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Accelerated Fixed Point Algorithms for Convex Optimization Using Extreme Learning Machine Models

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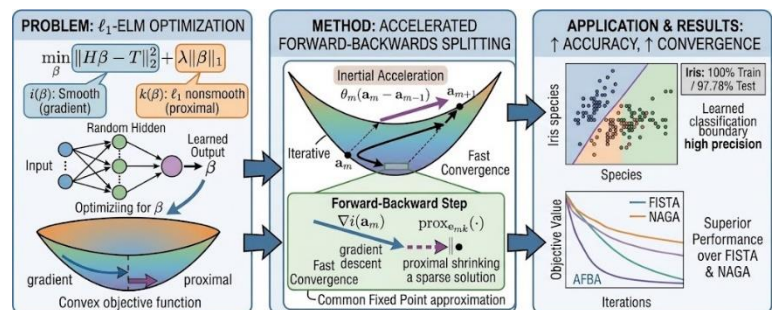
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ABSTRACT

In this paper, a common fixed point of a countable set of nonexpansive mappings in Hilbert space. Under appropriate regulatory circumstances, the weak convergence theorems are formulated. Nonsmoothed convex minimization problems, which are expressed as the union of two suitable convex lower semicontinuous functions, are tackled by applying the proposed method. The subsequent method provides a viable answer to structured convex optimization problems that arise in many engineering and scientific settings; it is constructed within the forward-backwards splitting paradigm. Also, using an extreme learning machine model with l_1 -regularization, we show that it works well for machine learning classification tasks. Improving forecast accuracy through rapid iterative refinement, the proposed method deftly computes output weights by tackling the basic convex optimization problem. Its effectiveness, durability, and improved classification accuracy compared to existing methods are confirmed by numerical experiments on many benchmark datasets.



Keywords: Accelerated fixed point algorithm; nonexpansive mappings; common fixed point; convex minimization

1. Introduction

The efficiency arises from combining a gradient step associated with i and a proximal step associated with k . Since the solution set can be characterized as the fixed-point set of a nonexpansive operator, fixed-point techniques provide a natural framework for designing effective optimization algorithms. Over the past decades, researchers have proposed numerous recursive techniques for approximating fixed points of nonexpansive operators, including the [1],[2],[3],[4],[5] and viscosity-type methods [6],[7],[8]. To further improve convergence speed, Polyak's inertial technique [9] inspired several accelerated algorithms. Among them, the Fast Iterative Shrinkage-Proximal Thresholding Algorithm (FISTA) [10] has achieved remarkable

success in convex optimization. Recently, accelerated fixed-point algorithms have attracted significant attention because of their strong theoretical properties and practical performance. Moreover, the rapid growth of machine learning applications has increased the demand for efficient optimization methods capable of handling large-scale datasets. Extreme Learning Machines (ELMs) require solving regularized optimization problems to determine output weights, making them suitable candidates for fixed-point and forward-backwards approaches.

Motivated by the growing interplay between fixed-point methods, convex optimization, and data-driven learning, we introduce an inventive accelerated fixed-point technique aimed at approximating a common fixed point for a measurable collection of nonexpansive operators in Hilbert spaces. The weak convergence



of the suggested method can be confirmed with minimal prerequisites. This approach is embedded within a forward-backwards splitting framework aimed at resolving composite convex optimization challenges of the kind $i + k$, where i is smooth and k is nonsmooth. The utility of the recommended technique is further exemplified by using sparse machine learning models that employ ℓ_1 -regularized Extreme Learning Machines.

The key contributions of this manuscript are detailed below:

- Development of an innovative accelerated fixed-point technique for measurable families of nonexpansive operators.
- Establishment of weak convergence in the setting of Hilbert spaces.
- Derivation of an accelerated forward-backwards fractionation approach for structured nonsmooth convex minimization issues.
- Application of the proposed framework to sparse classification models via ℓ_1 -regularized Extreme Learning Machines.
- Extensive numerical validation demonstrates the effectiveness, robustness, and computational advantages of the proposed approach on benchmark classification datasets.

