.

Keywords: Acceptance sampling; Multiple dependent state; Half normal distribution; Percentile based life test; Consumer's risk

1. Introduction

In modern manufacturing and supply chain operations, product quality verification constitutes a fundamental pillar of industrial competitiveness. Organizations face increasing pressure to deliver high-reliability products while simultaneously minimizing inspection costs and maintaining rigorous quality standards. This dual imperative has elevated the role of acceptance sampling plans (ASPs) as indispensable statistical tools for quality assurance across diverse industrial sectors. ASPs provide a systematic framework for making acceptance or rejection decisions on product lots based on examining a representative sample, achieving a pragmatic balance between the impracticality of zero inspection and the prohibitive costs of 100% inspection (Schilling & Neubauer, 2009) ; (Montgomery, 2020). The importance of such plans is further underscored by the substantial economic `consequences of undetected quality failures, including product recalls, warranty claims, and reputational damage, which have driven continuous refinement of lot disposition methodologies. Sampling inspection has therefore evolved as a widely favored approach, as it provides clear decision rules enabling practitioners to

sentence lots economically and efficiently while controlling inherent sampling risks (Wu, Shu, & Wang, 2025); (Zheng, Yang, & Zhao, 2025).
The fundamental challenge underlying any acceptance sampling plan lies in balancing two opposing risks: the producer's risk α(the probability of erroneously rejecting a good-quality lot) and the consumer's risk β(the probability of erroneously accepting a poor-quality lot). While early sampling inspection plans were primarily attribute-based, evaluating quality by counting defective items, recent manufacturing advancements have drastically improved process capabilities, reducing defect levels to parts per million and making attribute-based plans less suitable for modern high-yield production environments (Wang, Shu, Hsu, & Hsu, 2022). Consequently, variable sampling inspection, which bases decisions on measured quality data, has gained increasing attention. Researchers have increasingly combined variable sampling inspection with process capability indices to enhance decision-making reliability (Wu, Shu, & Wang, 2024); (Darmawan & Wu, 2025).
The evolution of ASPs reflects a sustained effort to align statistical theory with industrial practicalities. Single sampling plans (SSPs), while straightforward and easily implemented, often require large sample sizes to achieve desired risk protection levels. This limitation motivated the development of double sampling plans, where an optional second inspection stage enables a more favorable balance between cost and risk when initial results are inconclusive. More sophisticated schemes subsequently emerged, each offering distinct advantages for specific operational contexts. Sequential sampling plans allow for repeated assessments unit-by-unit until a definitive conclusion is reached, often substantially reducing the average sample number (ASN). Group sampling plans analyze subsets of items collectively, making them particularly suitable for high-throughput testing environments where units can be tested simultaneously. Resubmitted sampling plans provide a mechanism for re-evaluating rejected lots, offering flexibility in supplier-buyer relationships.
Among advanced sampling schemes, the multiple dependent state (MDS) sampling plan has demonstrated exceptional efficiency by incorporating information from preceding lots. Originally introduced by (Wortham & Baker, 1976) and later refined by (Balamurali & Jun, 2007), the MDS plan decides the disposition of the current lot based not only on the current sample but also on the quality status of a specified number of preceding lots. This historical information sharing enables substantial reductions in sample size without compromising risk protection. Recent developments have extended MDS plans to various lifetime distributions, including the exponentiated Weibull distribution (Gadde, Fulment, & Peter, 2021), the odd log-logistic generalized exponentiated (OLLGE) distribution (Fulment, Rao, & Peter, 2024), and the exponentiated Fréchet distribution (Gadde Srinivasa Rao, Jilani, & Peter, 2025).The MDS framework therefore represents a fertile ground for methodological innovation, particularly when paired with lifetime models that have not yet been explored within this scheme.
Traditional acceptance sampling under life testing has predominantly relied on the mean lifetime as the primary quality parameter. Truncated life tests, where experimentation is terminated at a pre-assigned time t_0 to minimize time and cost, have become standard practice. However, reliance on the mean lifetime presents several critical limitations. The mean does not adequately capture distributional features, particularly for skewed or heavy-tailed lifetime distributions commonly encountered in reliability engineering. Consider a scenario where the mean lifetime remains acceptable, yet variance increases significantly; in such cases, lower percentiles (e.g., the 5th or 10th percentile) may drop sharply, allowing products with high early failure propensity to pass inspection while the mean criterion remains satisfied.
This inadequacy has motivated a paradigm shift toward percentile-based acceptance criteria, which more accurately capture distribution tails and ensure consumer safety. In structural engineering applications, the 5th percentile of material strength often serves as the relevant benchmark, as failure below such thresholds can lead to catastrophic consequences. Similarly, in warranty analysis and reliability guarantee contexts, specific percentile lifetimes provide more meaningful quality assurance than average performance measures. The significance of percentile-based acceptance sampling was initially highlighted by (Lio, Tsai, & Wu, 2009), who introduced percentile-based ASPs for the Birnbaum–Saunders distribution under truncated life testing. This foundational work established the minimum sample sizes necessary to ensure specified percentiles while managing consumer risk, validated with real-world datasets. Subsequent research has extended percentile-based frameworks to numerous lifetime distributions, including the log-logistic distribution (G Srinivasa Rao & Kantam, 2010), the Marshall–Olkin extended exponential distribution (G Srinivasa Rao, 2013), the linear failure rate distribution (B. S. Rao, Priya, & Kantam, 2014), the extended generalized exponential distribution (Amer Ibrahim Al-Omari & Alomani, 2024), and the exponentiated generalized inverse Rayleigh distribution (Jayalakshmi & Aleesha, 2024).
The Half-Normal Distribution (HND) holds particular relevance for reliability modeling and acceptance sampling applications due to its analytical tractability and appropriate hazard properties. Formally defined as a truncated normal distribution constrained to non-negative values with a mean of zero and scale parameter λ, the HND possesses an increasing failure rate (IFR) property, making it particularly suitable for modeling wear-out failure mechanisms commonly observed in mechanical components, electronic devices, and aging systems where failure propensity increases over time. The HND can be derived from the absolute value of a normally distributed random variable, providing a natural connection to well-understood normal theory while accommodating non-negative lifetime data.

