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Two-stage group acceptance sampling for the half exponential power distribution: Design, optimization, and validation

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ABSTRACT

Acceptance sampling plans based on truncated life tests offer a cost-effective mechanism for lot-disposition decisions by terminating the experiment at a pre-specified time and recording only the failure count, while Group Acceptance Sampling Plans (GASPs) further enhance efficiency by testing multiple items simultaneously across groups. This paper proposes a Two-Stage GASP for the Half Exponential Power Distribution (HEPD) under truncated life tests, constructed under the two-point operating characteristic criterion that simultaneously controls the producer's and consumer's risks. The failure probability under the truncated test is derived as a closed-form expression in the mean ratio and termination multiplier, and is shown to be independent of the scale parameter, a property that renders the resulting design tables universally applicable. Optimal design parameters are obtained by minimizing the Average Sample Number (ASN) at the consumer's quality level subject to both risk constraints, and comprehensive design tables are provided across the practical ranges of termination multiplier, mean ratio, and group size. The proposed plan achieves consistent ASN reductions of 18.4% to 41.1% relative to the single-stage GASP, with the largest savings at the most stringent consumer's risk and narrowest quality margin, while the producer's acceptance probability consistently exceeds 0.95, confirming that these gains arise from the two-stage structure rather than any relaxation of the risk constraints. The plan's practical utility is further demonstrated on two real-life data sets, the stress rupture life of Kevlar 49/epoxy strands and the plasma ferritin concentration of Australian athletes, in which it delivers a sound quality decision with materially smaller inspection effort than the single-stage alternative.

Keywords: Acceptance sampling; Average sample number; Group acceptance sampling plan; Half exponential power distribution; Operating characteristic function; Truncated life test; Two-stage sampling

1. Introduction

Quality assurance is an indispensable function in modern industrial production. Across sectors ranging from electronics and pharmaceuticals to aerospace and consumer goods, manufacturers bear the responsibility of ensuring that outgoing products conform to agreed standards before delivery. When complete inspection of every item is economically impractical, physically destructive, or operationally infeasible, acceptance sampling provides a statistically principled and cost-effective alternative. Under an acceptance sampling scheme, a representative sample is drawn from a submitted lot, examined against a quality criterion, and the entire lot is accepted or rejected on the basis of the sample evidence alone (Montgomery, 2020).

The statistical performance of any sampling plan is characterized by two fundamental risk parameters: the producer's risk, defined as the probability that a conforming lot is incorrectly rejected, and the consumer's risk, defined as the probability that a non-conforming lot is incorrectly accepted. These two risks, together with the Operating Characteristic (OC) curve that traces their joint behavior across quality levels and the Average Sample Number (ASN) that quantifies expected inspection effort, form the complete statistical framework within which sampling plans are designed and evaluated. The overarching objective is to minimize the ASN and thereby reduce inspection cost and test duration while simultaneously satisfying prescribed upper bounds on both risks (Schilling & Neubauer, 2009).Since the pioneering sampling tables of (Dodge & Romig, 1929) and the early life test frameworks of (Epstein, 1954) and (Gupta, 1962),), the field of acceptance sampling has undergone sustained and far-reaching development. Single-stage attribute plans have been superseded or supplemented by progressively more sophisticated architectures: double sampling plans, sequential plans, skip-lot plans, variables plan based on process capability indices, and most recently plans adapted for fuzzy data, neutrosophic and ,imprecise environments.(Turanoğlu, Kaya, & Kahraman, 2012), (Afshari, Sadeghpour Gildeh, & Sarmad, 2017), (Aslam & Arif, 2018), (Naveed, Azam, Saeed, Khan, & Aslam, 2021),(Rao, Aslam, Josephat, Al-Husseini, & Albassam, 2024), (Shahzadi, Dar, Khan, Albassam, & Aslam, 2025), (Naveed, Iltaf, et al., 2025), (Naveed, Iltaf, Azam, & Khan, 2026). Within this expanding landscape, sampling plans based on truncated life tests have emerged as a particularly important strand of research, driven by the practical necessity of bounding the calendar duration of costly reliability experiments. In a time-truncated life test, the experiment is terminated at a pre-specified time t_0 expressed as a multiple of a quality benchmark typically the specified mean life mu_0, so that $t_0 = a * mu_0$, and the observed failure count is compared against an acceptance number to reach a lot disposition decision. This single modification transforms an open-ended life test into a bounded experiment compatible with production schedules, without sacrificing the statistical rigor of the accept/reject decision.

A further and practically decisive advance within the truncated life test paradigm is the Group Acceptance Sampling Plan (GASP), which exploits the capacity of modern testing equipment to accommodate multiple items simultaneously. Rather than testing items one at a time, a GASP places $n = g * r$ items on test in parallel, distributing r items across each of g testers or test stations. All items are exposed concurrently for the duration t_0, and the total number of failures d across all groups is compared to an acceptance number c: the lot is accepted if $d \leq c$ and rejected otherwise. This simultaneous testing reduces the calendar time required for inspection by a factor of r relative to sequential testing, a saving of direct economic value in industries where tester occupancy is a bottleneck.

(Aslam & Jun, 2009a) introduced the GASP for the Weibull distribution, establishing the foundational methodology. Subsequent contributions extended the GASP framework to the inverse Rayleigh and log-logistic distributions (Aslam & Jun, 2009b), the gamma distribution (Aslam, Jun, & Ahmad, 2009), the Birnbaum-Saunders distribution (Aslam, Jun, & Ahmad, 2011a), the generalized Rayleigh distribution (Aslam, Jun, Lio, & Ahmad, 2011), Dagum distribution under percentiles lifetime (Aslam, Shoaib, & Khan, 2011), the generalized exponential distribution (Aslam, Kundu, Jun, & Ahmad, 2011), and the Burr type XII distribution (Aslam, Jun, Lio, Ahmad, & Rasool, 2011).More recently, (Algarni, 2022) developed a GASP for a compounded three-parameter Weibull model,(Naz et al., 2023) developed a GASP based on flexible new Kumaraswamy exponential distribution,(Hussain et al., 2024) developed plan for the odd exponential-logarithmic Fréchet distribution, (Fayomi & Khan, 2024) constructed a GASP for the generalized transmuted-exponential distribution, (Ameeq et al., 2024) constructed a GASP for exponential logarithmic distribution,(Nadir, Aslam, Anyiam, Alshawarbeh, & Obulezi, 2025) constructed a GASP for for the Kumaraswamy Bell–Rayleigh distribution and (Ameeq et al., 2025) introduced a GASP for the modified power exponential distribution each study affirming the sustained productivity of this research line. (Qomi & Aslam, 2025) introduced a GASP for Weibull distribution with limited risks.

Despite the efficiency gains of the single-stage GASP, its binary decision structure imposes a limitation: a lot whose observed failures marginally exceed the acceptance number c is immediately rejected, even when the excess may reflect random sampling variation rather than genuine quality deterioration. The Two Stage Group Acceptance Sampling Plan (two stage GASP) addresses this deficiency by introducing a conditional zone that grants borderline lots a second opportunity before a final decision is rendered. In the two stage GASP, first-stage failures d_1 that are at most c_{1a} lead to immediate acceptance, while failures that reach or exceed c_{1r} lead to immediate rejection; failures in the intermediate zone $c_{1a} < d_1 < c_{1r}$ trigger a second independent sample of size $n_2 = g_2 * r$. At Stage 2, the lot is accepted if the combined failure count from both stages does not exceed c_{2a}, and rejected otherwise. This architecture yields a lower ASN than the single-stage GASP because the majority of lots those with clearly good or clearly poor quality are disposed of at Stage 1, and only the ambiguous fraction incurs the cost of a second sample. (Aslam, Jun, Rasool, & Ahmad, 2010) introduced the two stage GASP for the Weibull distribution based on individual-group failure counts; (Aslam, Jun, & Ahmad, 2011b) subsequently reformulated for general distribution.

(Aslam, Azam, Lio, & Jun, 2013) extended the plan to Burr type X percentiles, achieving ASN reductions of 15% to 35% over the single-stage counterpart.(Aslam, Srinivasa Rao, & Khan, 2021) developed single and two stage GASP for Weibull distribution under neutrosophic statistics. (Aslam, Rao, Khan, & Ahmad, 2022) developed two stage GASPusing process loss index under neutrosophic statistics. Collectively, these studies establish that the two stage GASP framework, consistently applied across diverse lifetime distributions, produces reliable gains in inspection efficiency and that extending it to new distributional families remains an active and productive research direction.

The lifetime distribution chosen as the basis for a sampling plan has a direct bearing on plan efficiency: a plan calibrated to the correct distributional model requires fewer test items to achieve the same risk protection as a plan based on a mis specified model. The Half Exponential Power Distribution (HEPD), introduced by Gui (2013b), is a non-negative family derived by truncating the symmetric exponential power distribution to the positive real line. Parameterized by a scale parameter $\sigma > 0$ and a shape parameter $\theta > 0$, the HEPD provides a flexible and analytically tractable lifetime model. The non-monotone hazard rate of HEPD, which distinguishes it from the exponential (constant hazard) and Weibull (monotone hazard) families and makes it suitable for products with mixed failure mechanisms such as fatigue, wear, corrosion, and thermal stress. (Gui, 2013b) established maximum likelihood and moment estimators for the HEPD parameters and demonstrated superior goodness-of-fit relative to competing half-distributions on several engineering datasets.