2. Preliminaries

This section introduces the notation, terminology, and auxiliary findings employed in the formulation and convergence analysis of the proposed algorithm. Let H be a real Hilbert space provided with the inner product $\langle \cdot, \cdot \rangle$ and the corresponding norm $\|\cdot\|$, and let Q be a nonempty closed convex subset of H .

An operator $T: Q \rightarrow Q$ is called **L -Lipschitz continuous** if there exists a constant $L > 0$ such that

$$\|Ta - Tb\| \leq L \|a - b\|, \forall a, b \in Q. \quad (1)$$

The operator T is said to be **nonexpansive** when $L = 1$. A point $a \in Q$ is called a fixed point of T whenever

$$Ta = a. \quad (2)$$

The collection of all fixed points of T is denoted by

$$\text{Fix}(T) := \{a \in Q : Ta = a\}. \quad (3)$$

Let $\{T_m\}_{m \geq 1}$ be a sequence of nonexpansive operators defined on Q , and let

$$\Gamma := \bigcap_{m=1}^{\infty} \text{Fix}(T_m) \quad (4)$$

denote the set of common fixed points of $\{T_m\}$. Throughout this paper, we assume that Γ is nonempty. For a bounded sequence $\{a_m\} \subset H$, let $\omega_w(a_m)$ represent the collection of all weak limit points of subsequences of $\{a_m\}$. The sequence $\{T_m\}$ is said to satisfy **condition (Z)** if, for any bounded sequence $\{a_m\} \subset Q$,

$$\|a_m - T_m a_m\| \rightarrow 0 \quad (5)$$

implies

$$\omega_w(a_m) \subset \Gamma. \quad (6)$$

We now introduce the optimization setting considered in this paper. Let $i: H \rightarrow \mathbb{R}$ be a convex differentiable function and let

$k: H \rightarrow (-\infty + \infty]$ be a proper, convex, and lower semicontinuous function. Suppose that the gradient mapping ∇i is L -Lipschitz continuous, namely,

$$\|\nabla i(a) - \nabla i(b)\| \leq L \|a - b\|, \forall a, b \in H. \quad (7)$$

The optimization task under investigation is the composite convex minimization problem.

$$\min_{a \in H} [i(a) + k(a)]. \quad (8)$$

Problems of this form arise in a wide range of applications, including sparse optimization, signal recovery, compressed sensing, regularized learning, and classification. Their composite structure naturally motivates the use of operator-splitting techniques, particularly forward-backwards splitting methods, which combine gradient and proximal operations to efficiently handle the smooth and nonsmooth components of the objective function.

A broad range of optimization and machine learning models can be formulated in this form. Examples include sparse regression, feature selection, compressed sensing, and regularized classification problems.

The proximal operator associated with k is defined by

$$\text{prox}_{ek}(a) = \arg \min_{b \in H} \left\{ k(b) + \frac{1}{2e} \|b - a\|^2 \right\}, e > 0. \quad (9)$$

It is well known that prox_{ek} is single-valued and firmly nonexpansive and therefore plays a central role in operator-splitting methods for nonsmooth convex optimization. The subdifferential of k at $a \in H$ is defined by

$$\partial k(a) := \{w \in H : k(b) \geq k(a) + \langle w, b - a \rangle \forall b \in H\}. \quad (10)$$

The set-valued operator ∂k is maximal monotone, and the optimality condition for a point $a^* \in H$ to solve the convex minimization problem is given by

$$0 \in \nabla i(a^*) + \partial k(a^*). \quad (11)$$

Equivalently, a^* is a solution if and only if it satisfies the fixed-point relation

$$a^* = \text{prox}_{ek}(a^* - e\nabla i(a^*)), e > 0, \quad (12)$$

This provides the foundation for forward-backwards splitting algorithms. A point $a^* \in H$ solves the optimization problem if and only if

$$0 \in \nabla i(a^*) + \partial k(a^*).$$

Equivalently,

$$a^* = \text{prox}_{ek}(I - e\nabla i)(a^*),$$

Where I denotes the identity operator. The mapping

$$\text{prox}_{ek}(I - e\nabla i)$$

is known as the forward-backwards operator. Consequently, solving the optimization problem can be viewed as finding a fixed point of a nonexpansive operator. This connection provides the



foundation for the accelerated algorithms proposed in the next section.