The utility of the HND has been demonstrated across various disciplines, including quality control, fatigue analysis, and strength-stress reliability. Foundational work by (Chou & Liu, 1998) established key properties of the HND, while (Castro, Gómez, & Valenzuela, 2012) illustrated its effectiveness in modeling fatigue-related data. Recent research has further validated the HND's practical relevance: a study comparing five two-parameter variants of folded-normal distributions found that the folded normal distribution (FND) and half-normal distribution (HND) provide better fitting to bus-motor failure data than existing models in terms of both model simplicity and goodness of fit (Jiang, Xue, & Cao, 2024). The power half-normal (PHN) lifetime model, incorporating an additional shape parameter into the traditional half-normal distribution, has been shown to provide superior fit compared to eight alternative models, including Weibull, gamma, and generalized exponential distributions, when applied to electrical appliance failure data (Alqasem & Elshahhat, 2025). The alpha power transformed half-normal distribution has similarly demonstrated highly flexible hazard function forms, enhancing its applicability to various lifetime and reliability datasets (Uke & Taye, 2025).The generalized half-normal (GHN) distribution, originally introduced by *(Cooray & Ananda, 2008)*, is a special case of the generalized gamma distribution capable of monotonically increasing, decreasing, or bathtub-shaped hazard rates. This flexibility has been leveraged in recent acceptance sampling methodologies (Qomi, Piadeh, Pérez-González, & Fernández, 2025), and its special case, the standard half-normal (HND) distribution, inherits the ability to model wear-out failures effectively.
Within the acceptance sampling literature, the HND has been extensively studied. (B. S. Rao, Kumar, & Rosaiah, 2013) developed single sampling plans for truncated life tests based on HND percentiles, establishing minimum sample sizes necessary to ensure specified percentiles while managing consumer risk. Subsequent extensions have included double acceptance sampling plans for time-truncated life tests based on the HND (Lu, Gui, & Yan, 2013),(Amer I Al-Omari, Al-Nasser, & Gogah, 2016), group acceptance sampling plans for HND percentiles (Azam, Aslam, Balamurali, & Javaid, 2015), two-stage group acceptance sampling plans for HND (Geetha, Jayabharathi, & Uddin, 2024), and hybrid group adoption sampling plans for HND lifetimes (B. S. Rao, Prasad, Rao, & Sudhakar, 2026). (Naveed et al., 2025) developed repetitive sampling plans for life tests using percentiles of the half-normal distribution, with applications to software reliability and device lifetime data. Their method improves sampling efficiency and provides practical decision rules for lot acceptance in reliability engineering. Recent methodological advances have also addressed optimal acceptance sampling plans for the generalized half-normal distribution under time-censoring with conventional and expected limited risks (Qomi et al., 2025) and improved time-censored group sampling schemes for GHN distributions (Naghizadeh & Mahdizadeh, 2026). Fuzzy theory has also been integrated with GHN distributions for acceptance sampling inspection plans (Tripathi, 2025).
A systematic review of the existing literature reveals a significant research gap. While MDS plans have been developed for various lifetime distributions, including the exponentiated Weibull distribution, odd log-logistic generalized exponentiated distribution, new Lomax Rayleigh distribution, and exponentiated Fréchet distribution, and percentile-based acceptance sampling plans have been extensively studied for the HND under single, double, and group sampling schemes, the specific combination of an MDS plan for percentile-based life tests assuming HND lifetimes has not been previously addressed. This gap is particularly notable given the demonstrated efficiency advantages of MDS plans (utilizing preceding lot information) and the robustness benefits of percentile-based criteria (capturing tail behavior). No existing work has developed a dedicated MDS plan for HND percentiles that leverages historical lot quality information to reduce inspection effort. The present study addresses this gap by introducing a novel MDS plan for truncated life tests based on percentiles of the Half-Normal Distribution. The proposed plan integrates three methodological innovations: (i) percentile-based life testing that captures distributional tail behavior and provides more robust quality assurance than mean-based criteria; (ii) the HND as the underlying lifetime model, which is analytically tractable and has an increasing failure rate suitable for wear-out mechanisms; and (iii) a multiple dependent state sampling framework that incorporates information from preceding lots to achieve substantial reductions in average sample number without compromising statistical risk protection.
The remainder of this paper is organized as follows. Section 2 presents the design of the proposed multiple dependent state (MDS) sampling plan along with the remaining operational work, including the mathematical formulation of the HND, the derivation of the failure probability for percentile-based truncated life tests, the operational procedure of the MDS plan, the derivation of the operating characteristic (OC) function, and the optimization model for determining the optimal design parameters. Section 3 then provides a comparative analysis that demonstrates the superior efficiency of the proposed MDS plan relative to the single sampling plan. Section 4 illustrates the real-world practical utility of the methodology through applications to software reliability data and device lifetime data. Finally, Section 5 concludes the paper with a summary of key findings and suggestions for future research.

2. Design of the Proposed Multiple Dependent State Sampling Plan for Half-Normal Distribution Percentiles

This section develops a multiple dependent state (MDS) sampling plan for life tests wherein the product lifetime follows a half-normal distribution (HND). The plan employs a percentile-based quality criterion and truncates the life test at a predetermined time. The distributional assumptions, failure probability derivation, operating characteristic (OC) function, and the optimization procedure for determining the plan parameters are presented sequentially.

2.1 The Half-Normal Distribution for Lifetime Modelling

The half-normal distribution is a flexible positively skewed distribution defined on non-negative support. It is particularly suitable for modelling lifetimes that exhibit an increasing failure rate, characteristic of wear-out mechanisms in mechanical and electronic components. Let T be a non-negative random variable representing product lifetime. The probability density function (PDF) of the HND is

$$f(t) = \frac{2}{\lambda\sqrt{\pi}} exp\left(-\frac{t^2}{\lambda^2 \pi}\right) \quad t \geq 0, \lambda > 0 \tag{1}$$

where λ is a scale parameter. A larger λ indicates greater dispersion of lifetimes, corresponding to a longer-lived product on average. The cumulative distribution function (CDF), which gives the probability of failure by time t, is expressed via the error function $erf(\cdot)$:

$$F(t) = erf\left(\frac{t}{\lambda\sqrt{\pi}}\right) \tag{2}$$

The error function, $erf(\cdot)$, is a standard mathematical function defined as:

$$erf(z) = \frac{2}{\sqrt{\pi}} \int_0^z \exp(-x^2)\, dx \tag{3}$$

The mean μ and variance σ^2 are

$$\mu = \frac{\lambda\sqrt{\pi}}{2} \tag{4}$$

$$\sigma^2 = \lambda^2 \left(\frac{\pi}{2} - 1\right) \tag{5}$$

The closed-form CDF of the HND and its increasing failure rate make it analytically tractable for designing truncated life test sampling plans.

2.2 Percentile Lifetime and Failure Probability in a Truncated Life Test

Quality assurance often requires that a certain proportion of the population survives beyond a specified time, equivalently, that a low percentile $(e.g., the\ 5th\ or\ 10th)$ exceeds a standard. Let t_q denote the $100\ q - th$ percentile of the lifetime distribution, i.e., $F(t_q) = q$. From the HND-CDF,

$$erf\left(\frac{t_q}{\lambda\sqrt{\pi}}\right) = q \tag{6}$$

$$t_q = \lambda\sqrt{\pi} erf^{-1}(q) \qquad 0 < \mathrm{q} < 1 \tag{7}$$

Thus, the scale parameter can be written as

$$\lambda = \frac{t_q}{\sqrt{\pi} erf^{-1}(q)} \tag{8}$$

For a given required percentile standard ${t_q}^0$ (specified by the consumer or a regulatory standard), a lot is considered acceptable if its true percentile t_q is at least ${t_q}^0$

$$\text{Define the quality ratio } d = \frac{t_q}{{t_q}^0} \tag{9}$$

so that $d \geq 1$ corresponds to conforming quality. In a truncated life test, the experiment is stopped at a predetermined time t_0. To connect t_0 with the percentile requirement, we set $t_0 = \delta {t_q}^0$, where $\delta \in (0,1]$ is a truncation multiplier (often chosen as 0.5 or 1.0). The probability that a single unit fails before t_0 is

$$p = F(t_0) = erf\left(\frac{t_0}{\lambda\sqrt{\pi}}\right) \tag{10}$$

Substituting λ from equation (8) and $t_0 = \delta {t_q}^0$ yields

$$p = erf\left(\frac{\delta {t_q}^0}{\left(\frac{t_q}{\sqrt{\pi} erf^{-1}(q)}\right)\sqrt{\pi}}\right)$$

Simplifying this expression:

$$p = erf\left(\frac{\delta {t_q}^0}{\left(\frac{t_q}{erf^{-1}(q)}\right)}\right)$$

$$p = erf\left(\frac{\delta t_q^0 erf^{-1}(q)}{t_q}\right)$$

Finally, we introduce the ratio $d = \frac{t_q}{{t_q}^0}$

$$p = erf\left(\frac{\delta erf^{-1}(q)}{\frac{t_q}{{t_q}^0}}\right)$$

$$\mathrm{p} = \mathrm{erf}\left(\frac{\delta \mathrm{erf}^{-1}(\mathrm{q})}{\mathrm{d}}\right) \tag{11}$$

Equation (11) is fundamental: it expresses the failure probability in a truncated life test directly in terms of the quality ratio d (true percentile divided by the standard), the truncation multiplier δ, and the chosen percentile level q. For a given d, the failure probability decreases as d increases (better quality) and increases as δincreases (longer test duration).