The HEPD has drawn increasing interest in statistical quality control. (Gui, 2013a) introduced the first single-stage acceptance sampling plan for the HEPD under truncated life tests, providing tables of minimum sample sizes for specified consumer's risk and mean ratio levels. (Gui & Xu, 2015) later extended this work to a double acceptance sampling plan (DASP) for the HEPD, demonstrating notable sample size savings compared with the single-stage plan. In the area of control charts, (Naveed et al., 2023) proposed attribute control charts for processes following both half-normal and half-exponential power distributions. Subsequently, (Naveed et al., 2024) incorporated repetitive sampling into these charts and showed that the HEPD-based chart achieves smaller average run length (ARL) values and therefore faster shift detection than its half-normal and exponential counterparts. (Naveed, Azam, Aslam, Khan, & Saeed, 2025) further advanced this line of research by developing control charts under multiple dependent state (MDS) sampling, reinforcing the superiority of the HEPD as a distributional foundation for quality monitoring whenever process lifetimes depart from the half-normal form.

Despite these contributions, a significant gap remains in the literature. No study has combined the group testing structure, which reduces calendar test time by a factor of r, with the two-stage architecture for the HEPD, nor has any study established a closed-form, HEPD-specific failure probability for such a design, nor provided tabulations of optimal design parameters across the full practical range of termination multipliers, mean ratios, and group sizes. The present paper fills this gap by developing a Two-Stage Group Acceptance Sampling Plan for the HEPD under truncated life tests, using the mean lifetime as the quality benchmark and adopting the two-point approach that simultaneously satisfies both risks. The contribution of this paper is threefold. First, this is the first work to combine the group-testing architecture with the two-stage conditional-zone architecture specifically for the HEPD; prior HEPD acceptance-sampling work (Gui, 2013a; Gui & Xu, 2015) is single-stage and tests individual items rather than groups. Second, the closed-form, scale-invariant failure probability derived in equation (4) is HEPD-specific, since it depends on Gui's particular gamma-function parameterization of the HEPD mean, and is not a routine substitution into a generic two-stage GASP formula. Third, the comprehensive joint tabulation across $\beta, a, r_1, and\ r$ constitutes the first such resource for this distribution under this architecture.

The specific contributions of this paper are as follows: (i) the failure probability under HEPD truncated life tests is derived analytically in closed form as a function of the mean ratio r_1 and termination multiplier a, for the shape parameter $\theta = 1$ adopted throughout this study;; (ii) the lot acceptance probability and OC function for the two-stage GASP are established; (iii) the ASN minimization problem subject to both risk constraints is formulated and solved by simulation; (iv) comprehensive design parameter tables are provided for $\beta \in \{0.25, 0.10, 0.05, 0.01\}$ termination multipliers $a \in \{0.3, 0.5\}$, mean ratios $r_1 \in \{2, 4, 6, 8\}$, and group sizes $r \in \{5, 10\}$; (v) the proposed plan is compared with the existing single-stage GASP and shown to achieve ASN reductions of 18.4% to 41.1%; and (vi) the plan is validated on two real industrial datasets.

The remainder of this paper is organized as follows. Section 2 presents the design of the proposed plan. Section 3 describes the algorithm for generating the design parameter tables. Section 4 reports the design parameter tables and discusses the results. Section 5 provides a comparison with the single-stage GASP. Section 6 illustrates the plan with real data applications. Section 7 concludes the paper.

2. Design of the Proposed Sampling Plans

This section develops the Two-Stage Group Acceptance Sampling Plan (Two-Stage GASP) under the assumption that product lifetimes follow the Half Exponential Power Distribution (HEPD). The plan is constructed under the two-point operating characteristic (OC) curve criterion to simultaneously satisfy the producer's and consumer's risk requirements, with design parameters obtained by minimizing the Average Sample Number (ASN). The HEPD and its key properties are first introduced; the failure probability under a truncated test is then derived; and the design of the Two-Stage GASP, including its OC function and ASN, is subsequently presented.

2.1 The Half Exponential Power Distribution

Let the lifetime X of a product follow the HEPD with scale parameter $\sigma > 0$ and shape parameter $\theta > 0$, denoted $X \sim HEPD(\sigma, \theta)$. The probability density function (PDF) is:

$$f(x) = \frac{\theta^{1-\frac{1}{\theta}}}{\sigma\,\Gamma\left(\frac{1}{\theta}\right)} e^{-\frac{x^{\theta}}{\theta\sigma^{\theta}}}; x \geq 0, \sigma > 0, \theta > 0. \tag{1}$$

where $\Gamma(\cdot)$ is the complete gamma function. The cumulative distribution function (CDF) is

$$F(x) = \frac{\gamma\left(\frac{1}{\theta}, \frac{x^{\theta}}{\theta\sigma^{\theta}}\right)}{\Gamma\left({}^{1}/_{\theta}\right)} \tag{2}$$

where $\gamma(s, x) = \int_0^x t^{s-1} e^{-t} dt$ denotes the lower incomplete gamma function. The mean of the HEPD is

$$\mu = E(x) = \frac{\sigma\left(\theta^{\frac{1}{\theta}}\right)\left\{\Gamma\left({}^{2}/_{\theta}\right)\right\}}{\Gamma\left({}^{1}/_{\theta}\right)} \tag{3}$$

The HEPD provides an analytically tractable lifetime model with a non-monotone hazard rate, making it suitable for products that exhibit mixed failure patterns including fatigue, wear, corrosion, and thermal stress. This distributional property distinguishes it from the exponential (constant hazard) and Weibull (monotone hazard) families and motivates its use as the lifetime model in the present study.

2.2 Failure Probability under Truncated Life Tests

A truncated life test terminates the experiment at a pre-specified time t_0, recording only the number of items that have failed before t_0 rather than waiting for all items to fail. This approach substantially reduces test duration while retaining the statistical information needed for a lot-disposition decision. Three quantities define the truncated test: the specified mean life μ_0, which is the minimum acceptable mean lifetime required by the design standard; the termination multiplier $a > 0$, which sets the test duration as $t_0 = a\mu_0$; and the mean ratio $r_1 = \mu/\mu_0$, which measures lot quality as the ratio of the true mean life to the specified mean life. When $r_1 > 1$ the lot exceeds the specification; when $r_1 = 1$ the lot just meets it. This section also states the data requirement, not just the quality interpretation: Consistent with this truncated-test design, the proposed Two-Stage GASP requires only the number of items that have failed by t_0 at each stage d_1 in Stage 1 and, where triggered, d_2 in Stage 2 and does not require the exact failure time of any individual item.

2.3 Derivation of the Failure Probability

We assume that the lifetime X follows the $HEPD(\sigma, \theta)$ with PDF (1) and CDF (2). The scale parameter σ is not directly given, but from the mean formula (3) we can express it in terms of the true mean μ:

$\sigma = \frac{\mu}{K(\theta)}$

$$\text{Where } K(\theta) = \frac{\left(\theta^{\frac{1}{\theta}}\right)\left\{\Gamma\left({}^{2}/_{\theta}\right)\right\}}{\Gamma\left({}^{1}/_{\theta}\right)}$$

The probability that a single item fails before time $t_0$is simply the CDF evaluated at t_0:

$$p = F(t_0; \sigma, \theta) = \frac{\gamma\left(\frac{1}{\theta}, \frac{t_0{}^{\theta}}{\theta\sigma^{\theta}}\right)}{\Gamma\left({}^{1}/_{\theta}\right)}$$

Now substitute $t_0 = a\mu_0$ and $\sigma = \frac{\mu}{K(\theta)}$, and also use $\mu = r_1\mu_0$:

$$\frac{t_0}{\sigma} = \frac{a\mu_0}{\frac{\mu}{K(\theta)}} = \frac{(a\mu_0)\{K(\theta)\}}{r_1\mu_0} = \frac{aK(\theta)}{r_1}$$

Therefore

$$\frac{t_0{}^{\theta}}{\theta\sigma^{\theta}} = \frac{1}{\theta}\left(\frac{t_0}{\sigma}\right)^{\theta} = \frac{1}{\theta}\left(\frac{aK(\theta)}{r_1}\right)^{\theta}$$

Inserting the explicit form of $K(\theta)$,

$$\frac{t_0{}^{\theta}}{\theta\sigma^{\theta}} = \left[\frac{a}{r_1} . \frac{\Gamma\left({}^{2}/_{\theta}\right)}{\Gamma\left({}^{1}/_{\theta}\right)}\right]^{\theta}$$

Thus, the failure probability under the truncated test is

$$p = F(t_0; \sigma, \theta) = \frac{\gamma\left(\frac{1}{\theta}, \left[\frac{a}{r_1} . \frac{\Gamma\left({}^{2}/_{\theta}\right)}{\Gamma\left({}^{1}/_{\theta}\right)}\right]^{\theta}\right)}{\Gamma\left({}^{1}/_{\theta}\right)} \tag{4}$$

This expression depends only on the shape parameter θ, the termination multiplier a, and the mean ratio r_1, it is completely independent of the scale parameter σ. This scale-invariance property ensures that design parameter tables are universally applicable for any product with a given shape parameter and mean ratio.