2.1. Cocystal Preparation and Its Structure Analysis

This section summarizes several auxiliary results that are instrumental in establishing the convergence properties of the presented technique.

Lemma 1 ([12]). Let H be a real Hilbert space, let $g: H \rightarrow \mathbb{R} \cup \{+\infty\}$ be a suitable, convex, and lower semicontinuous function, and let $i: H \rightarrow \mathbb{R}$ be a convex, smooth function with an L -Lipschitz continuous gradient. Assume that $\{T_m\}$ is a sequence of forward-backwards operators associated with parameters $\{e_m\} \subset (0, 2/L)$ such that $e_m \rightarrow e$ for some $e \in (0, 2/L)$. Then the sequence $\{T_m\}$ satisfies the NST-condition (I) with respect to the forward-backwards operator corresponding to e .

Lemma 2 ([13]). Let H be a real Hilbert space. Then, for every $a, b \in H$ and $l \in [0, 1]$, the following identities hold:

$$\|la + (1-l)b\|^2 + l(1-l)\|a-b\|^2 = l\|a\|^2 + (1-l)\|b\|^2 \quad (13)$$

$$\|a \pm b\|^2 = \|a\|^2 \pm 2\langle a, b \rangle + \|b\|^2. \quad (14)$$

Lemma 3 ([14]). Let $\{p_m\}$, $\{r_m\}$, and $\{\gamma_m\}$ be nonnegative sequences satisfying

$$p_{m+1} \leq (1 + \gamma_m)p_m + r_m, \quad m \geq 1. \quad (15)$$

If

$$\sum_{m=1}^{\infty} \gamma_m < \infty \text{ and } \sum_{m=1}^{\infty} r_m < \infty, \quad (16)$$

Then the sequence $\{p_m\}$ converges. In particular,

$$\lim_{m \rightarrow \infty} p_m \quad (17)$$

exists.

3. Proposed Algorithms and Applications

This section presents the proposed accelerated algorithms and their applications. We first develop an innovative accelerated fixed-point framework for a countable family of nonexpansive mappings in Hilbert spaces and establish its weak-convergence properties. We then extend the proposed framework to convex optimization problems and demonstrate its effectiveness in machine learning classification tasks.

3.1. Accelerated Fixed Point Algorithm

Consider a real Hilbert space H together with a sequence $\{T_m\}$ of nonexpansive mappings defined on H . Define

$$\Gamma := \bigcap_{m=1}^{\infty} \text{Fix}(T_m), \quad (18)$$

and assume that $\Gamma \neq \emptyset$.

Many existing iterative methods achieve convergence but often require a large number of iterations. Motivated by recent advances in inertial and accelerated techniques, we propose an innovative accelerated fixed-point algorithm that combines extrapolation and multi-step averaging strategies. The proposed method aims to improve practical performance while preserving convergence guarantees.

Algorithm 3.1 (Accelerated Fixed Point Algorithm (AFPA))

Initialization: Choose $a_0, a_{-1} \in H$. Let $\{\alpha_m\}$, $\{\beta_m\}$, $\{\mu_m\} \subset [0, 1]$ and $\{\gamma_m\} \subset (0, \infty)$.

Iterative Step: For each $m \geq 1$, perform:

Step 1. Inertial extrapolation

$$\theta_m = \begin{cases} \min\left\{\mu_m, \frac{\gamma_m}{\|a_m - a_{m-1}\|}\right\}, & a_m \neq a_{m-1}, \\ \mu_m, & \text{otherwise,} \end{cases}$$

$$b_m = a_m + \theta_m(a_m - a_{m-1}).$$

Step 2. Fixed point updates

$$b_m = a_m + \theta_m(a_m - a_{m-1}), \quad (19)$$

$$o_m = (1 - \beta_m)a_m + \beta_m T_m b_m, \quad (20)$$

$$w_m = (1 - \alpha_m)T_m b_m + \alpha_m T_m o_m, \quad (21)$$

$$a_{m+1} = T_m w_m. \quad (22)$$

Set $m := m + 1$ and repeat.