2.3 Multiple Dependent State Sampling Scheme

The proposed MDS plan uses both the current sample and the disposition of the most recent m lots. The operational procedure is as follows:

Step 1: Draw a random sample of n units from the lot. Place them on a life test for a duration $t_0 = \delta {t_q}^0$. Count the number of failures X by time t_0.

Step 2: If $X \leq c_1$, accept the lot (conclude $t_0 \geq {t_q}^0$).where c_1 is acceptance number.

Step 3: If $X > c_2$, reject the lot (conclude $t_0 < {t_q}^0$). where c_2 is rejection number such that $c_2 > c_1$.

Step 4: If $c_1 < X \leq c_2$, the decision depends on the preceding lots. If the previous m lots were all accepted (each under the condition $X \leq c_1$for that lot), accept the current lot; otherwise, reject the current lot.

The plan parameters are n, c_1, c_2 and m. When $m = 0$ or $c_1 = c_2$, the MDS plan reduces to a traditional single sampling plan.

2.4 Operating Characteristic (OC) Function

Let p denote the probability that an individual unit fails before the test duration t_0. This probability depends on the quality ratio d through equation (11). The OC function $L(p)$ gives the probability of accepting a lot under the MDS plan.

For the proposed MDS scheme, the acceptance probability is derived as follows. A lot is accepted if either (i) the number of failures in the current sample is at most c_1, or (ii) the number of failures falls into the ambiguous interval $c_1 < X \leq c_2$ and all of the preceding m lots were accepted. Therefore,

$$L(p) = P(X \leq c_1) + P(c_1 < X \leq c_2).[P(X \leq c_1)]^m \tag{12}$$

If $X > c_2$, the lot is rejected directly, contributing no acceptance probability. Because the life test is attribute-based (each unit is classified as fail or pass at time t_0), the probabilities involved are computed from the binomial distribution:

$$P(X \leq c_1) = \sum_{i=0}^{c_1} \binom{n}{i} \mathrm{p}^i (1-\mathrm{p})^{n-i} \qquad 13)$$

$$P(c_1 < X \leq c_2) = \sum_{i=c_1+1}^{c_2} \binom{n}{i} \mathrm{p}^i (1-\mathrm{p})^{n-i} \tag{14}$$

Substituting equations (13-14) into equation (12) yields the OC function as an explicit function of p:

$$L(p) = \sum_{i=0}^{c_1} \binom{n}{i} \mathrm{p}^i (1-\mathrm{p})^{n-i} + \sum_{i=c_1+1}^{c_2} \binom{n}{i} \mathrm{p}^i (1-\mathrm{p})^{n-i}.\left[\sum_{i=0}^{c_1} \binom{n}{i} \mathrm{p}^i (1-\mathrm{p})^{n-i}\right]^m \tag{15}$$

Finally, using the relation $\mathrm{p} = \mathrm{erf}\left(\frac{\delta \mathrm{erf}^{-1}(\mathrm{q})}{\mathrm{d}}\right)$ from (11), the acceptance probability can be expressed directly in terms of the quality ratio d for fixed design parameters $n, c_1, c_2, m, \delta, q$.

2.5 Optimization Model for Plan Parameters

The design of the proposed MDS sampling plan requires selecting the three parameters n, c_1 and c_2 such that the plan simultaneously protects the interests of both the producer and the consumer. This is achieved by imposing probabilistic constraints on the operating characteristic (OC) function at two quality levels: the acceptable quality level (AQL) and the limiting quality level (LQL).

2.5.1 Quality Levels and Associated Risks

Let the quality of a lot be expressed through the quality ratio $d = \frac{t_q}{{t_q}^0}$ from equation (9), where t_q is the true $100q - th$ percentile lifetime of the product and ${t_q}^0$ is the required standard. A lot is considered conforming when $d \geq 1$.

- Acceptable Quality Level (AQL): A high-quality lot is defined by $d = d_0$, where $d_0 \in \{2,4,6,8\}$. That is, the true percentile is 2, 4, 6, or 8 times the minimum required standard. For such lots, the producer expects a high probability of acceptance.
- Limiting Quality Level (LQL): The poorest quality that is still marginally acceptable is defined by $d = 1$, i.e., the product exactly meets the percentile standard. Lots with $d < 1$ are considered unsatisfactory, but the consumer wishes to limit the probability of accepting lots at the borderline $d = 1$.

The two types of risks are:

- Producer's risk (α): The probability of rejecting a lot of AQL quality. Typically, $\alpha = 0.05$ (5%).

- Consumer's risk (β) : The probability of accepting a lot of LQL quality. Common values are $\beta = 0.25, 0.10, 0.05, 0.01$.

2.5.2 Failure Probabilities at AQL and LQL

From equation (11), the probability that a single unit fails before the truncated test duration $t_0 = \delta t_q{}^0$ is given by:

$$\mathrm{p} = \mathrm{erf}\left(\frac{\delta \mathrm{erf}^{-1}(\mathrm{q})}{\mathrm{d}}\right) \quad (11)$$

where δ is the truncation multiplier (e.g., 0.5 or 1.0), q is the percentile of interest (e.g., 0.10 for the $10th$ percentile), and d is the quality ratio.

Substituting the AQL quality ratio $d = d_0$ and the LQL quality ratio $d = 1$ directly into (11) yields the corresponding failure probabilities:

$$\mathrm{p}_{AQL} = \mathrm{erf}\left(\frac{\delta \mathrm{erf}^{-1}(\mathrm{q})}{\mathrm{d_0}}\right) \quad (16)$$

$$\mathrm{p}_{LQL} = \mathrm{erf}\left(\frac{\delta \mathrm{erf}^{-1}(\mathrm{q})}{\mathrm{d{=}1}}\right) \quad (17)$$

These values represent the probability that an individual item fails before the test termination time when the lot is of AQL quality or LQL quality, respectively. Note that because $\mathrm{d_0} \geq 1$, we have $\mathrm{p}_{AQL} \leq \mathrm{p}_{LQL}$; a higher-quality lot yields a lower individual failure probability.

2.5.3 Risk Constraints Using the OC Function

The OC function must satisfy two conditions:

- Producer's condition: Lots of AQL quality should be accepted with probability at least $1 - \alpha$. Hence
$$L(\mathrm{p}_{AQL}) \geq 1 - \alpha. \quad (18)$$
- Consumer's condition: Lots of LQL quality should be accepted with probability at most β. Hence
$$L(\mathrm{p}_{LQL}) \leq \beta. \quad (19)$$

2.5.4 Optimization Problem Statement

The objective is to minimize the sample size n (economic goal) while satisfying the two risk constraints. The decision variables are the integer parameters $n, c_1, and\ c_2$. The optimization problem is formally stated as follows. For a fixed integer $m \geq 0$ $(e.g., m = 1,2,3)$, the optimisation problem is:

$$\text{minimize } n \quad (20)$$

subject to

$$L(\mathrm{p}_{AQL}) \geq 1 - \alpha, \quad (21)$$

$$L(p_{LQL}) \leq \beta, \quad (22)$$

2.6 Solution Algorithm

Because the decision variables are discrete and the OC function involves binomial sums, a closed-form solution is not available. An exhaustive search over feasible combinations is therefore performed. The algorithm proceeds as follows:

Input:

Design constants $\alpha, \beta, \delta, q, m$(typically $m = 1,2,3$) and the set of AQL ratios $d_0 \in \{2,4,6,8\}$.(Here LQL corresponds to d=1.)

Initialize:

Set feasible integer ranges for the design parameters $(n, c1, c2)$ such that $0 \leq c1 < c2 < n$.

(Note: m is fixed and not optimised over.)