2.4 Two-Point Quality Levels

In an acceptance sampling plan, the failure probability p is evaluated at two specific values of the mean ratio r_1, corresponding to two distinct quality levels:

- **Acceptable Quality Level (AQL)**: at this quality level, the product is considered good and the lot should be accepted with high probability. The true mean life is substantially larger than the specified mean, so $r_1 > 1$. The corresponding failure probability is:$p_1 = p(r_1 > 1)$. This probability is small because items whose true mean life greatly exceeds μ_0 are unlikely to fail before the truncation time t_0. The sampling plan must ensure that lots with failure probability p_1 are accepted with probability at least $1 - \alpha$, thereby protecting the producer against the risk of having good lots incorrectly rejected.
- **Lot Tolerance Quality Level (LTQL)**: at this quality level, the product is considered poor and the lot should be rejected with high probability. The true mean just meets the specified mean, so $r_1 = 1$. The corresponding failure probability is:$p_2 = p(r_1 = 1)$. This probability is larger because items whose true mean life barely meets the specification are more likely to fail before t_0. The sampling plan must ensure that lots with failure probability p_2 are accepted with probability no greater than β, thereby protecting the consumer against the risk of accepting bad lots.

The sampling plan is designed to simultaneously satisfy both conditions, high acceptance probability for good lots and low acceptance probability for bad lots, through the two-point optimization procedure described in the following sections.

2.5 Two-Stage Group Acceptance Sampling Plan (Two-Stage GASP)

The Two-Stage GASP extends the single-stage GASP by introducing an intermediate conditional zone between outright acceptance and outright rejection. Lots with few Stage-1 failures are accepted immediately, lots with many Stage-1 failures are rejected immediately, and borderline lots (where Stage 1 evidence is insufficient for a confident decision) proceed to Stage 2 inspection. This selective use of the second stage reduces the expected total number of items inspected, averaged over many lots, while maintaining the same producer's and consumer's risk guarantees as the single-stage plan.

2.5.1 Plan Procedure

The Two-Stage GASP under truncated life tests proceeds through the following steps.

Stage 1.

Step 1: First sample selection and group allocation. Draw a random sample of $n_1 = g_1 \times r$ items from the submitted lot. Randomly assign exactly r items to each of g_1 independent groups so that all groups are tested simultaneously. Place all n_1 items on life test and run for the pre-specified truncation time $t_0 = a\mu_0$. Let d_1 denote the total number of failures observed across all g_1 groups by the end of the test.

Step 2: Stage 1 decision rule. Three outcomes are possible based on d_1 and two pre-determined critical numbers, the Stage 1 acceptance number c_{1a} and the Stage 1 rejection number c_{1r}, where $c_{1r} > c_{1a}$.

- If $d_1 \le c_{1a}$: the number of failures is sufficiently small to conclude that the lot meets the required quality standard. Accept the lot unconditionally. The inspection process terminates after Stage 1, and no second sample is required.
- If $d_1 \ge c_{1r}$: the number of observed failures is too large, providing strong evidence that the lot fails to meet the required quality standard. The lot is therefore rejected immediately. In practice, the test may be truncated as soon as the c_{1r}-th failure is observed, saving further testing time. The inspection process terminates after Stage 1 without proceeding to Stage 2.
- If $c_{1a} < d_1 < c_{1r}$: the evidence from Stage 1 is inconclusive. The failure count is neither low enough to accept nor high enough to reject. The lot enters the conditional zone and proceeds to Stage 2 for further inspection.

Stage 2.

- **Step 3: Second sample selection and group allocation.** When Stage 2 is triggered, draw a fresh independent random sample of $n_2 = g_2 \times r$ items from the same lot. This second sample must be drawn independently of the first, items already tested in Stage 1 are not reused. Randomly assign exactly r items to each of g_2 independent groups and place all n_2 items on life test for the same truncation time $t_0 = a\mu_0$. Let d_2 denote the total number of failures observed across all g_2 groups in Stage 2.

Step 4: Stage 2 combined decision rule. The final decision is based on the combined evidence from both stages specifically, the total failure count $(d_1 + d_2)$ from all items inspected across both stages.

- If $(d_1 + d_2) \le c_{2a}$: the combined failure count is acceptably small. Accept the lot.
- If $(d_1 + d_2) > c_{2a}$: the combined failure count is too large. Reject the lot.

No further sampling is carried out beyond Stage 2. The five design parameters of the Two-Stage GASP are therefore $(g_1, g_2, c_{1a}, c_{1r}, c_{2a})$. Two boundary conditions must hold: $c_{1r} > c_{1a}$ to ensure a non-trivial conditional zone, and $c_{2a} \geq c_{1a}$ to ensure that a lot already acceptable at Stage 1 would also be acceptable at Stage 2. When $g_2 = 0$ the plan reduces to the Single GASP, confirming that the Single GASP is a special case of the Two-Stage GASP.

2.5.2 Operating Characteristic Function

The total number of failures from g_1 groups in Stage 1 is denoted by d_1, which follows a Binomial distribution with parameters $n_1 = rg_1 \text{ and } p$, where p is the probability that a single item fails before the truncation time t_0, as derived in equation (4). Similarly, the total number of failures from g_2 groups in Stage 2 is denoted by d_2, which follows a Binomial distribution with parameters $n_2 = rg_2 \text{ and } p$. Since Stage 2 draws a completely fresh and independent sample from the lot, $d_1 \text{ and } d_2$ are independent of each other. The OC function of the Two-Stage GASP is derived as follows.

Stage 1 Acceptance Probability

After placing $n_1 = g_1 r$ items on test across g_1 groups for duration t_0, the total number of failures d_1 is observed. The lot is accepted immediately at Stage 1 without any need for a second sample, if d_1 does not exceed the pre-specified Stage 1 acceptance number c_{1a}. Since each item fails independently with probability p, d_1 follows a Binomial distribution and the Stage 1 acceptance probability is:

$$P_a^{1}(p) = P(d_1 \leq c_{1a}) = \sum_{j=0}^{c_{1a}} C(g_1 r, j)\, p^j\, (1-p)^{(g_1 r - j)} \quad (5)$$

A larger value of $P_a^{1}(p)$ means more lots are resolved at Stage 1 without requiring a second sample, which directly reduces the ASN.

Stage 1 Rejection Probability

The lot is rejected immediately at Stage 1 without proceeding to Stage 2, if the total number of failures d_1 reaches or exceeds the pre-specified Stage 1 rejection number c_{1r}. The Stage 1 rejection probability is:

$$P_r^{1}(p) = P(d_1 \geq c_{1r}) = \sum_{j=c_{1r}}^{n_1} C(n_1, j)\, p^j\, (1-p)^{(n_1 - j)} \quad (6)$$

The constraint $c_{1r} > c_{1a}$ must always hold to ensure that the conditional zone between acceptance and rejection is non-trivial. If $c_{1r} = c_{1a} + 1$, no conditional zone exists and the Two-Stage GASP reduces to the Single GASP.

Conditional Zone Probability

When the Stage 1 failure count d_1 falls strictly between $c_{1a} \text{ and } c_{1r}$ that is, when the evidence from Stage 1 alone is neither strong enough to accept nor strong enough to reject, the lot enters the conditional zone and proceeds to Stage 2. The probability that a randomly selected lot enters this conditional zone is:

$$P\_mid(p) = 1 - P_a^{1}(p) - P_r^{1}(p) = P(c_{1a} < d_1 < c_{1r}) \quad (7)$$

This probability plays a central role in determining the efficiency of the Two-Stage GASP. When $P\_mid(p)$ is small meaning most lots are either clearly good or clearly bad and are resolved at Stage, the saving in inspection effort over the Single GASP is greatest and the ASN is close to n_1. When $P\_mid(p)$ is large, meaning many lots fall in the inconclusive zone and proceed to Stage 2, the ASN approaches $n_1 + n_2$ and the efficiency advantage of the two-stage structure diminishes.

Stage 2 Acceptance Probability

Given that Stage 2 has been entered with exactly $d_1 = j$ failures already observed in Stage 1, a fresh independent sample of $n_2 = g_2 r$ items is drawn from the same lot and tested for the same duration t_0. Let d_2 denote the total number of failures observed across all g_2 groups in Stage 2. The final decision is based on the combined failure total from both stages. The lot is accepted at Stage 2 if:

$$d_1 + d_2 \leq c_{2a}$$

Since $d_1 = j$ is already known from Stage 1, this condition simplifies to:

$$d_2 \leq c_{2a} - j$$

This remaining allowance $c_{2a} - j$ decreases as j increases, the more failures that occurred in Stage 1, the fewer additional failures are permitted in Stage 2 ensuring that the overall combined acceptance standard c_{2a} is always maintained regardless of how failures are distributed between the two stages. The conditional Stage 2 acceptance probability, given that exactly j failures occurred in Stage 1, is:

$$P_a^{2}(p \mid d_1 = j) = P(d_2 \leq c_{2a} - j) = \sum_{i=0}^{c_{2a}-j} C(g_2 r, i)\, p^i\, (1-p)^{(g_2 r - i)} \quad (8)$$

Summing this conditional probability over all possible values of j in the conditional zone that is, over all values of j from $c_{1a} + 1 \text{ to } c_1 r - 1$ gives the overall Stage 2 acceptance probability:

$$P_a^{2}(p) = \sum_{j=c_{1a}+1}^{c_1 r - 1} P(d_1 = j) \times P(d_2 \leq c_{2a} - j)$$

$$P_a^{2}(p) = \sum_{j=c_{1a}+1}^{c_1 r - 1} C(g_1 r, j)\, p^j\, (1-p)^{(g_1 r - j)} \times \left[\sum_{i=0}^{c_{2a}-j} C(g_2 r, i)\, p^i\, (1-p)^{(g_2 r - i)}\right] \quad (9)$$

Overall Lot Acceptance Probability(OC Function)
The overall lot acceptance probability $L(p)$ is the sum of two mutually exclusive components: the probability that the lot is accepted outright at Stage 1, and the probability that the lot enters the conditional zone at Stage 1 but is ultimately accepted at Stage 2. These two components together cover all possible ways in which a lot can be accepted under the Two-Stage GASP:

$$L(p) = P_{\mathrm{a}}{}^{1}(p) + P_{\mathrm{a}}{}^{2}(p) \tag{10}$$

This OC function is a decreasing function of p: as p increases, reflecting worse lot quality, the acceptance probability decreases. It is evaluated at the two quality levels to verify both risk constraints:

$L(p_1) \geq 1 - \alpha$ (Producer's risk constraint)

$L(p_2) \leq \beta$ (Consumer's risk constraint)

where $p_1 = p(r_1 = r_1{}^{\wedge}AQL)$ is the failure probability at the acceptable quality level and $p_2 = p(r_1 = 1)$ is the failure probability at the lot tolerance quality level, both computed from equation (4).