The inertial term θ_m accelerates the iteration by exploiting previous information. The intermediate points o_m and w_m enhance stability and improve the approximation of common fixed points. The following theorem establishes the weak convergence of AFPA.

Theorem 3.1

Let $\{a_m\}$ be generated by Algorithm 3.1. Assume that

(Q1) $0 < \rho < \theta_m \leq 1$ for all $m \geq 1$;

(Q2) $0 < r < \beta_m < d < 1$ for all $m \geq 1$;

(Q3) $\sum_{m=1}^{\infty} \gamma_m < \infty$.

Then $\{a_m\}$ weakly converges to a point $a^* \in \Gamma$. The demonstration follows by establishing the existence of $\lim_{m \rightarrow \infty} \|a_m - a^*\|$, showing that $\|b_m - T_m b_m\| \rightarrow 0$, and applying the condition (Z) together with Opial's lemma. Consequently,

$$a_m \rightharpoonup a^* \in \Gamma.$$

3.2. Application to Convex Minimization

We next apply AFPA to the convex optimization problem.

$$\min_{a \in H} \{i(a) + k(a)\}, \quad (23)$$

Where $i: H \rightarrow \mathbb{R}$ is convex and smooth with an L -Lipschitz continuous gradient and $k: H \rightarrow (-\infty, +\infty]$ is a lower-bounded, convex, and lower semicontinuous mapping.

Let

$$\Omega := \text{argmin}(i + k)$$

denote the set of minimizers of (23).

It is well known that the forward-backwards operator.

$$T := \text{prox}_{ek}(I - e\nabla i)$$

is nonexpansive for $e \in (0, \frac{2}{L})$ and satisfies



$$\text{Fix}(T) = \Omega.$$

Replacing T_m in Algorithm 3.1 by

$$T_m := \text{prox}_{e_m k}(I - e_m \nabla i),$$

yields the following accelerated forward-backwards algorithm.

Algorithm 3.2 (Accelerated Forward-Backward Algorithm (AFBA))

Initialization: Choose $a_0, a_{-1} \in H$ and a sequence $\{e_m\} \subset (0, 2/L)$ such that $e_m \rightarrow e$.

Step 1. Compute

$$\theta_m = \begin{cases} \min \left\{ \mu_m, \frac{\gamma_m}{\|a_m - a_{m-1}\|} \right\}, & a_m \neq a_{m-1}, \\ \mu_m, & \text{otherwise.} \end{cases}$$

Step 2. Compute

$$b_m = a_m + \theta_m(a_m - a_{m-1}), \quad (24)$$

$$o_m = (1 - \beta_m)a_m + \beta_m \text{prox}_{e_m k}(I - e_m \nabla i)(b_m), \quad (25)$$

$$w_m = (1 - \alpha_m)\text{prox}_{e_m k}(I - e_m \nabla i)(b_m) \quad (26)$$

$$+ \alpha_m \text{prox}_{e_m k}(I - e_m \nabla i)(o_m), \quad (27)$$

$$a_{m+1} = \text{prox}_{e_m k}(I - e_m \nabla i)(w_m). \quad (28)$$

Set $m := m + 1$ and repeat.

The proposed AFBA incorporates inertial extrapolation into the forward-backwards framework. As a result, AFBA can efficiently handle nonsmooth convex optimization problems that arise in sparse learning and regularized machine learning models.

Theorem 3.2

Let $\{a_m\}$ be generated by Algorithm 3.2. If conditions (Q1)-(Q3) hold, then

$$a_m \rightarrow a^* \in \Omega.$$

Proof. For each $m \in \mathbb{N}$, define

$$T_m := \text{prox}_{e_m k}(I - e_m \nabla i).$$

Then T_m is nonexpansive and

$$\text{Fix}(T_m) = \text{Argmin}(i + k) = \Omega.$$

Moreover, $\{T_m\}$ satisfies condition (Z). Applying Theorem 3.1 completes the proof.