Iterative Search:

a. For each combination of $(n, c1, c2)$, compute the acceptance probability using the MDS OC function (Equation (15)):

$$L(p) = \sum_{i=0}^{c_1} \binom{n}{i} \mathrm{p}^i (1-\mathrm{p})^{n-i} + \sum_{i=c_1+1}^{c_2} \binom{n}{i} \mathrm{p}^i (1-\mathrm{p})^{n-i} . \left[\sum_{i=0}^{c_1} \binom{n}{i} \mathrm{p}^i (1-\mathrm{p})^{n-i}\right]^m$$

b. Evaluate the OC function at both quality levels:

At AQL $(d = d0)$: $\mathrm{p}_{AQL} = \mathrm{erf}\left(\frac{\delta \mathrm{erf}^{-1}(\mathrm{q})}{\mathrm{d_0}}\right)$.

At LQL (d=1): $\mathrm{p}_{LQL} = \mathrm{erf}\left(\frac{\delta \mathrm{erf}^{-1}(\mathrm{q})}{\mathrm{d{=}1}}\right)$

c. Check the risk constraints:

$L(\mathrm{p}_{AQL}) \geq 1 - \alpha$, and $L(p_{LQL}) \leq \beta$.

Retain only those parameter sets that satisfy both inequalities.

Optimization (Efficiency Criterion):

For the MDS plan without resampling, the sample size n is the primary measure of efficiency (smaller n implies lower inspection cost). Therefore, select the combination (n, c_1, c_2) that yields the **minimum sample size** n among all feasible solutions. If multiple combinations have the same minimal n, choose the one with the smallest $c_1 + c_2$ (or smallest c_2 as a tie-breaker).

Output:

Report the optimal plan parameters (n, c_1, c_2) for the given m, along with the associated OC values $L(p_{AQL})$ and $L(p_{LQL})$.

Results and Discussion of the Proposed MDS Plan (m = 2)

The optimal design parameters for the proposed multiple dependent state (MDS) sampling plan, obtained by solving the optimization problem described in above Section, are presented in Tables 2 through 5. These tables provide the minimum sample size n along with the corresponding acceptance number c_1 and rejection number c_2 that satisfy both the producer's risk constraint $L(\mathrm{p}_{AQL}) \geq 1 - \alpha)$ and the consumer's risk constraint $L(\mathrm{p}_{LQL}) \leq \beta)$ for a fixed number of preceding lots $m = 2$. The results are generated under various combinations of the consumer's risk $\beta \in \{0.25, 0.10, 0.05, 0.01\}$,the AQL quality ratio $d_0 \in \{2,4,6,8\}$, the truncation multiplier $\delta \in \{0.5, 1.0\}$, and the percentile level $q \in \{0.25, 0.50\}$. The producer's risk is conventionally set at $\alpha = 0.05$ $(i.e.,\ 1 - \alpha = 0.95)$, so the reported $L(\mathrm{p}_{AQL})$ values represent the actual acceptance probabilities achieved at AQL, all of which exceed 0.95 as required.

Justification for choosing $\boldsymbol{m = 2}$: While larger values of m $(e.g.,\ m = 3)$ can be considered, they result in substantially larger sample sizes for the same risk protection. This phenomenon is consistent with the findings of (Balamurali & Jun, 2007), who observed that increasing the number of dependent states increases the required sample size, and consequently they tabulated plans for $m = 1,2,3$ but noted that larger m yields diminishing returns in efficiency. To conserve testing time and cost, and to avoid excessively large samples, we adopt $m = 2$ as a practical choice that captures the benefits of dependency without undue inflation of n. For reference, the reader is directed to (Balamurali & Jun, 2007), where similar trade-offs are discussed for variables MDS plans under normal distributions. A complete list of the notation used throughout the paper is provided in Table 1.

Tables 2–5 present the optimal parameters (n, c_1, c_2) of the proposed MDS plan for $m = 2$ under four distinct combinations of the truncation multiplier δ and the percentile level q. Specifically:

- Table 2: $\delta = 0.5,\ q = 0.25$ (test truncated at half the standard time; $25th$ percentile criterion).
- Table 3: $\delta = 0.5, q = 0.50$ (half test duration; median criterion).
- Table 4: $\delta = 1.0, q = 0.25$ (full test duration; $25th$ percentile).
- Table 5: $\delta = 1.0, q = 0.50$ (full test duration; median).

Across all tables, the producer's risk is fixed at $\alpha = 0.05$ $(i.e., 1 - \alpha = 0.95)$, and the consumer's risk β takes values $0.25, 0.10, 0.05, 0.01$. The AQL quality ratio d ranges over $\{2,4,6,8\}$. The following patterns are consistently observed.

Effect of consumer's risk $\boldsymbol{\beta}$

Stricter consumer protection (smaller β) demands a larger sample size to reduce the probability of accepting poor-quality lots.

- Example from Table 2 $(\delta = 0.5, q = 0.25)$: For $d = 2$, as β decreases from 0.25 to 0.01, the required sample size increases from $n = 68$ to $n = 245$.

Example from Table 5 $(\delta = 1.0, q = 0.50)$: For $d = 4$, when $\beta = 0.25$ we have $n = 5$, whereas at $\beta = 0.01$ the sample size rises to $n = 19$. This dramatic increase illustrates the universal trade-off between consumer protection and sampling effort. The effect of β on sample size is also presented in Figure 1.

Effect of AQL quality ratio $\boldsymbol{d}$

Higher AQL quality (larger d) leads to smaller individual failure probabilities $pAQL$, making it easier to satisfy the producer's risk constraint with fewer samples.

- Example from Table 3 $(\delta = 0.5, q = 0.50)$: For $\beta = 0.10$, increasing d from 22 to 66 reduces n from 52 to 14.

Example from Table 4 $(\delta = 1.0, q = 0.25)$: For $\beta = 0.05$, raising d from 22 to 88 lowers the sample size from 74 to 18. The effect of d on sample size is also presented in Figure 2.

Effect of truncation multiplier $\boldsymbol{\delta}$

A longer test duration $(\delta = 1.0$ $versus$ $\delta = 0.5)$ provides more failure information per unit, thereby reducing the required sample size for the same risk levels.

- Comparison between Table 2 $(\delta = 0.5, q = 0.25)$ and Table 3 $(\delta = 1.0, q = 0.25)$: At $\beta = 0.10, d = 2$, n drops from 120 (Table 1) to 55 (Table 3).

Comparison between Table 3 $(\delta = 0.5, q = 0.50)$ and Table 5 $(\delta = 1.0, q = 0.50)$: For $\beta = 0.05, d = 4$, the sample size decreases from 27 (Table 2) to 13 (Table 5). These reductions confirm that extending the test duration is an effective way to economies on sample size, provided that practical testing constraints allow it. The effect of truncation multiplier δ on sample size is also presented in Figure 3.

Effect of percentile level $\boldsymbol{q}$

Using a higher percentile ($e.g., q = 0.50$ $instead\ of$ $q = 0.25$) makes the quality criterion less stringent because the median is larger than a lower percentile for the same scale parameter. Consequently, smaller samples are sufficient to achieve the desired protection.

- Comparison between Table 2 ($q = 0.25$) and Table 3 ($q = 0.50$) for $\delta = 0.5$: At $\beta = 0.10, d = 2, n = 120$ when $q = 0.25$ (Table 2), whereas $n = 52$ when $q = 0.50$ (Table 3).

Comparison between Table 4 ($q = 0.25$) and Table 5 ($q = 0.50$) for $\delta = 1.0$: For $\beta = 0.01, d = 4$, the sample size is 43 at $q = 0.25$ (Table 4) but only 19 at $q = 0.50$ (Table 5). These results highlight that choosing a higher percentile as the quality benchmark can dramatically reduce inspection costs. The effect of percentile level q on sample size is also presented in Figure 4.

Producer's risk protection

In every scenario, the realized acceptance probability at AQL, $L(\mathrm{p}_{AQL})$, exceeds the nominal $1 - \alpha = 0.95$, confirming that the producer's constraint is satisfied.