2.5.3 Average Sample Number

A key advantage of the Two-Stage GASP over the Single GASP is that many lots are resolved at Stage 1 without ever requiring a second sample. Lots that are clearly acceptable are accepted immediately, lots that are clearly unacceptable are rejected immediately, and only borderline lots those falling in the conditional zone, incur the additional cost of Stage 2 inspection. The expected total number of items inspected per lot, known as the Average Sample Number, quantifies this efficiency advantage and is given by:

$$ASN(p) = g_1 r + g_2 r \times (1 - P_{\mathrm{a}}{}^{1}(p) - P_{\mathrm{r}}{}^{1}(p)) \tag{11}$$

The ASN is evaluated at the consumer's quality level p_2, corresponding to the lot tolerance quality level where $r_1 = 1$. At this quality level the conditional zone probability $P\_mid(p_2)$ is at its largest, representing the worst case expected inspection burden. Minimizing the ASN at p_2 therefore provides the most conservative and practically meaningful efficiency guarantee, ensuring that the Two-Stage GASP outperforms the Single GASP even under the most demanding inspection conditions.

2.5.4 Two-Point Optimization

The optimal design parameters $(g_1, g_2, c_{1\mathrm{a}}, c_{1r}, c_{2\mathrm{a}})$ are determined by solving the following constrained minimisation problem:

$$\textit{Minimize: } ASN(p_2) = g_1 r + g_2 r \times \left(1 - P_{\mathrm{a}}{}^{1}(p) - P_{\mathrm{r}}{}^{1}(p)\right) \tag{12}$$

$$\textit{Subject to: } P_{\mathrm{a}}(p_1) \geq 1 - \alpha = 0.95 \ \textit{(Producer's risk constraint)} \tag{13}$$

$$P_{\mathrm{a}}(p_2) \leq \beta \quad \text{(Consumer's risk constraint)} \tag{14}$$

The producer's risk constraint ensures that a lot of acceptable quality, where the true mean life meets or exceeds the specification is accepted with probability of at least 0.95. The consumer's risk constraint ensures that a lot of poor quality, where the true mean life just meets the specification is accepted with probability no greater than β. Among all combinations of $(g_1, g_2, c_{1\mathrm{a}}, c_{1r}, c_{2\mathrm{a}})$ satisfying both constraints simultaneously, the combination that yields the minimum ASN is selected as the optimal Two-Stage GASP.

3. Algorithm for Generating Design Parameter Tables

This section presents the computational algorithms used to generate the design parameter tables for the proposed Single GASP and Two-Stage GASP under the HEPD. The algorithms is implemented in R.

Algorithm for the Two-Stage GASP

The Two-Stage GASP requires finding the optimal combination of five parameters $(g_1, g_2, c_{1\mathrm{a}}, c_1 r, c_{2\mathrm{a}})$ that minimizes the ASN at the consumer's quality level p_2 while satisfying both risk constraints. The acceptance and rejection numbers are not pre-specified; they are search variables in the optimization. Among all $(g_1, g_2, c_{1\mathrm{a}}, c_{1r}, c_{2\mathrm{a}})$ combinations satisfying the boundary conditions and both risk constraints (13)-(14), the combination with minimum $ASN(p_2)$ is reported in Tables 1-4 as the optimal plan, the algorithm proceeds as follows:

Step 1: Specify Input Parameters

Set the same input values as in Section 3.1.

Set the following input values $\theta, r, a, r_1, \beta, \alpha, g_1, g_2$

Step 2: Compute Failure Probabilities

For each combination of (θ, a, r_1), compute p_1 and p_2 using equation (4):

- $p_1 = p(r_1 = r_1{}^{\wedge}AQL)$, failure probability at the producer's quality level
- $p_2 = p(r_1 = 1)$, failure probability at the consumer's quality level

Step 3: Search Over Candidate Parameters

For each candidate combination of $(g_1, g_2, c_{1\mathrm{a}}, c_{1r}, c_{2\mathrm{a}})$ satisfying $c_{1r} > c_{1\mathrm{a}}$ and $c_{2\mathrm{a}} \geq c_{1\mathrm{a}}$, compute:

$$P_a^{1}(p) = P(d_1 \le c_{1a}) = \sum_{j=0}^{c_{1a}} C(g_1 r, j)\, p^{j}\, (1-p)^{(g_1 r - j)}$$

$$P_r^{1}(p) = P(d_1 \ge c_{1r}) = \sum_{j=c_{1r}}^{n_1} C(n_1, j)\, p^{j}\, (1-p)^{(n_1 - j)}$$

$$P\_mid(p) = 1 - P_a^{1}(p) - P_r^{1}(p)$$

$$P_a^{2}(p) = \sum_{j=c_{1a}+1}^{c_1 r - 1} C(g_1 r, j)\, p^{j}\, (1-p)^{(g_1 r - j)} \times \left[\sum_{i=0}^{c_{2a}-j} C(g_2 r, i)\, p^{i}\, (1-p)^{(g_2 r - i)} \right]$$

$$L(p) = P_a^{1}(p) + P_a^{2}(p)$$

Step 4: Check Risk Constraints

Verify both constraints:

- $L(p_1) \ge 1 - \alpha = 0.95$ (Producer's risk)
- $L(p_2) \le \beta$ (Consumer's risk)

Step 5: Compute ASN

For all combinations satisfying both constraints, compute:

$ASN(p_2) = g_1 r + g_2 r \times P\_mid(p_2)$

Step 6: Select Optimal Parameters

Select the combination $(g_1, g_2, c_{1a}, c_{1r}, c_{2a})$ yielding the minimum $ASN(p_2)$. Record $(c_{1a}, c_{1r}, c_{2a}, g_1, g_2, ASN, L(p_1))$.

Step 7: Repeat

Repeat Steps 2 to 6 for all combinations of $(\theta, a, r, r_1, \beta)$ to build the complete tables. The search is implemented in R using 10,000 random samples with replacement per iteration, repeated 1,000 times.

4. Results and Discussion

This paper presents the results of the proposed Two-Stage Group Acceptance Sampling Plan (GASP) developed under truncated life tests, assuming that the lifetime of the product follows the Half Exponential Power Distribution (HEPD) with shape parameter $\theta = 1.0$. The general failure probability derivation in equation (4) holds for any fixed shape parameter θ; the present design tables, comparisons, and real-data applications are restricted to $\theta = 1.0$, with extension to other shape parameter values identified as a direction for future research. The plan is designed to simultaneously satisfy the producer's risk constraint, fixed at $\alpha = 0.05$, and varying levels of consumer's risk $\beta \in \{0.25, 0.10, 0.05, 0.01\}$. The four consumer's risk levels β ∈ {0.25, 0.10, 0.05, 0.01} follow the convention established in the GASP literature (e.g., (Aslam & Jun, 2009a), (Aslam & Jun, 2009b)), spanning lenient to stringent consumer protection and enabling direct comparison with existing single- and two-stage GASP designs for other distributions. Tables 1 through 4 present the optimal design parameters $(g_1, g_2, c_{1a}, c_{1r}, c_{2a})$ along with the corresponding Average Sample Number (ASN) and the lot acceptance probability at the producer's quality level $L(p_1)$, for two group sizes $r \in \{5, 10\}$ and two termination time multipliers $a \in \{0.3, 0.5\}$. The results are discussed below with reference to the effects of consumer's risk, mean ratio, group size, and termination time multiplier on the ASN.

4.1 Effect of Consumer's Risk (β)

One of the most prominent patterns observed across all tables is the direct relationship between consumer's risk β and the Average Sample Number. As β decreases, indicating a more stringent consumer protection requirement, the ASN increases substantially. This is consistent with statistical theory: a lower β demands that the sampling plan be more capable of rejecting genuinely poor-quality lots, which inherently requires a larger inspection effort.

For example, in Table 1 ($r = 5, a = 0.3$), when the mean ratio $\boldsymbol{r_1} = 2$, the ASN increases from 50.90 at $\beta = 0.25$ to 81.39 at $\beta = 0.10$, further to 89.62 at $\beta = 0.05$, and reaches 116.69 at $\beta = 0.01$. This represents a more than two-fold increase in inspection effort as β moves from the most lenient to the most stringent level. The same monotone increasing trend is consistently observed across all combinations of group size and termination time multiplier, confirming that consumer's risk is one of the primary drivers of ASN in the proposed plan.