4. Machine Learning Classification

Machine learning algorithms often require solving large-scale optimization problems. Classification models rely on efficient optimization strategies to achieve high predictive accuracy and robust generalization performance. Since AFBA is designed for nonsmooth convex optimization, it can naturally be applied to machine learning tasks. To assess the practical performance of the proposed AFBA, we apply it to the training of an Extreme Learning Machine (ELM) classifier. ELM is a single-hidden-layer feedforward neural network characterized by its rapid learning process and favorable generalization performance. In contrast to conventional neural networks, the hidden-layer parameters are randomly assigned and remain fixed, while only the output weights are determined through an optimization procedure.

Let

$$D = \{(a_v, l_v) : a_v \in \mathbb{R}^m, l_v \in \mathbb{R}^m, v = 1 \dots N\}$$

denote a training dataset, where a_v represents the input sample and l_v denotes the corresponding target vector. The ELM model is formulated as

$$H\beta = T,$$

Where H is the matrix of hidden-layer outputs, β is the matrix of output weights, and T is the matrix of targets. Determining β leads to the optimization problem

$$\min_{\beta} \|H\beta - T\|_2^2. \quad (29)$$

To improve sparsity and reduce overfitting, we consider the LASSO formulation.

$$\min_{\beta} \|H\beta - T\|_2^2 + \lambda \|\beta\|_1, \quad (30)$$

Where $\lambda > 0$ is the regularization parameter.

Problem (30) is a composite convex optimization problem of the form.

$$i(\beta) + k(\beta),$$

Where

$$i(\beta) = \|H\beta - T\|_2^2$$

is smooth and convex, while

$$k(\beta) = \lambda \|\beta\|_1$$

is convex but nonsmooth. Therefore, AFBA provides a suitable computational framework for solving (30). In the experimental study, AFBA computes the optimal output weights of the ELM classifier and is evaluated on benchmark classification datasets. We compare its performance with FISTA and the accelerated fixed-point method proposed in [11]. The obtained results demonstrate that AFBA achieves competitive classification accuracy, stable convergence behavior, and improved optimization performance. These findings confirm the potential of accelerated fixed-point techniques for modern machine learning and data-driven applications.

4.1. Data Classification

To assess the practical effectiveness of the presented technique, we apply Algorithm 3.2 to several supervised classification challenges based on the Extreme Learning Machine (ELM) framework. The objective is to investigate whether the accelerated optimization mechanism can improve classification performance while maintaining stable convergence. We compare the proposed approach with FISTA and NAGA. All experiments were conducted in MATLAB using an Intel Core i7 12th Generation processor with 32 GB RAM under Windows 11. The sigmoid function was employed as the hidden-layer activation function. For the Iris and Hypertension Disease UCI datasets, the number of hidden neurons was fixed at $M = 100$, the regularization parameter was set to $\lambda = 10^{-5}$. For the Hypertension dataset, an 80%-20% training-testing split was adopted. The proposed AFBA-based ELM used $M = 300$, $\lambda = 10^{-3}$, $\alpha_m =$



0.7, $\beta_m = 0.3$, $\mu_m = 0.95$, $\gamma_m = 0.1$. We considered three benchmark datasets frequently used in classification research.

(i) Iris Dataset: This dataset contains 150 samples from three iris species, namely *Iris Setosa*, *Iris Versicolor*, and *Iris Virginica*. Each sample is described by four botanical features. The task is to identify the correct species.

(ii) Hypertension Dataset: This dataset consists of clinical and physiological measurements, including age, sex, daily temperature, pulse rate, respiratory rate, oxygen saturation, body weight, height, and body mass index. The objective is to classify patients as hypertensive or non-hypertensive. Table 1 summarizes the characteristics of the datasets used in the experiments.

Table 1:
Summary of the datasets used in the classification experiments.

Dataset	Attributes	Training Samples	Testing Samples
Iris	4	105	45
Hypertension	9	4886	1222

The model accuracy was computed.

$$\text{Accuracy} = \frac{\text{No of Correct Data for Predictions}}{\text{Total No of Samples}} \times 100.$$

Table 2 reports the training and testing accuracies obtained by the compared methods.