- From Table 2: For $d = 8, \beta = 0.25$, $L(\mathrm{p}_{AQL}) = 0.9956$, indicating that lots of exceptional quality are almost always accepted.
- From Table 5: For $d = 6, \beta = 0.10$, $L(\mathrm{p}_{AQL}) = 0.9711$, still well above 0.95 despite the much smaller sample size $(n = 7)$. This demonstrates that the MDS plan effectively safeguards producer interests even under highly economical sampling.

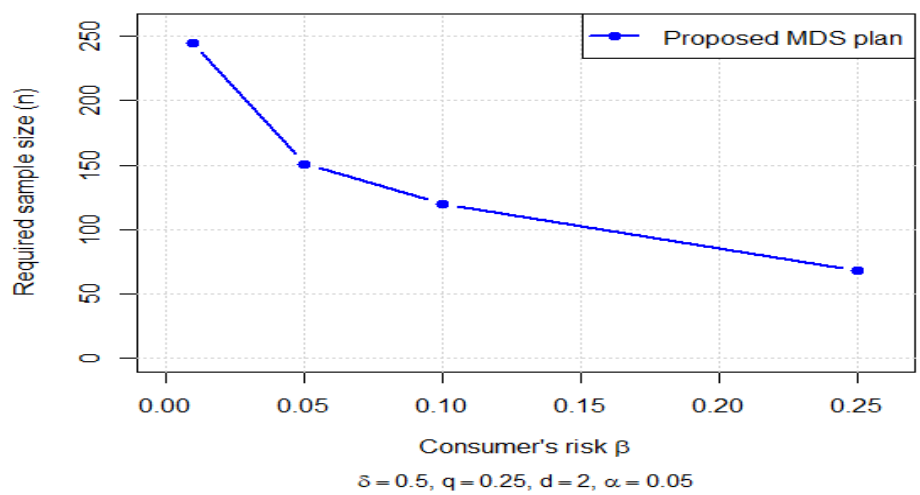

Figure 1: Effect of consumer's risk β

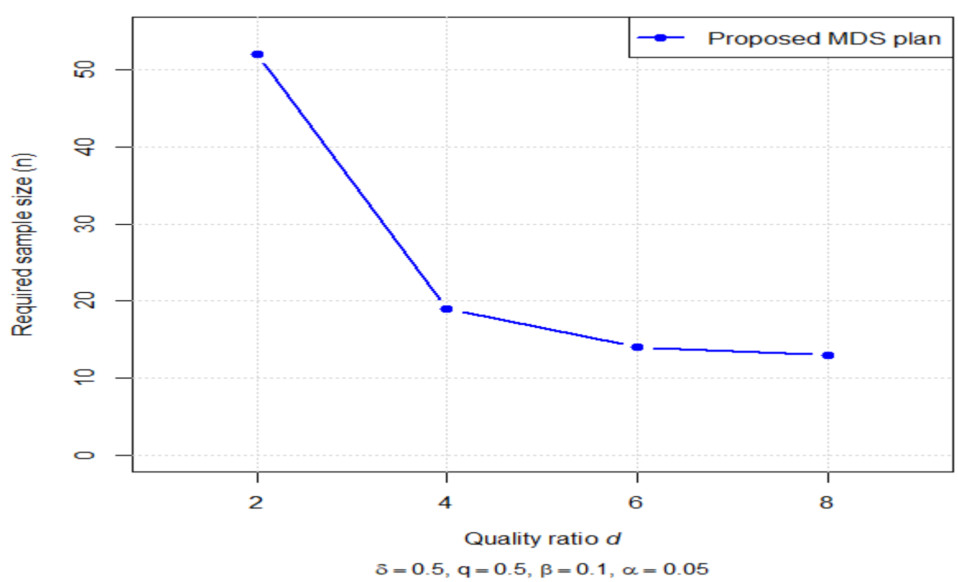

Figure 2: Effect of quality ratio d

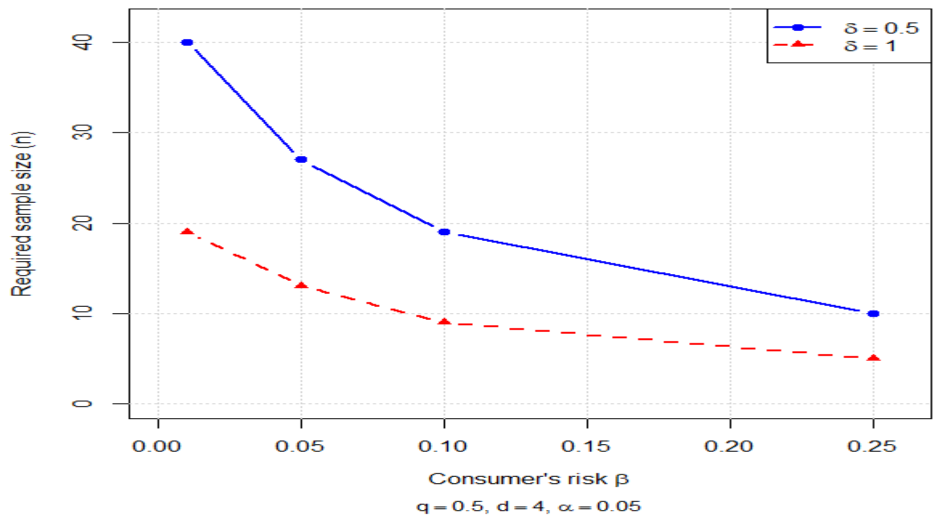

Figure 3: Effect of truncation multiplier δ

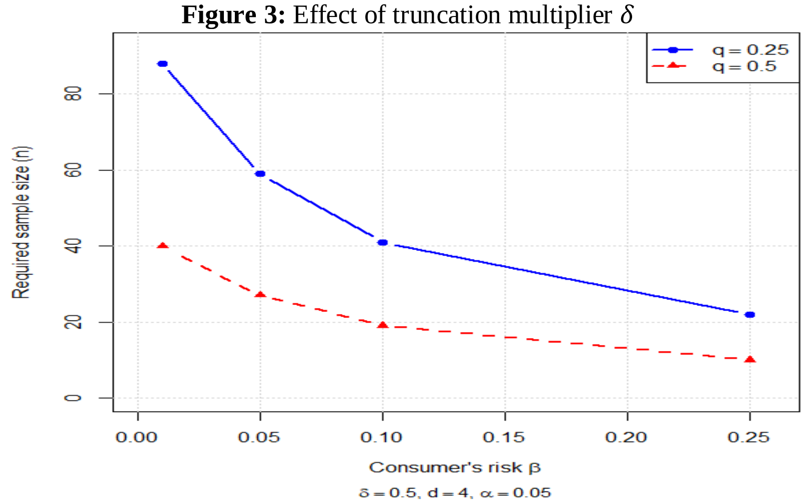

Figure 4: Effect of percentile level q

Table 1: List of Notations

Symbol	Meaning
n	Sample size (number of units drawn from a lot)
c_1	Acceptance number
c_2	Rejection number ($c_2 > c_1$)
m	Number of preceding lots considered in the MDS rule
d	Quality ratio = $\frac{t_q}{t_q{}^0}$
d_0	Quality ratio at the AQL $d_0 \in \{2,4,6,8\}$
q	Percentile level (e.g., 0.25 for the 25th percentile)
δ	Truncation multiplier

Table 2: Design Parameters of the Proposed MDS plan when($m = 2, \delta = 0.5, q = 0.25$)

β	d	c_1	c_2	n	$L(p_{AQL})$
0.25	2	6	9	68	0.9560
	4	1	13	22	0.9563
	6	1	30	22	0.9881
	8	1	13	22	0.9956
0.10	2	10	15	120	0.9600
	4	2	6	41	0.9629
	6	1	16	30	0.9672
	8	1	38	30	0.9871
0.05	2	12	34	151	0.9505
	4	3	6	59	0.9718
	6	2	38	48	0.9872
	8	1	32	36	0.9764
	2	19	25	245	0.9525