Similarly, in Table 2 ($r = 10, a = 0.3$), the ASN at $\boldsymbol{r_1} = 2$ rises from 53.30 ($\beta = 0.25$) to 119.79 ($\beta = 0.01$), and in Table 3 ($r = 5, a = 0.5$), it rises from 36.75 to 85.06 over the same range of β. This behaviour underscores that stricter quality protection, while desirable from the consumer's perspective, must be balanced against the increased cost and time of inspection.

4.2 Effect of Mean Ratio ($\boldsymbol{r_1} = \boldsymbol{\mu}/\boldsymbol{\mu}_0$)

The mean ratio $\boldsymbol{r_1} = \mu/\mu_0$ represents the ratio of the true mean life of the product to the specified minimum mean life. A larger mean ratio indicates that the product's actual lifetime is substantially greater than the minimum requirement, meaning the lot is of higher quality relative to the acceptance threshold. Across all four tables, a clear and consistent inverse relationship is observed between $\boldsymbol{r_1}$ and the ASN: as $\boldsymbol{r_1}$ increases, the ASN decreases, sometimes dramatically.

This pattern is physically intuitive. When the true mean life is much larger than μ_0 (i.e., high $\boldsymbol{r_1}$), the failure probability within the truncated test period is very small, making it easy for the plan to identify the lot as acceptable with fewer groups inspected. Conversely, at $r_1 = 2$, the quality margin is narrow, and more inspection is required to confidently accept or reject the lot.

For instance, in Table 1 at $\beta = 0.10$, the ASN drops from 81.39 at $r_1 = 2$ to 25.38 at $r_1 = 4$, and further to 17.01 and 16.01 at $r_1 = 6\, and\, 8$ respectively. This represents an approximately 80% reduction in sampling effort as the mean ratio increases from 2 to 8. The corresponding reduction in the number of groups required in Stage 1 (g_1) and Stage 2 (g_2) further confirms that higher-quality lots are efficiently identified and accepted at the very first stage of inspection, with Stage 2 rarely being invoked.

4.3 Effect of Group Size ($\boldsymbol{r}$)

The group size r refers to the number of items placed simultaneously on a single tester or inspection unit, and the total sample size at each stage is the product of the number of groups and the group size ($n = g \times r$). Tables 1 and 2 ($a = 0.3$) and Tables 3 and 4 (a = 0.5) permit a direct comparison of the ASN under $r = 5\, and\, r = 10$ respectively.

The results reveal that increasing the group size from 5 to 10 generally increases the ASN, particularly at smaller mean ratios. For example, at $\beta = 0.10\, and\, r_1 = 2\, with\, a = 0.3$, the ASN rises from 81.39 ($r = 5, Table\ 1$) to 90.33 ($r = 10, Table\ 2$). At $\beta = 0.01\, and\, r_1 = 2$, the ASN increases from 116.69 ($r = 5$) to 119.79 ($r = 10$). This occurs because with a larger group size, each stage tests more items simultaneously, but since the total number of groups required is typically fewer, the rounding constraint imposed by the discrete group structure can push the total sample size slightly upward.

However, the differences in ASN between $r = 5\, and\, r = 10$ are relatively modest in magnitude, particularly at higher mean ratios. At $r_1 = 6\, and\, r_1 = 8$, the ASN values under both group sizes are often similar or even converge. The practical implication is that the choice of group size should be guided primarily by the physical testing infrastructure available, the number of testers or testing stations, rather than solely by ASN minimization. A larger group size reduces the number of groups needed ($g_1 + g_2$) and hence reduces the logistical complexity of the plan, which is an important practical advantage in industrial life testing.

4.4 Effect of Termination Time Multiplier ($\boldsymbol{a}$)

The termination time multiplier a determines the length of the life test relative to the specified mean life, such that the truncation time is $t_0 = a\mu_0$. A larger value of a implies a longer experiment duration, during which more failures are expected to occur, thereby providing stronger statistical evidence about lot quality. Comparing Tables 1 and 3 ($both\ at\ r = 5$) and Tables 2 and 4 ($both\ at\ r = 10$) reveals the consistent effect of a on ASN.

As a increases from 0.3 to 0.5, the ASN decreases substantially across all combinations of $\beta\, and\, r_1$. This reduction occurs because a longer test period produces a higher failure probability within the test window, making it easier to discriminate between acceptable and unacceptable lots with fewer groups. For instance, at $\beta = 0.10, r_1 = 2\, and\, r = 5$, the ASN decreases from 81.39 ($a = 0.3, Table\ 1$) to 53.63 ($a = 0.5, Table\ 3$), a reduction of approximately 34%. At $\beta = 0.01\, and\, r_1 = 2\, with\, r = 5$, the ASN drops more markedly from 116.69 to 85.06, a reduction of over 27%.

The same pattern holds for $r = 10$: comparing Tables 2 and 4, the ASN at $\beta = 0.01$ and $r_1 = 2$ decreases from 119.79 to 87.07 as a increases from 0.3 to 0.5. This finding has a direct practical interpretation: if the testing environment permits a slightly longer experiment, a larger value of a can substantially reduce the number of items inspected, leading to significant savings in cost and time without compromising the integrity of either the producer's or consumer's risk constraints.

4.5 Producer's Acceptance Probability $\boldsymbol{L(p_1)}$

Across all four tables, the producer's lot acceptance probability $L(p_1)$, evaluated at the producer's quality level corresponding to the specified mean ratio, consistently exceeds 0.95 in all reported cases. The values range between approximately 0.9511 and 0.9854, confirming that the proposed plan reliably satisfies the producer's risk constraint of $\alpha = 0.05$. This robustness across all parameter combinations demonstrates that the optimization procedure correctly identifies plans that simultaneously protect both the producer and the consumer.

It is further observed that $L(p_1)$ tends to be higher at larger mean ratios r_1, which is expected since higher-quality lots are inherently easier to accept with high probability. The consistency of $L(p_1)$ values above 0.95 across all scenarios provides strong evidence that the proposed two-stage GASP maintains the desired operating characteristic behaviour throughout the range of parameters considered.

4.6 Overall Summary

In summary, the results presented in Tables 1 through 4, and illustrated in Figures 1 through 4, collectively demonstrate that the proposed Two-Stage Group Acceptance Sampling Plan under HEPD with $\theta = 1.0$ provides a flexible, efficient, and reliable framework for acceptance decisions under truncated life testing. The ASN is primarily driven by the consumer's risk β and the mean ratio r_1, while the termination time multiplier a offers an important lever for reducing inspection effort when a longer test duration is feasible. The group size r has a comparatively modest effect on ASN but plays an important practical role in determining the logistical structure of the sampling plan. All plans satisfy the producer's risk constraint with $L(p_1) > 0.95$ throughout, and achieve meaningful reductions in sample size relative to single-stage GASP, confirming the efficiency advantage of the two-stage design.

Table.1: Proposed Two Stage Group Sampling Plan for Half Exponential Power Distribution $with\, r = 5, \theta = 1.0\, and\, a = 0.3$

β	r_1	c_{1r}	c_{1a}	c_{2a}	g_1	g_2	ASN	$L(p_1)$
0.25	2	10	6	17	8	7	50.90	0.9511
	4	4	1	4	3	2	18.59	0.9630
	6	3	0	2	2	1	12.24	0.9667
	8	3	0	2	2	1	12.22	0.9841
0.10	2	16	10	23	13	10	81.39	0.9675
	4	5	1	5	4	3	25.38	0.9603
	6	3	0	3	3	2	17.01	0.9514
	8	3	0	2	3	1	16.01	0.9647
0.05	2	18	12	25	16	10	89.62	0.9543
	4	6	2	6	5	4	31.27	0.9614
	6	4	1	4	4	3	22.68	0.9698
	8	3	1	4	4	2	20.50	0.9638
0.01	2	23	12	34	21	16	116.69	0.9566
	4	7	1	8	7	6	39.80	0.9535
	6	5	1	5	6	4	31.50	0.9626
	8	4	1	4	5	4	26.50	0.9730

Table.2: Proposed Two Stage Group Sampling Plan for Half Exponential Power Distribution $with\, r = 10, \theta = 1.0\, and\, a = 0.3$

β	r_1	c_{1r}	c_{1a}	c_{2a}	g_1	g_2	ASN	$L(p_1)$
0.25	2	12	10	16	5	3	53.30	0.9515
	4	5	3	4	2	1	21.80	0.9668
	6	4	2	8	2	1	21.21	0.9854
	8	3	1	4	2	1	20.58	0.9638
0.10	2	18	0	25	8	5	90.33	0.9530
	4	6	4	5	3	1	30.92	0.9586
	6	4	2	3	2	1	21.21	0.9631
	8	3	1	7	2	1	20.58	0.9644
0.05	2	18	11	30	8	7	94.00	0.9667
	4	6	3	7	3	2	32.86	0.9683
	6	5	3	4	3	1	30.51	0.9692
	8	3	1	4	2	1	20.58	0.9638
0.01	2	23	4	35	10	9	119.79	0.9630
	4	7	3	9	4	3	42.19	0.9607
	6	5	1	5	3	2	31.57	0.9626
	8	4	2	3	3	1	30.21	0.9536

Table 3: Proposed Two Stage Group Sampling Plan for Half Exponential Power Distribution $with\, r = 5, \theta = 1.0\, and\, a = 0.5$