Table 2:
Summary of the datasets used in the classification experiments.

Dataset	AFBA Train (%)	AFBA Test (%)	NAGA Train (%)	NAGA Test (%)	FISTA Train (%)	FISTA Test (%)
Iris	100.00	97.78	71.45	70.56	82.86	64.44
Hypertension	90.30	88.13	79.37	78.48	88.83	87.89

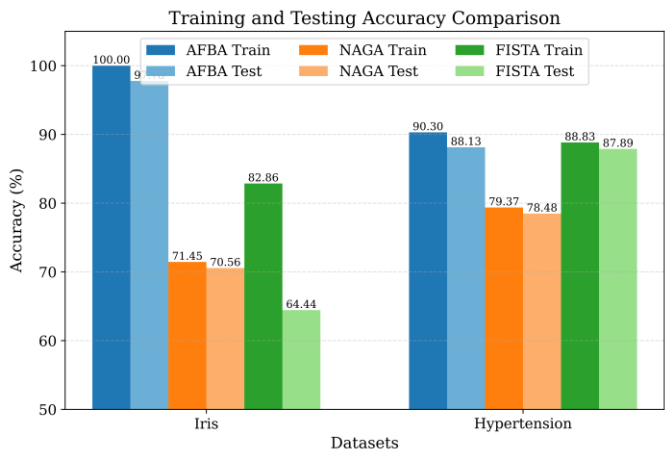


Figure 1: Comparison of training and testing accuracies of AFBA, NAGA, and FISTA on the Iris and Hypertension datasets, demonstrating the superior performance of AFBA over competing methods.

The classification accuracy results presented in Figure 1 demonstrate the effectiveness of the proposed Accelerated

Forward-Backward Algorithm (AFBA) for training Extreme Learning Machine (ELM) classifiers. AFBA consistently achieved the highest training and testing accuracy across both the Iris and Hypertension datasets. On the Iris dataset, AFBA attained 100.00% training accuracy and 97.78% testing accuracy, outperforming NAGA (71.45% and 70.56%) and FISTA (82.86% and 64.44%). Similarly, for the Hypertension dataset, AFBA achieved 90.30% training accuracy and 88.13% testing accuracy, providing superior performance compared with NAGA (79.37% and 78.48%) and competitive results relative to FISTA (88.83% and 87.89%). These results indicate that the proposed acceleration mechanism enhances the optimization process and improves the quality of the learned model parameters. The small gap between training and testing accuracies observed for AFBA suggests strong generalization capability and robustness against overfitting.

In contrast, NAGA exhibited lower predictive performance on both datasets, while FISTA showed a significant reduction in testing accuracy for the Iris dataset, indicating weaker generalization. The AFBA delivered the most reliable and accurate classification performance across the considered benchmark datasets. The experimental findings demonstrate that integrating accelerated forward-backwards splitting with adaptive update strategies leads to more effective optimization and enhanced predictive performance. Therefore, AFBA represents a promising approach for solving regularized machine learning problems and training sparse ELM classifiers, particularly in applications requiring high classification accuracy and stable generalization performance.

5. Conclusion

This paper proposes an iterative approach for solving composite convex optimization challenges and approximating common fixed points of countable families of nonexpansive mappings. By combining inertial extrapolation with forward-backwards splitting techniques, the proposed scheme yields enhanced convergence characteristics while preserving the underlying operator-theoretic structure. A weak convergence theorem for the generated sequence is established in the setting of a real Hilbert space under appropriate control conditions. To demonstrate its practical applicability, we employed the proposed algorithm to train ℓ_1 -regularized Extreme Learning Machine classifiers. Experimental results on the Iris and Hypertension datasets confirmed the effectiveness of the proposed approach. In particular, the algorithm achieved testing accuracies of 97.78%, 68.82%, and 88.13%, respectively, outperforming NAGA and FISTA on most datasets.

Declaration

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the manuscript. All authors reviewed and approved the final version of the manuscript.

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