0.01	4	4	46	88	0.9591
	6	2	32	64	0.9560
	8	2	20	64	0.9871

Table 3: Design Parameters of the Proposed MDS plan when $(m = 2, \delta = 0.5, q = 0.5)$

β	d	c_1	c_2	n	$L(p_{AQL})$
0.25	2	5	37	28	0.9513
	4	1	34	10	0.9625
	6	1	28	9	0.9902
	8	0	2	5	0.9542
0.10	2	9	50	52	0.9581
	4	2	40	19	0.9676
	6	1	14	14	0.9693
	8	1	43	13	0.9881
0.05	2	12	17	70	0.9617
	4	3	6	27	0.9781
	6	2	48	22	0.9896
	8	1	39	17	0.9771
0.01	2	17	31	105	0.952
	4	4	13	40	0.9691
	6	2	31	29	0.9641
	8	2	35	28	0.9898

Table 4: Design Parameters of the Proposed MDS plan when $(m = 2, \delta = 1, q = 0.25)$

β	d	c_1	c_2	n	$L(p_{AQL})$
0.25	2	6	11	34	0.9676
	4	1	11	11	0.9575
	6	1	2	11	0.9804
	8	1	23	11	0.9958
0.10	2	9	18	55	0.9575
	4	2	6	20	0.9681
	6	1	21	15	0.9679
	8	1	19	15	0.9875
0.05	2	12	17	74	0.9614
	4	3	31	29	0.9775
	6	2	3	23	0.9772
	8	1	25	18	0.9769
0.01	2	17	33	111	0.9519
	4	4	46	43	0.9659
	6	2	24	31	0.9623
	8	2	48	31	0.9893

Table 5: Design Parameters of the Proposed MDS plan when $(m = 2, \delta = 1, q = 0.5)$

β	d	c_1	c_2	n	$L(p_{AQL})$
0.25	2	5	18	14	0.9645
	4	1	47	5	0.9655
	6	1	48	5	0.9913
	8	1	18	5	0.9969
0.10	2	8	46	24	0.9541
	4	2	26	9	0.9778
	6	1	49	7	0.9711
	8	1	34	7	0.9891
0.05	2	10	13	30	0.9519
	4	3	24	13	0.9862
	6	1	18	8	0.9549
	8	1	35	8	0.9823
0.01	2	15	42	47	0.9565
	4	4	47	19	0.9811
	6	2	8	14	0.9711
	8	4	44	14	0.9922

3. Efficiency Comparison: Proposed MDS Plan vs. Single Sampling Plan

A comparative analysis is conducted to evaluate the practical efficiency of the proposed multiple dependent state (MDS) sampling plan against the established single sampling plan for life tests based on percentiles of the half-normal distribution, as developed by (Rao et al., 2013). The comparison metric is the required sample size n per lot, which directly reflects the inspection effort. The results, summarised in Table 6 for a fixed producer's risk ($\alpha = 0.05$), truncation multiplier $\delta = 0.5$, and the $50th$ percentile ($q = 0.5$), demonstrate a decisive advantage for the MDS framework. Across all considered consumer's risk levels ($\beta = 0.25, 0.10, 0.05, 0.01$) and increasing values of the quality ratio (d), the proposed plan consistently yields a smaller sample size than its single-sampling counterpart.

The results in Table 6 reveal a clear and consistent advantage of the proposed MDS plan over the single sampling plan. For every combination of β and d, the MDS plan requires a substantially smaller sample size while satisfying the same producer and consumer risk constraints.

- At $\beta = 0.25$: For $d = 2$, the MDS plan needs $n = 28$ compared to $n = 48$ for the single plan, reduction of 41.67%. At $d = 8$, the sample size drops from 10 (single) to 5 (MDS), a 50% reduction.
- At $\beta = 0.10$: For $d = 2$, the MDS plan requires $n = 52$ while the single plan requires $n = 82$ (a 36.6% saving). Even at $d = 8$, the MDS plan achieves $n = 13$ versus $n = 19$ for the single plan.
- At $\beta = 0.05$: For $d = 2$, the sample size is reduced from 101 (single) to 70 (MDS), saving of 31 units. For $d = 6$, the MDS plan uses $n = 22$ while the single plan uses $n = 27$.
- At the most stringent consumer protection ($\beta = 0.01$): For $d = 2$, the MDS plan requires $n = 105$ versus $n = 151$ for the single plan, a reduction of 46 units. For $d = 8$, the MDS plan achieves $n = 28$ while the single plan requires $n = 35$.

The comparison between the proposed MDS plan and the single sampling plan is also presented in Figure 5, which visually illustrates the sample size requirements for the two plans at a representative consumer's risk level of $\beta = 0.05$. In this figure, the sample size n is plotted against the quality ratio d for both the proposed MDS plan and the single sampling plan. The curve for the MDS plan lies uniformly below that of the single sampling plan across the entire range of d, confirming its superior efficiency. The gap between the two curves is widest at lower d values (e.g., at $d = 2$, the difference is 31 units) and narrows as d increases, but the MDS plan consistently maintains a lower sample size.

Table 6: Comparison of Proposed MDS with Single Sampling Plan using HND

$\delta = 0.5, q = 0.5$			
		Proposed sampling plan	Single sampling plan proposed by (Rao et al., 2013)
$\boldsymbol{\beta}$	d	n	n
0.25	2	28	48
	4	10	19
	6	9	14
	8	5	10
0.10	2	52	82
	4	19	29
	6	14	24
	8	13	19
0.05	2	70	101
	4	27	37
	6	22	27
	8	17	22
0.01	2	105	151
	4	40	56
	6	29	40
	8	28	35

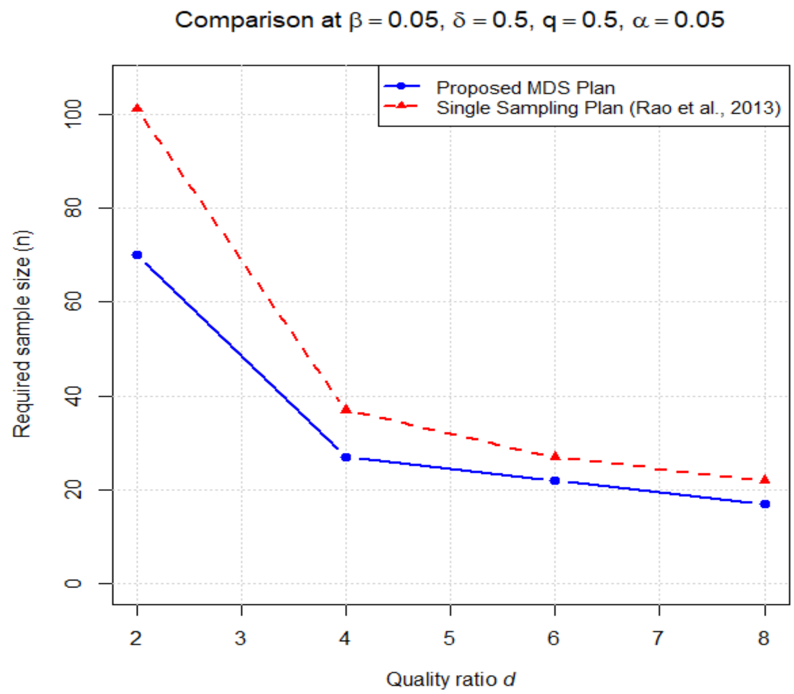

Figure 5: Graph of the comparison proposed plan

4. Real-World Implementation of the Proposed MDS Plan

The operational value of the proposed multiple dependent state (MDS) sampling scheme under the half-normal distribution (HND) is demonstrated through two real-world reliability datasets drawn from distinct engineering domains. One dataset records failure time of software systems, which characterize complex digital degradation processes. The other dataset captures lifetimes of electronic components, reflecting conventional mechanical or material wear-out behavior. By selecting these two heterogeneous examples, we show that the MDS plan retains its flexibility and decision-making power across different failure mechanisms while rigorously maintaining the pre-specified producer and consumer risks.