β	r_1	c_{1r}	c_{1a}	c_{2a}	g_1	g_2	ASN	$L(p_1)$
0.25	2	13	11	15	7	3	36.75	0.9529
	4	4	1	4	2	1	11.74	0.9658
	6	3	1	4	2	1	10.64	0.9593
	8	3	1	5	2	1	10.60	0.9807
0.10	2	17	13	24	10	5	53.63	0.9582
	4	5	2	6	3	2	17.02	0.9651
	6	3	1	3	2	1	10.63	0.952
	8	3	1	3	2	1	10.61	0.9774
0.05	2	19	12	29	11	8	62.57	0.9643
	4	6	3	7	4	2	21.2	0.968
	6	4	1	4	3	1	15.47	0.9671
	8	4	2	3	3	1	15.34	0.9765
	2	26	1	41	16	12	85.06	0.968

0.01	4	7	3	8	5	3	26.2	0.954
	6	5	2	5	4	2	20.53	0.9662
	8	4	2	4	4	1	20.07	0.9673

Table 4: Proposed Two Stage Group Sampling Plan for Half Exponential Power Distribution $with\ r = 10, \theta = 1.0\ and\ a = 0.5$

β	r_1	c_{1r}	c_{1a}	c_{2a}	g_1	g_2	ASN	$L(p_1)$
0.25	2	12	9	16	3	2	38.29	0.9636
	4	6	4	6	2	1	20.81	0.9586
	6	5	3	5	2	1	20.39	0.9719
	8	4	2	7	2	1	20.14	0.9701
0.10	2	15	8	22	4	3	55.22	0.9639
	4	6	4	7	2	1	20.81	0.9708
	6	5	3	9	2	1	20.39	0.9817
	8	4	2	4	2	1	20.14	0.9596
0.05	2	18	13	26	5	4	64.3	0.9516
	4	6	3	7	2	1	22.50	0.968
	6	5	3	8	2	1	20.39	0.9817
	8	4	2	10	2	1	20.14	0.9701
0.01	2	27	8	34	8	4	87.07	0.9543
	4	8	4	8	3	1	30.49	0.9574
	6	5	2	5	2	1	22.53	0.9662
	8	4	2	5	2	1	20.14	0.9684

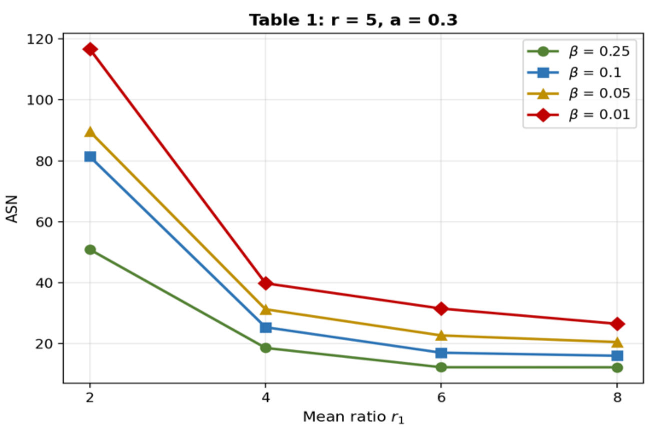

Figure 1: ASN as a function of the mean ratio r_1, for consumer's risk $\beta \in \{0.25, 0.10, 0.05, 0.01\}$, at group size $r = 5$ and termination multiplier $a = 0.3$ (corresponding to Table 1).

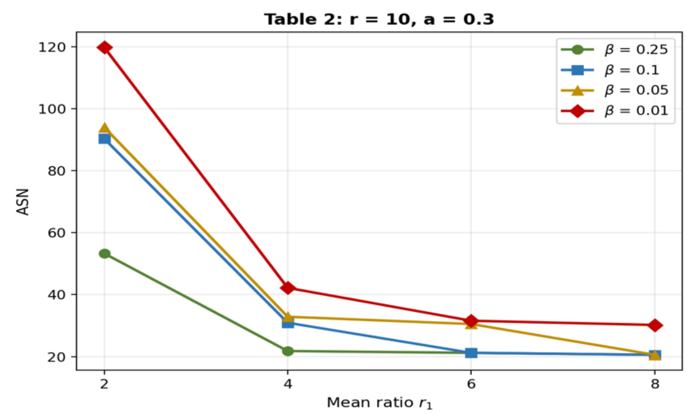

Figure 2: ASN as a function of the mean ratio r_1, for consumer's risk $\beta \in \{0.25, 0.10, 0.05, 0.01\}$, at group size $r = 10$ and termination multiplier $a = 0.3$ (corresponding to Table 2).

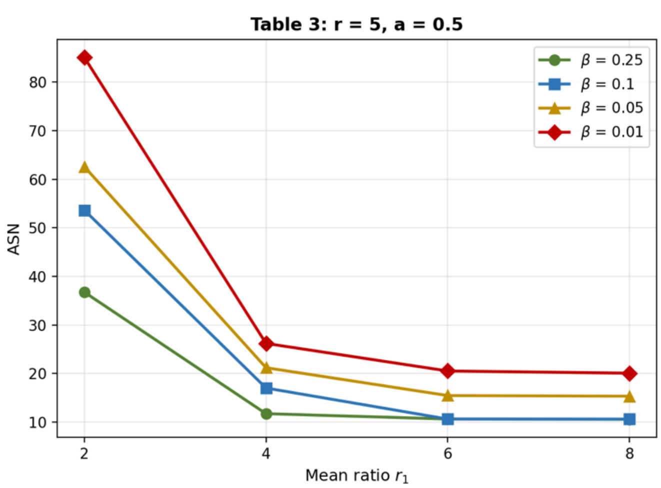

Figure 3: ASN as a function of the mean ratio r_1, for consumer's risk $\beta \in \{0.25, 0.10, 0.05, 0.01\}$, at group size $r = 5$ and termination multiplier $a = 0.5$ (corresponding to Table 3).

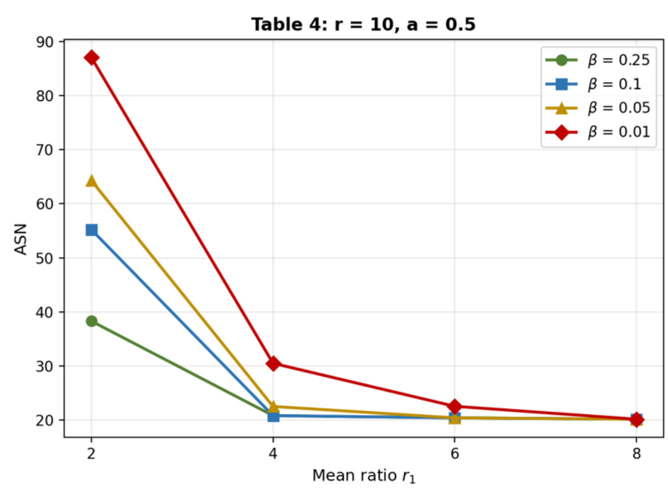

Figure 4: ASN as a function of the mean ratio r_1, for consumer's risk $\beta \in \{0.25, 0.10, 0.05, 0.01\}$, at group size $r = 10$ and termination multiplier $a = 0.5$ (corresponding to Table 4).

5. Comparison of the proposed two-stage GASP with single GASP

A fundamental motivation for proposing a two-stage group acceptance sampling plan is its ability to reduce the average inspection effort compared to the conventional single-stage GASP, while simultaneously satisfying both the producer's and consumer's risk constraints. In a single-stage GASP, a fixed number of items $n = g \times r$ is always inspected regardless of the observed number of failures, and the accept/reject decision is made in one step. By contrast, the proposed two-stage plan allows an early decision at Stage 1 whenever the evidence is sufficiently conclusive, invoking Stage 2 only in ambiguous cases. This section presents a systematic comparison of the two plans for $\theta = 1.0, a = 0.3$ and group size $r = 5$ across all combinations of consumer's risk $\beta \in \{0.25, 0.10, 0.05, 0.01\}$ and mean ratio $r_1 \in \{2, 4, 6, 8\}$. The comparison is summarized in Table 5 and illustrated graphically in Figure 5.

Table 5: Comparison of Single GASP and Proposed Two-Stage GASP under HEPD (θ = 1.0, a = 0.3, r = 5)

β	r_1	Single GASP		Proposed Two-Stage GASP							
		n	$L(p_1)$	g_1	g_2	c_{1a}	c_{1r}	c_{2a}	ASN	$L(p_1)$	Reduction%
0.25	2	70	0.9707	8	7	6	10	17	50.90	0.9511	27.3%
	4	25	0.9690	3	2	1	4	4	18.59	0.9630	25.6%
	6	15	0.9660	2	1	0	3	2	12.24	0.9667	18.4%
	8	15	0.9837	2	1	0	3	2	12.22	0.9841	18.4%
0.10	2	105	0.9687	13	10	10	16	23	81.39	0.9675	22.5%
	4	35	0.9623	4	3	1	5	5	25.38	0.9603	27.5%
	6	25	0.9685	3	2	0	3	3	17.01	0.9514	32.0%
	8	20	0.9644	3	1	0	3	2	16.01	0.9647	19.9%
0.05	2	130	0.9648	16	10	12	18	25	89.62	0.9543	31.1%
	4	45	0.9586	5	4	2	6	6	31.27	0.9614	30.5%
	6	35	0.9736	4	3	1	4	4	22.68	0.9698	35.2%
	8	30	0.9766	4	2	1	3	4	20.58	0.9638	31.4%
0.01	2	190	0.9670	21	16	12	23	34	116.69	0.9566	38.6%
	4	65	0.9565	7	6	1	7	8	39.80	0.9535	38.8%
	6	50	0.9659	6	4	1	5	5	31.57	0.9626	36.9%
	8	45	0.9757	5	4	1	4	4	26.50	0.9730	41.1%

Discussion of Comparison Results

The results in Table 5 demonstrate that the proposed two-stage GASP consistently achieves a lower average sample number than the single-stage GASP across all parameter combinations. The percentage reduction in inspection effort computed as $(n - ASN)/n \times 100$, ranges from approximately 18.4% to over 41.1%, depending on the values of $\beta \, and \, r_1$. Crucially, this reduction is achieved without compromising either risk constraint: $L(p_1) > 0.95$ in every case for both plans, confirming that the efficiency gains arise from the adaptive two-stage architecture rather than from any relaxation of the statistical guarantees.

The efficiency advantage of the two-stage plan is most pronounced at small mean ratios ($r_1 = 2$) and stringent consumer's risk levels ($\beta = 0.01$). For instance, at $\beta = 0.01 \, and \, r_1 = 2$, the single-stage GASP requires $n = 190$ items, whereas the two-stage plan achieves the same objectives with an ASN of only 116.69, a reduction of approximately 38.6%. This advantage arises because at $r_1 = 2$ the quality margin between acceptable and marginal lots is narrow, and the two plans differ most substantially: the single-stage plan must always commit to inspecting all 190 items, whereas the two-stage plan frequently reaches a definitive decision at Stage 1 with only $n_1 = g_1 \times r = 13 \times 5 = 65$ items, invoking Stage 2 only when the Stage 1 failure count falls in the inconclusive zone.

At larger mean ratios, the two plans converge in terms of sample size because high-quality lots are easily identified with fewer groups, leaving little room for further reduction. At $r_1 = 8$ across all β levels, the ASN values of the two-stage plan range between 12.24 and 26.50, compared to single-stage values of 15 to 45, still consistently lower, but with a narrower absolute gap. The percentage reductions at $r_1 = 8$ remain meaningful, reaching up to 41.1% at $\beta = 0.01$.

The producer's acceptance probability $L(p_1)$ is maintained above 0.95 for both plans in every row of Table 5, providing direct confirmation that adopting the two-stage structure does not inflate the producer's risk. The two-stage plan therefore offers a strictly superior balance of inspection economy and risk protection relative to the single-stage GASP: it reduces the expected inspection burden by 18.4% to 41.1% while delivering the same statistical guarantees to both the producer and the consumer.

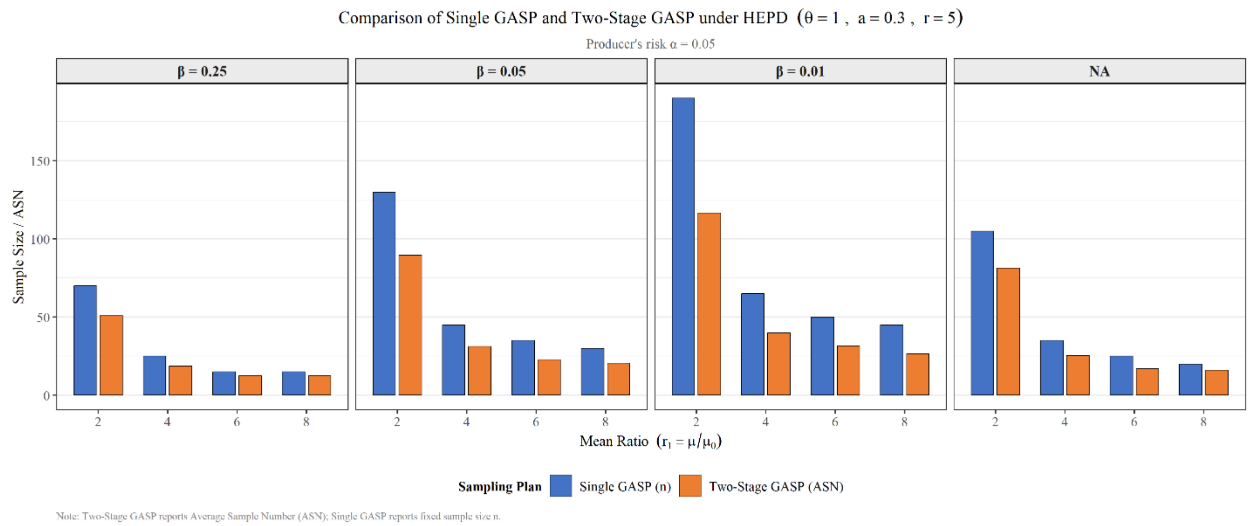

Figure 5. Comparison of Single GASP VS Two-Stage GASP under ($\boldsymbol{\theta = 1.0, a = 0.3, r = 5}$)

6. Real-Life Applications

To demonstrate the practical applicability of the proposed Two-Stage Group Acceptance Sampling Plan (GASP) under the HEPD, two real-life data sets are analyzed in this section. For each data set, the Kolmogorov–Smirnov (KS) goodness-of-fit test is first applied to verify conformity to the HEPD with θ = 1.0, after which the appropriate design parameters are extracted from Tables 1 through 4 and the sampling plan is implemented step by step. The two examples span different applied domains, materials engineering and sports medicine, demonstrating the breadth of contexts in which the proposed plan can serve as a rigorous and operationally efficient quality assurance tool.

6.1 Example 1: Stress Rupture Life of Kevlar 49/Epoxy Strands

The first example uses the stress rupture life data of Kevlar 49/epoxy strands under 90% stress level, originally studied by (Cooray & Ananda, 2008) and subsequently by (Olmos, Varela, Gómez, & Bolfarine, 2012), (Naveed et al., 2023). The dataset contains 101 observations of failure time (in hours) under sustained pressure.

To verify distributional conformity, the KS goodness-of-fit test was applied to the square-root-transformed data using R The null hypothesis H_0 states that the data follow the HEPD with $\theta = 1.0$. The test yielded a statistic $D = 0.073$ and an approximate $p-value > 0.15$, which exceeds the significance level $\alpha = 0.05$. Accordingly, H_0 is not rejected, and the transformed data are accepted as following the HEPD with $\theta = 1.0$ and estimated scale parameter $\sigma = 33.07$.

Suppose a manufacturer wants to assure that the mean stress rupture life of Kevlar strands is at least $\mu_0 = 600$ hours. The life test is conducted under a truncated scheme with termination time multiplier $a = 0.5$, so the test is ended at $t_0 = 0.5 \times 600 = 300$ hours. Based on historical performance, the true mean life is believed to be approximately 1,200 hours, giving a mean ratio of $r_1 = \mu/\mu_0 = 1200/600 = 2$. The producer's risk is fixed at $\alpha = 0.05$ and the consumer's risk at $\beta = 0.10$. Testing equipment accommodates $r = 5$ strands per tester simultaneously. With $\theta = 1.0, a = 0.5, r = 5, \beta = 0.10$ *and* $r_1 = 2$, the design parameters are read directly from Table 3 of the proposed plan:

g_1	g_2	c_{1a}	c_{1r}	c_{2a}	ASN	$L(p_1)$
10	5	13	17	24	53.63	0.9582

The plan is implemented in two stages as follows:

Stage 1: Draw a random sample of $n_1 = g_1 \times r = 10 \times 5 = 50$ Kevlar strands from the production lot. Allocate 5 strands to each of 10 testers and subject all items to 90% stress loading for 300 hours. Let d_1 denote the total number of failures across all groups. Accept the lot unconditionally if $d_1 \le c_{1a} = 13$. Reject the lot immediately if $d_1 \ge c_{1r} = 17$. If $d_1 \in \{14, 15, 16\}$, the evidence is inconclusive; proceed to Stage 2.

Stage 2: Draw a fresh independent sample of $n_2 = g_2 \times r = 5 \times 5 = 25$ strands and allocate them to 5 testers. Run the same life test for 300 hours. Accept the lot if the combined failure count from both stages satisfies $d_1 + d_2 \le c_{2a} = 24$. Otherwise, reject the lot.

The minimum ASN of 53.63 indicates that, on average, approximately 54 strands are inspected per lot. The producer's acceptance probability $L(p_1) = 0.9582$ confirms that conforming lots are accepted with probability 95.82%, satisfying the required lower bound of $1 - \alpha = 0.95$. Relative to the corresponding single-stage GASP, which requires a fixed sample of $n = 65$ items under identical parameter settings, the two-stage plan reduces the expected inspection burden by approximately 17.5%, representing a meaningful saving in both testing time and material cost.

6.2 Example 2: Plasma Ferritin Concentration of Athletes

The second data set records the plasma ferritin concentration (ng/mL) of 202 athletes 102 male and 100 female, from the Australian Institute of Sports, originally published by (Cook & Weisberg, 2009) and subsequently by (Elal-Olivero, Olivares-Pacheco, Gómez, & Bolfarine, 2009), (Gui, 2013), (Naveed et al., 2023) and (Naveed et al., 2024).. Ferritin is a blood protein that reflects total body iron storage and is a recognized indicator of athletic health and endurance capacity. Low ferritin levels are associated with iron deficiency, fatigue, and impaired physical performance.