4.1 Industrial Application: Software Failure Time Application

The practical implementation of the MDS plan is first demonstrated using a well-known software reliability dataset comprising failure times (in hours): 519, 968, 1430, 1893, 2490, 3058, 3625, 4422, and 5218. Before applying the sampling plan, the suitability of the HND for this data was rigorously validated. The HND assumption was tested using the Kolmogorov–Smirnov (KS) procedure. The computed KS statistic is $D = 0.152$ with a corresponding $p - value = 0.095$, indicating that the HND cannot be rejected at the 5% significance level. Additionally, a $Q - Q$ plot (Rao et al., 2013) gave a correlation coefficient $R = 0.9287$, further confirming the adequacy of the HND.

Implementation scenario:

A quality engineer requires that the $50th$ percentile lifetime ($t_{0.5}$) of a software lot be at least 300 hours. Producer's risk $\alpha = 0.05$, consumer's risk $\beta = 0.25$. The life test is truncated at 150 hours, giving the termination time ratio $\delta = 150/300 = 0.5$. For $q = 0.5,\ \delta = 0.5, and\ m = 2$, the MDS plan parameters are selected from Table 3 (with $m = 2, \beta = 0.25,\ d = 2$):

$$n = 28, c_1 = 5, c_2 = 37$$

Decision rule:

- Draw a random sample of 28 items and test until the truncation time (i.e., until $\delta \times t_{0.5} = 150$ hours). Let D be the number of failures.
- Accept the lot if $D \leq c_1 = 5$.
- Reject the lot if $D > c_2 = 37$.
- If $5 < D \leq 37$, accept the lot only if the two immediately preceding lots were both accepted; otherwise reject.

Application:

No failure occurs before the truncation time (first failure at 519 hours). Thus $D = 0$. Since $0 \leq 5$, the lot is **accepted** on the first sample. The plan satisfies the specified producer's and consumer's risk levels.

4.2 Device Lifetime Data Application

To demonstrate the broad applicability and practical implementation of the proposed methodology across different industrial domains, we apply the MDS plan to the well-established Aarset dataset comprising 50 device lifetimes. The dataset consists of failure times (in hours): 0.1, 0.2, 1, 1, 1, 1, 1, 2, 3, 6, 7, 11, 12, 18, 18, 18, 18, 18, 21, 32, 36, 40, 45, 46, 47, 50, 55, 60, 63, 63, 67, 67, 67, 67, 72, 75, 79, 82, 82, 83, 84, 84, 84, 85, 85, 85, 85, 85, 86, and 86. The fit of the half-normal distribution was rigorously tested. The Kolmogorov–Smirnov test yielded $D = 0.089$ with a $p - value = 0.078$, indicating a good statistical fit and supporting the validity of the HND assumption ($p > 0.05$). Furthermore, the $Q - Q$ plot analysis (Rao et al., 2013) showed a high correlation coefficient ($R = 0.9175$), reinforcing the suitability of the HND.

Implementation of the MDS Plan

Consider a practical scenario in component manufacturing where quality specifications require that the $25th$ percentile lifetime ($t_{0.25}$) of submitted devices exceeds 10 hours. The producer and consumer agree on risk thresholds of $\alpha = 0.05$ and $\beta = 0.10$, respectively. The life test is truncated at 5 hours, resulting in a termination time ratio $\delta = 0.5$. For these parameters ($q = 0.25, \delta = 0.5, \alpha = 0.05,\ \beta = 0.10,\ d = 4$), the optimal parameters for the proposed MDS plan are obtained from Table 2 ($m = 2, \delta = 0.5, q = 0.25$) as:

$$n = 41, c_1 = 2, c_2 = 6.$$

Decision rule:

- Sample 41 devices, test to the truncation time (5 hours), count failures D.
- Accept if $D \leq 2$.
- Reject if $D > 6$.
- If $2 < D \leq 6$, accept only if the previous two lots were both accepted.

Application:

Failures before the truncation time occur at times: $0.1, 0.2, 1, 1, 1, 1, 1, 2, 3 \ \rightarrow \ D = 9$.

Since $9 > 6$, the lot is rejected immediately on the first sample.

Summary

The two examples demonstrate the balanced performance of the proposed MDS plan: the conforming software lot is accepted, and the non-conforming device lot is rejected, all while strictly maintaining the pre-specified producer's and consumer's risk thresholds ($\alpha = 0.05,\ \beta = 0.25$ for software; $\alpha = 0.05,\ \beta = 0.10$ for devices). The plan's dependency on the quality history of preceding lots (via the parameters $m,\ c_1, c_2$) provides flexible risk control without increasing the required sample size.

5. Conclusion

This paper introduced a multiple dependent state (MDS) sampling plan for truncated life tests under the half-normal distribution (HND) using percentile-based quality criteria. Unlike mean-based plans, the proposed scheme is more sensitive to distributional tails and early failure risks. It leverages the quality history of preceding lots (via parameter m) to reduce sample size without compromising producer and consumer risks.

The main contributions are threefold. First, we derived a closed-form failure probability expression for a truncated HND life test as a function of quality ratio d, truncation multiplier δ, and percentile q. Second, we solved an optimization problem to generate comprehensive tables of optimal parameters $(n, c_1, c_2)\ for\ m = 2$ covering practical risk levels ($\beta = 0.25, 0.10, 0.05, 0.01$), quality ratios ($d = 2,4,6,8$), test durations ($\delta = 0.5, 1.0$), and percentiles ($q = 0.25, 0.50$). Third, we validated the plan using two real-world datasets (software failure times and device lifetimes), achieving correct decisions with substantially lower sample sizes than conventional single sampling plans.

Key insights: stricter consumer protection (smaller β) and lower d increase sample size; longer test duration ($\delta = 1.0$) reduces sample size but must balance practical constraints; using a higher percentile ($q = 0.50$) reduces inspection effort. Comparative analysis confirmed sample size reductions up to 50% over single sampling plans.

Limitations and future work include extending the framework to more flexible distributions (e.g., generalized HND), exploring other sampling schemes such as multiple dependent state repetitive sampling plans to further enhance efficiency, and integrating accelerated life testing for high-reliability products Moreover, the proposed MDS percentile-based framework could be adapted to other lifetime distributions (e.g., generalized gamma, exponentiated Weibull) and to alternative censoring schemes such as progressive Type-I censoring.

In summary, the proposed MDS plan for HND percentiles offers a practical, efficient, and statistically rigorous tool for reliability engineering. The provided tables and case studies serve as a ready-to-use reference for industrial implementation.

References

Al-Omari, A. I., Al-Nasser, A. D., & Gogah, F. S. (2016). Double acceptance sampling plan for time-truncated life tests based on half normal distribution. *Economic Quality Control, 31*(2), 93-99.

Al-Omari, A. I., & Alomani, G. A. (2024). Acceptance sampling plans based on percentiles for extended generalized exponential distribution with real data application. *Journal of Radiation Research and Applied Sciences, 17*(4), 101081.

Alqasem, O. A., & Elshahhat, A. (2025). Reliability Analysis of the Newly Power Half-Normal Model via Improving Adaptive Progressive Type II Censoring and Its Applications. *Quality and Reliability Engineering International, 41*(7), 2928-2951.

Azam, M., Aslam, M., Balamurali, S., & Javaid, A. (2015). Two stage group acceptance sampling plan for half normal percentiles. *Journal of King Saud University-Science, 27*(3), 239-243.

Balamurali, S., & Jun, C.-H. (2007). Multiple dependent state sampling plans for lot acceptance based on measurement data. *European Journal of Operational Research, 180*(3), 1221-1230.

Castro, L. M., Gómez, H. W., & Valenzuela, M. (2012). Epsilon half-normal model: Properties and inference. *Computational Statistics & Data Analysis, 56*(12), 4338-4347.