The KS goodness-of-fit test was applied to the appropriately transformed data using R, with H_0 stating that the data follow the HEPD with $\theta = 1.0$. The test yielded $D = 0.061$ and an approximate $p - value = 0.069$, which exceeds the significance level $\alpha = 0.05$. Accordingly, H_0 is not rejected, and the transformed ferritin data are accepted as following the HEPD with $\theta = 1.0$.

Suppose a sports scientist or team physician wants to verify that the mean ferritin level across athletes is at least $\mu_0 = 50$ ng/mL. The screening test is truncated at $t_0 = a\mu_0 = 0.3 \times 50 = 15$ ng/mL (representing a lower threshold screening). The true mean ferritin level is assumed to be $\mu = 100$ ng/mL, giving a mean ratio $r_1 = 100/50 = 2$. Testing (screening) is conducted in groups of $r = 10$ athletes per session. The producer's risk (trainer or medical officer's risk) is $\alpha = 0.05$ and the consumer's risk (team management's risk) is $\beta = 0.25$.

With $\theta = 1.0, a = 0.3, r = 10, \beta = 0.25 \, and \, r_1 = 2$, the design parameters are taken directly from Table 2 of the proposed plan:

g_1	g_2	c_{1a}	c_{1r}	c_{2a}	ASN	$L(p_1)$
5	3	10	12	16	53.30	0.9515

The screening plan proceeds as follows:

Stage 1: Randomly select 50 athletes from the squad ($n_1 = g_1 \times r = 5 \times 10 = 50$) and conduct ferritin screening in 5 sessions of 10 athletes each. A test outcome is classified as a "failure" if an athlete's ferritin level falls below the threshold of 15 ng/mL. Accept the entire group if total failures $d_1 \leq c_{1\mathrm{a}} = 10$. . Reject immediately if failures $\mathrm{d}_1 \geq \mathrm{c}_{1\mathrm{r}} = 12$. If $d_1 = 11$, the evidence is inconclusive; proceed to Stage 2

Stage 2: Screen an additional 30 athletes ($n_2 = g_2 \times r = 3 \times 10 = 30$) in 3 further sessions. Accept the group if the combined failures from both stages are $d_1 + d_2 \leq c_{2\mathrm{a}} = 16$. Otherwise, the group is rejected and further individual follow-up assessments are recommended.

The minimum ASN is 53.30, indicating that on average 53 athletes need to be screened per assessment round. The producer's acceptance probability $L(p_1) = 0.9515$ ensures that a genuinely healthy group with mean ferritin equal to twice the threshold is correctly cleared with probability 95.15%. This plan offers a structured and statistically rigorous approach to group-level health screening, reducing the burden of testing every individual in the squad while maintaining strict risk control for both the trainer and team management. The two-stage design also provides flexibility: if Stage 1 results are conclusive, as they will be in the majority of cases when the group quality is either clearly good or clearly poor, the additional Stage 2 screening is avoided entirely, further reducing the average screening effort.

7. Concluding Remarks

This paper developed a Two-Stage Group Acceptance Sampling Plan (GASP) for products whose lifetimes follow the Half Exponential Power Distribution (HEPD) with shape parameter θ = 1.0, under truncated life tests and the two-point operating characteristic (OC) criterion. The plan simultaneously satisfies both the producer's risk (α = 0.05) and consumer's risk (β ∈ {0.25, 0.10, 0.05, 0.01}), and its performance was systematically compared with the conventional single-stage GASP across a comprehensive range of design parameters.

7.1 Main Contributions

The principal contributions of this paper are as follows. First, the failure probability under the HEPD truncated life test was derived analytically in closed form as a function of the mean ratio $r_1 = \mu/\mu_0$ and the termination multiplier a for $\theta = 1.0$, and was shown to be entirely independent of the scale parameter σ. This scale-invariance property ensures that the resulting design tables are universally applicable to any product governed by the HEPD, regardless of its absolute scale.

Second, the optimal design parameters ($g_1, g_2, c_{1\mathrm{a}}, c_{1\mathrm{r}}, c_{2\mathrm{a}}$) were obtained by minimizing the ASN at the consumer's quality level subject to both risk constraints simultaneously, and comprehensive design tables were provided for termination multipliers $a \in \{0.3, 0.5\}$, mean ratios $r_1 \in \{2, 4, 6, 8\}$, and group sizes $r \in \{5, 10\}$. To the best of the authors' knowledge, this represents the first tabulation for the HEPD under a two-stage group testing architecture with dual risk control.

Third, systematic comparison with the single-stage GASP demonstrated consistent ASN reductions of 18.4% to 41.1% across all parameter combinations examined, with the producer's acceptance probability $L(p_1)$ exceeding 0.95 in every case, confirming that the efficiency gains are structural and do not arise from any relaxation of the risk constraints.

Fourth, the practical applicability of the proposed plan was validated on two real-life data sets, the stress rupture life of Kevlar 49/epoxy strands (Cooray & Ananda, 2008) and the plasma ferritin concentration of Australian athletes (Cook & Weisberg, 2009), both confirmed to follow the HEPD with $\theta = 1.0$ by the Kolmogorov–Smirnov goodness-of-fit test ($D = 0.073, p-value > 0.15$ and $D = 0.061, p-value = 0.069$, respectively). In both cases, the two-stage plan delivered a statistically sound quality decision with a materially smaller expected inspection effort than the single-stage alternative.

7.2 Key Findings

Four consistent patterns emerge from the numerical results. First, the ASN increases monotonically as the consumer's risk β decreases at $\beta = 0.01$ the ASN is more than double its value at $\beta = 0.25$ under otherwise identical conditions, reflecting the fundamental statistical trade-off between the stringency of consumer protection and the magnitude of the inspection effort required to enforce it. Second, the ASN decreases sharply as r_1 increases; at $\beta = 0.01$, raising r_1 from 2 to 8 reduces the ASN by approximately 77%, since high-quality lots are identified and accepted conclusively at Stage 1 with very few failures, rarely triggering Stage 2. Third, increasing the termination multiplier from $a = 0.3$ to $a = 0.5$ reduces the ASN by 27% to 34% across comparable scenarios, confirming that a modestly longer test duration, when operationally feasible, is among the most effective levers available to the quality engineer for reducing inspection cost without compromising either risk guarantee. Fourth, the group size r exerts a comparatively modest influence on the ASN; its selection should therefore be guided primarily by the available testing infrastructure rather than by ASN minimization alone, since a larger group size reduces the total number of groups required and thereby simplifies the logistical administration of the plan.

7.3 Future Research

Several directions merit further investigation. A natural extension is to develop the proposed framework using the median lifetime, or other percentile-based quality benchmarks, in place of the mean ratio r_1 adopted here; such a reformulation would require re-deriving the failure probability and recalibrating the design tables, but has precedent elsewhere in the GASP literature (Aslam, Shoaib, & Khan, 2011). The present derivation and design tables are restricted to the shape parameter θ = 1.0; extending the failure probability, design tables, and comparisons to other values of θ is a further direction worth pursuing, since the closed-form expression in equation (4) holds for any fixed θ. The framework can also be adapted for neutrosophic or fuzzy environments to handle ambiguous failure classifications, and Bayesian extensions could incorporate prior quality information for further efficiency gains. Variable sampling plans based on measured lifetime values, rather than failure counts alone, and multi-stage or sequential generalizations are additional avenues of practical interest.

In summary, the proposed Two-Stage GASP under the HEPD with $\theta = 1.0$ provides a statistically rigorous, operationally efficient, and practically implementable framework for acceptance decisions in truncated life testing, achieving meaningful reductions in inspection effort relative to the single-stage GASP without compromising either risk guarantee.

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Appendix

The following is a list of abbreviations and key notation used throughout the manuscript, together with their full terms or definitions, to assist the reader in following the derivations and design tables presented in Sections 2 through 6.

Abbreviation / Symbol	Full Term / Description
GASP	Group Acceptance Sampling Plan
HEPD	Half Exponential Power Distribution
OC	Operating Characteristic
ASN	Average Sample Number
AQL	Acceptable Quality Level
LTQL	Lot Tolerance Quality Level
KS	Kolmogorov–Smirnov (goodness-of-fit test)
PDF	Probability Density Function
CDF	Cumulative Distribution Function
g_1	Number of groups in Stage 1
g_2	Number of groups in Stage 2
r	Group size (number of items per group/tester)
n_1	Stage 1 sample size ($n_1 = g_1 \times r$)
n_2	Stage 2 sample size ($n_2 = g_2 \times r$)
d_1	Number of failures observed in Stage 1
d_2	Number of failures observed in Stage 2
c_{1a}	Stage 1 acceptance number
c_{1r}	Stage 1 rejection number
c_{2a}	Stage 2 (combined) acceptance number
t_0	Truncation (test termination) time
a	Termination time multiplier ($t_0 = a\mu_0$)
μ_0	Specified (minimum acceptable) mean life
μ	True mean life of the product
r_1	Mean ratio ($r_1 = \mu/\mu_0$)
σ	Scale parameter of the HEPD
θ	Shape parameter of the HEPD
p	Failure probability under the truncated test
p_1	Failure probability at the producer's (AQL) quality level
p_2	Failure probability at the consumer's (LTQL) quality level
α	Producer's risk
β	Consumer's risk
$L(p)$	Lot acceptance probability (OC function)