Chou, C.-Y., & Liu, H.-R. (1998). Properties of the half-normal distribution and its application to quality control. *Journal of Industrial Technology, 14*(3), 4-7.

Cooray, K., & Ananda, M. M. (2008). A generalization of the half-normal distribution with applications to lifetime data. *Communications in Statistics—Theory and Methods, 37*(9), 1323-1337.

Darmawan, A., & Wu, C.-W. (2025). Designing an enhanced acceptance sampling strategy with the process loss index. *European Journal of Industrial Engineering, 20*(2), 157-182.

Fulment, A. K., Rao, G. S., & Peter, J. K. (2024). Sampling plan for lab extracted beverages data with multiple dependent states using the odd log-logistic distribution. *J Stat Appl Probab, 13*, 157-168.

Gadde, S. R., Fulment, A. K., & Peter, J. K. (2021). Design of Multiple Dependent State Sampling Plan Application for COVID-19 Data Using Exponentiated Weibull Distribution. *Complexity, 2021*(1), 2795078.

Geetha, C., Jayabharathi, S., & Uddin, M. A. (2024). TWO-STAGE GROUP ACCEPTANCE SAMPLING PLAN FOR HALF-NORMAL DISTRIBUTION. *Reliability: Theory & Applications, 19*(4 (80)), 835-841.

Jayalakshmi, S., & Aleesha, A. (2024). STUDY ON ACCEPTANCE SAMPLING PLAN BASED ON PERCENTILES FOR EXPONENTIATED GENERALIZED INVERSE RAYLEIGH DISTRIBUTION. *Reliability: Theory & Applications, 19*(2 (78)), 202-208.

Jiang, R., Xue, W., & Cao, Y. (2024). *Five 2-parameter variants of folded-normal distribution and their application in reliability modeling.* Paper presented at the 2024 6th International Conference on System Reliability and Safety Engineering (SRSE).

Lio, Y., Tsai, T.-R., & Wu, S.-J. (2009). Acceptance sampling plans from truncated life tests based on the Birnbaum–Saunders distribution for percentiles. *Communications in Statistics-Simulation and Computation, 39*(1), 119-136.

Lu, X., Gui, W., & Yan, J. (2013). Acceptance sampling plans for half-normal distribution under truncated life tests. *American Journal of Mathematical and Management Sciences, 32*(2), 133-144.

Naghizadeh, M., & Mahdizadeh, Z. (2026). Improved time- censored group sampling schemes based on generalized half-normal distribution. *Caspian Journal of Mathematical Sciences*.

Naveed, M., Iltaf, M., Azam, M., Khan, N., Saeed, M., & Akbar, M. Z. (2025). Repetitive Sampling Plans for Life Tests Based on Percentiles of the Half-Normal Distribution with Applications to Software Reliability and Device Lifetime Data. *SCOPUA Journal of Applied Statistical Research, 1*(3).

Qomi, M. N., Piadeh, M., Pérez-González, C. J., & Fernández, A. J. (2025). Optimal acceptance sampling plans based on generalized half-normal distribution under time-censoring with conventional and expected limited risks. *Communications in Statistics-Simulation and Computation*, 1-20.

Rao, B. S., Kumar, C. S., & Rosaiah, K. (2013). Acceptance sampling plans from life tests based on percentiles of half normal distribution. *Journal of Quality and Reliability Engineering, 2013*(1), 302469.

Rao, B. S., Prasad, J. S. R., Rao, B. V. P., & Sudhakar, M. (2026). Application of Half Normal Distribution in Hybrid Group Adoption Sampling Plan for Life Tests. *Thailand Statistician, 24*(1), 26-33.

Rao, B. S., Priya, M. C., & Kantam, R. (2014). Acceptance sampling plans for percentiles assuming the linear failure rate distribution. *Stochastics and Quality Control, 29*(1), 1.

Rao, G. S. (2013). Acceptance sampling plans from truncated life tests based on the Marshall–Olkin extended exponential distribution for percentiles.

Rao, G. S., Jilani, S., & Peter, J. K. (2025). Designing of multiple dependent state sampling plan for Exponentiated Frechet Distribution. *Research in Statistics, 3*(1), 2531810.

Rao, G. S., & Kantam, R. (2010). Acceptance sampling plans from truncated life tests based on the log-logistic distributions for percentiles.

Schilling, E. G., & Neubauer, D. V. (2009). *Acceptance sampling in quality control*: Chapman and Hall/CRC.

Tripathi, H. (2025). Acceptance sampling inspection plan under fuzzy theory for generalized half-normal distribution. *Life Cycle Reliability and Safety Engineering*, 1-11.

Uke, T., & Taye, A. (2025). Alpha Power Transformed Half Normal Distribution with Its Properties and Applications. *East African Journal of Biophysical and Computational Sciences, 6*(2), 25-37.

Wang, T.-C., Shu, M.-H., Hsu, B.-M., & Hsu, C.-W. (2022). Adjustable variables multiple-dependent-state sampling plans based on a process capability index. *Journal of the Operational Research Society, 73*(12), 2626-2639.

Wortham, A., & Baker, R. (1976). Multiple deferred state sampling inspection. *The International Journal of Production Research, 14*(6), 719-731.

Wu, C.-W., Shu, M.-H., & Wang, T.-C. (2024). Designing a variables two-plan sampling system with adjustable acceptance criteria for lot disposition. *Quality Engineering, 36*(3), 521-533.

Wu, C.-W., Shu, M.-H., & Wang, T.-C. (2025). Optimal design of variables switch-based sampling scheme for verifying Weibull distributed product lifetimes. *Journal of Applied Statistics, 52*(1), 119-134.

Zheng, H., Yang, J., & Zhao, Y. (2025). Accelerated acceptance sampling plan for degraded products based on inverse Gaussian process considering the acceleration factor uncertainty. *Computers & Industrial Engineering, 203*, 111052.

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Author(s) Bio / Authors' Note

Muhammad Naveed *is an Assistant Professor in the Higher Education Department, Punjab, Pakistan. His research interests include statistical quality control and related areas. Email:* mnaveedakbar09@gmail.com

Muhammad Iltaf *earned his MPhil in Statistics from the National College of Business Administration and Economics, Lahore. His area of interest is statistical quality control. Email:* ranaaltaf123@gmail.com

Muhammad Azam *earned his Master's degree in Statistics from the Islamia University of Bahawalpur (Pakistan) in 1996, graduating with distinction as a Gold Medalist. He completed his MPhil in Statistics at Quaid-i-Azam University, Islamabad, in 2006, and his PhD at the University of Innsbruck, Austria, in 2010. With over 28 years of teaching and research experience, Dr. Azam began his academic career as a Lecturer in Statistics with the Punjab Education Department, where he served for 13 years. In 2010, he joined Forman Christian College (A Chartered University), Lahore, as an Assistant Professor, and in 2015, he moved to the University of Veterinary and Animal Sciences (UVAS), Lahore, as an Associate Professor. He was promoted to Professor of Statistics in January 2018. He has twice served as Dean, Faculty of Life Sciences Business Management, UVAS Lahore—from March 2018 to March 2021, and again from June 2022 to June 2025—showcasing his leadership in academic administration. Dr. Azam has published over 110 research papers, primarily in high impact international journals. His research interests include survey sampling, statistical quality control, decision trees, and ensemble classifiers. He has successfully supervised four PhD and twenty-six MPhil scholars, contributing significantly to the advancement of statistical research and education. Email:* mazam@uvas.edu.pk

Nasrullah Khan *is an Associate Professor in the Department of Statistical Sciences at the University of the Punjab, Lahore, Pakistan. His research interests span statistical modeling, machine learning, climate data analysis, and quality control. He has contributed in methodological and applied statistics, focusing on the integration of classical and modern data-driven approaches for real-world problem solving. Email:* nasrullah.stat@pu.edu.pk